

Religious Beliefs, Longevity, and the Costs of Fundraising

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Abstract: We develop a simple life cycle model with endogenous longevity and religious beliefs where religious firms influence beliefs using donations as an input. Because longevity is endogenous, wealth changes produce wealth effect and a substitution effect by raising the relative price of donations. We show that the balance of the two effects determines whether development is associated with secularization as is true for most countries or religiosity as is true for the United States. We also show that the United States exception of high wealth, high religiosity, but unexceptional life expectancy can also be explained by competition for religious donations. In particular, we show that increased competition by religious firms for donations reduces the relative price of donations and results in a substitution effect that increases donations and religious beliefs at the expense of consumption and longevity. We also analyze the effects exogenous increases in faith and mortality risks on lifetime consumption, longevity, and religiosity.

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For most of the world wealth is positively associated with longevity and negatively associated with religiosity. However, not all countries conform with the overall pattern. As shown in Figures 1 and 2, the United States presents a very visible exception. If wealth is measured by real per capita GDP, longevity is measured by average life expectancy at birth, and religiosity is measured by belief in an afterlife, then by these measures the US is very wealthy but also very religious compared to its OECD peers, wealthy and religious but not especially long-lived. While some aspects in Figures 1 and 2 have been explained by economists and sociologists, no satisfactory explanation exists for the general patterns and the United States exception in each of the Figures and no explanation exists at all that connects the two Figures. We address this gap by developing a simple life cycle model with endogenous longevity and endogenous beliefs or faith.

Our model addresses several ideas that have recently been put forth for understanding the role of religion in economic performance and for understanding the economics of life extension. According to Iannacone (1995), the literature on the socioeconomics of religion can be loosely grouped into theories that emphasize the demand-side of religion (or secularization forces) and those that emphasize the supply-side of religion (or market forces), while the longevity production approach has emphasized the demand side (Philipson and Becker, 1998). On the demand side is the controversial secularization hypothesis that has been associated with Weber (1905) though other versions exist as well (cite). The secularization hypothesis ties increased secularization to development, which explains all but the US. Azzi and Ehrenberg (1975) present an economic rationale for the secularization hypothesis that religious participation falls as wages and the opportunity costs of time rise. While it is argued that high wage individuals substitute cash for time, the literature on charitable contributions suggests that religious contributions fall as income increases, leading some to call religious donations an anomaly (Andreoni, 2004). Related work has continued to focus on religious participation, assuming that participation and religious beliefs are somehow tied but not making the connection explicit. However, the emphasis of the literature has shifted from the demand side to the supply side of religion as represented by the markets theory of religion (Adam Smith, 1776). According to this theory, competition among religions increases religiosity much like competition increases the production of any commodity. While this theory can explain why the United States differs from other countries in Figure 1, it also takes belief formation as given and is silent about connections to other dimensions of

Figure 1: Belief in Heaven and Per Capita GDP in 1990

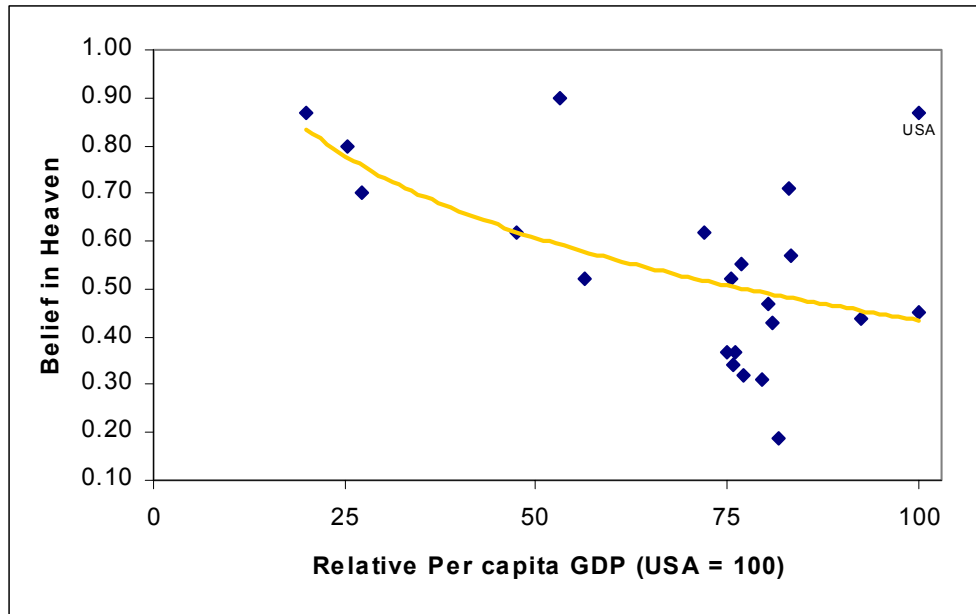
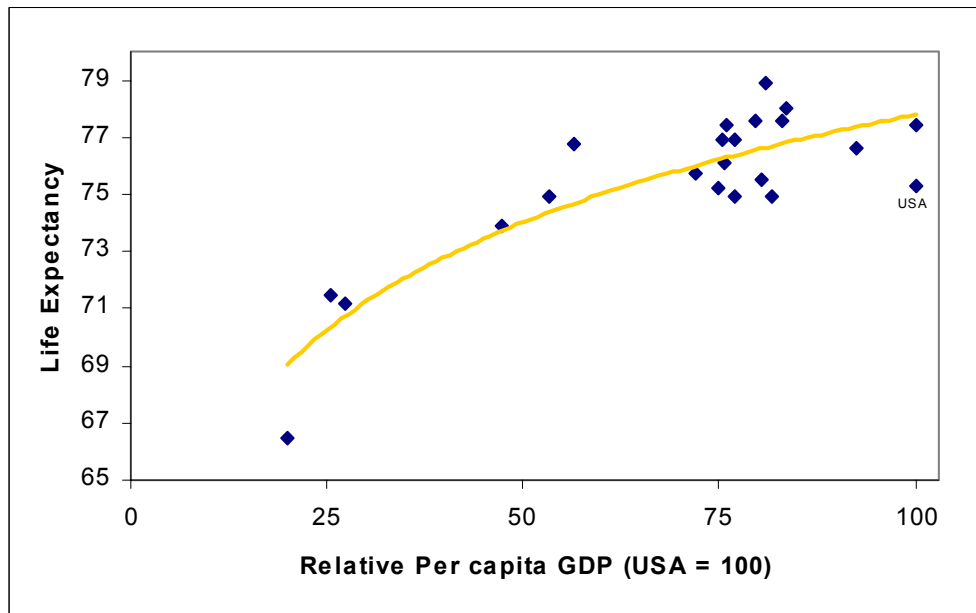


Figure 2: Life Expectancy at Birth and Per Capita GDP in 1990



Notes: Countries in sample: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Japan, Mexico, Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.
 Source: Barro and McCleary (2003) and OECD.

Table 1:

individuals life cycle choices such as longevity. According to the longevity production approach by Philipson and Becker (1998) life expectancies vary positively with wealth, which leaves open why the United States has a lower life expectancy than its OECD peers despite higher wealth and religiosity.

Our framework starts with the perspective that (on a very simple level) religions emphasize good works on earth and certitude about an afterlife. Good works can involve many things such as the giving of time and money, but at its simplest it involves money given to a religious organization or firm. More interesting for our purposes is that religions produce credence goods (Iannacone, 1998) by promoting certainty of beliefs about an uncertain and unverifiable afterlife. However, as Iannacone stresses in his survey of the economics of religion literature, there has been very little research in the area of religious belief formation and how beliefs are affected by competing religions. As Barro and McCleary (2003) have shown, religious beliefs may be important for economic behavior with beliefs about hell, heaven, and an afterlife positively associated with economic growth. We model the role of beliefs in a very simple equilibrium model with endogenous lifespans along the lines of Philipson and Becker (1998), where the afterlife payoff is determined by a lifetime of good works but there is uncertainty about the afterlife.¹ Religious organizations are assumed to control the technology that produces faith. Faith is modeled simply as greater certainty about the afterlife payoff and reduction of doubt.

While one theme of our paper is the role of religious beliefs for lifetime behavior, another theme is the role of how these beliefs are financed. Financing occurs through donations to religious organizations who transform these donations into belief production. The cost of donations is determined by the costs of fundraising, which in turn depend critically on the nature of charitable good, the competitive landscape of religious organizations, and whether a market exists for donations and fundraising. We distinguish between involuntary donations (a tax by religious organizations) and voluntary donations where donations are for a purely private charitable good. The difference between involuntary and voluntary donations is analogous to the difference between persuasive advertising that directly affects preferences without consent and the theory of advertising put forth by Becker and Murphy

¹Our approach is complementary to the work following Azzi and Ehrenberg (1975) that models the afterlife payoff as a function of religious capital built up through an exogenous lifetime of sacrifice. Azzi and Ehrenberg do not focus on the uncertainty associated with the afterlife and the role of religious organizations in alleviating this uncertainty. Also see **Iannacone** (1990).

(1993) where a market for advertising exists that reconciles individuals' willingness to pay for advertising and firms costs of promoting their goods. Because donations ultimately are used to produce religious certainty they act much like advertising, though some differences remain. Whereas normally advertising complements the product being sold, religious advertising is the product. and while advertising is not explicitly priced by consumers of the product, religious advertising is priced by individuals' willingness to pay for contributions. Finally, the literature emphasizes the competitiveness of the market for religious goods, we emphasize the competitiveness of the market for inputs (donations) that are used to produce a homogenous output (beliefs).

Our framework implies that market competition by religious organizations (and technological progress in the form of lower communication costs) strengthen faith and religious beliefs. While progress and competition increase fundraising and donations and thus religious faith, longevity falls, and consumption may rise or fall. We also find that economic development and higher wealth increase consumption and longevity but donations and faith may rise or fall. The intuition depends on the balance of wealth and substitution effects that arise when wealth and competition increase and on behavior of two relative prices: the relative price of longevity and the relative price of donations or fundraising. We show that our model can easily explain Figures 1 and 2. We also investigate the role of exogenous shocks of religiosity (epiphanies) and exogenous changes in mortality risk. We find that religiosity has a positive effect on longevity and religious donations and a negative effect on consumption. While increased mortality risk tends to increase consumption, the effects on longevity and donations are much less clear. **(Complete this paragraph)**

1 The Individual's Decision Problem

Individuals are assumed to maximize a conventional lifetime utility function to which we add an explicit afterlife payoff. If we only consider stationary outcomes, then expected lifetime utility is

$$Tu(c) + \pi a(T) \tag{1}$$

where T is the endogenous length of life, c is annual consumption, and $u(\cdot)$ is the value placed on consumption. We assume the typical regularity conditions for $u(\cdot)$ that guarantee an interior solution for consumption. We abstract from discounting for now and an uncertain

length of life, but revisit these issues later.

The afterlife is uncertain and described by a payoff $a(T)$ if heaven occurs and 0 otherwise and π is the probability that heaven will occur.² For the time being we will assume that this probability is exogenous to the individual, but this assumption is key and the subject of the analysis. More on π will be said in the next section where we describe religious organizations. Under our notation, faith is measured on a continuous scale with an atheist having $\pi = 0$ and a true believer having $\pi = 1$. We assume that $a(T)$ increases with T or that the heavenly payoff increases with the length of time good works have been performed. In other words, we assume a Dantean paradise. Though we assume $a'(T) > 0$, because afterlife is fundamentally non-verifiable, we have no strong position on the sign of $a''(T)$ and for the most part we will simply assume $a''(T) \leq 0$. Normally, an economist would simply take it on faith that there are diminishing returns, which though plausible for most earthly relationships, may not be plausible for the transcendental. One might also consider the sign of $a''(T)$ an empirical matter, but direct tests for an afterlife have yet to be devised. However, as our comparative statics results indicate, the assumption of $a''(T) \leq 0$ is consistent with what we know about earthly behavior.³

Individuals maximize expected utility subject to a wealth constraint

$$T(c + \tau) = Z \tag{2}$$

where Z is lifetime wealth and τ is annual contributions to the religion, where these donations can be either involuntary or voluntary in which case they are interpreted as good works. We note that the length of life is determined by its value in lifetime utility and constrained by lifetime wealth. We do not explicitly consider the connection between health and longevity. Philipson and Becker (1998) introduce health production by assuming a concave lump-sum cost of longevity $M(T)$ that is subtracted from wealth. We do not follow this path here, but instead focus on the effect of having religious beliefs on an afterlife. The inessential difference between the two approaches is that religious beliefs affect the marginal rate of

²We do not assume there is a hell, for simplicity. Barro and McCleary (2003) show that the belief in heaven and hell may each have distinct implications for economic behavior.

³We note that πa could be interpreted more secularly as a quality of life indicator with π an index of health that is produced by the health sector. Donations would then be contributions towards the health sector. One **difference** with this interpretation is that health services are priced in a market while religious belief is not explicitly traded and priced.

substitution between longevity and consumption, whereas longevity production affects the relative price between longevity and consumption.

Optimizing individuals set the marginal benefit of living longer equal to the marginal cost of living longer and they do the same for annual consumption (and also donations when these are endogenous). Combining these two optimality conditions, we obtain a tangency condition that equates the marginal rate of substitution between longevity and consumption and the price of longevity relative to consumption

$$\frac{u(c) + \pi a'(T)}{u'(c)} = c + \tau \quad (3)$$

The relative price of longevity is endogenous and a positive function of $c + \tau$, because the wealth constraint is a non-linear function of consumption and longevity. The optimality condition reflects a tangency of convex indifference curves in (c, T) space with a wealth constraint that is also convex (for related work see Edlefsen, 1981). We put aside the issue of multiple solutions, and only consider the case of a unique interior solution. We also note that an increase in faith π increases the subjective valuation of longevity relative to consumption. When faced with such a taste shock, individuals seek to reestablish the optimality condition by balancing increases in consumption that raise the relative price of longevity with decreases in consumption that lower the subjective valuation of longevity.

2 Religious Organizations and the Market Structure

To close the model we assume a particularly simple form for the religious organizations and the market conditions they operate under. More general forms are certainly possible, but may obscure the central point of this analysis. We assume first, that religious organizations act as perfect competitors and then compare the outcome with the outcome when religious organizations have monopsony power and when there is free entry of non-profit religious organizations. Initially, we assume that τ is extracted involuntarily from individuals, but later we will let individuals choose how much τ to supply and firms

Religious organizations choose τ to maximize production of religious certainty less the cost of acquiring funds τ . The unit cost of acquiring funds is assumed to be a convex function $\theta(\tau)$ that can be interpreted as the cost of fundraising per unit of τ . Production of faith is assumed to take the following simple form $\pi(\tau)$ with $\pi \in [0, 1]$ and $\pi' > 0 > \pi''$ or

diminishing marginal output because faith is a credence good and credibility is hard fought. In other words, we assume that $\pi(\cdot)$ has the shape of a cumulative distribution function for a continuous random variable. Thus, the religious organization chooses τ to maximize

$$\pi(\tau) - \theta(\tau)\tau \quad (4)$$

The details of the maximization depend critically on the competitive landscape. When the religious organization raises funds in a competitive manner, θ is treated as exogenous by the organization and we have

$$\pi'(\tau) = \theta X^* \quad \text{where } X^* = 1$$

or that funds received by the organization vary inversely with unit proselytizing costs. Intuitively, higher unit costs require higher marginal productivity which occurs when funds decline. Fewer funds in turn imply that less faith is produced for the individual.

For comparison, suppose the religious organization has monopsony power with control over its fundraising costs. This assumption is related to the observation that in many countries with state religions, religious organizations act monopolistically (Tollison, Iannacone). If we assume convex costs $\theta(\tau)$ with $\theta'(\tau) > 0$ and $\theta''(\tau) \geq 0$, then profit maximization implies

$$\pi'(\tau) = \theta X^m \quad \text{where } X^m = 1 + \varepsilon_\theta \equiv 1 + \frac{\tau\theta'(\tau)}{\theta(\tau)} > 1$$

Since $X^m > X^*$ and $\pi''(\tau) < 0$ as well as $\theta'(\tau) > 0$, we have $\tau^m < \tau^*$. In other words, greater competitiveness requires greater fundraising effort in order to satisfy the firm's optimality condition and more fundraising leads to more donations τ and stronger beliefs π .

We, also, consider the case of non-profit religions under free entry, because many countries give religious charitable organizations tax exemptions and non-profit status. Free entry implies that individual profits are driven down to zero with θ again treated as exogenous, or

$$\frac{\pi(\tau)}{\tau} = \theta$$

which can be rewritten as

$$\pi'(\tau) = \theta X^n \quad \text{where } X^n = \varepsilon_\pi \equiv \frac{\tau\pi'(\tau)}{\pi(\tau)} < 1$$

Since $X^* > X^n$, we have $\tau^n > \tau^*$. Thus, free entry and the non-profit condition imply more fundraising and stronger beliefs than under pure competition. This situation may as a short

hand be sometimes referred to as “more competitive” than perfect competition in the sense of greater fundraising effort.

We summarize the firm optimality and entry conditions with

$$\pi'(\tau) = \theta(\tau) X^i \tag{5}$$

where

$$X^m = 1 + \varepsilon_\theta > X^* = 1 > X^n = \varepsilon_\pi$$

Because $X^m > X^* > X^n$, we have $\tau^n > \tau^* > \tau^m$ as well as $\pi(\tau^n) > \pi(\tau^*) > \pi(\tau^m)$. In other words, a more “competitive” landscape for religions leads to more fundraising and ultimately to stronger beliefs.

We can also show that technological progress is in many ways similar to a more competitive market structure. Technological progress can be in the form of a more efficient belief production technology (higher marginal product π' for all τ) or lower fundraising costs (lower unit cost θ for all τ) by which we mean lower communication and distribution costs. In either case, progress leads to higher τ to satisfy (5) and thus to stronger beliefs and more religious certainty. This might explain the puzzle that a wealthy country like the U.S. is so religious. Not only is religion very competitive (and afforded non-profit status) but also there are low communication costs. Both factors encourage faith and more religious certainty under our assumptions. The model also implies that secularism in other wealthy countries may be explained by lack of competition among religions or relatively high communication costs either for technological reasons or market interventions (perhaps through state control of communication outlets). The competition interpretation would be in line with the traditional markets theory of religion originally put forth by Smith, whereas the technological progress interpretation could be thought of as a Schumpeterian view of the spread of religion..

3 Comparative Statics with Involuntary Donations

Before we extend the model to deal with a market for donations, we analyze the model’s response when involuntary donations can be extracted. Involuntary donations can be raised through the government when a portion of income taxes are channeled to religious organizations. Dedicated taxes as in Germany where a given percent goes to religion would not

be considered involuntary donations because the individual always has the option of not paying the religious tax. Involuntary donations may also be extracted directly by religions. However, religious organizations differ from the government in the sense that they can not extract unlimited resources and that the amount of resources extracted is dictated by the market structure that the religious organizations are assumed to operate under. In other words, lower X^i or greater competitiveness requires more fundraising effort to satisfy the firm optimality and entry condition (5). Since donations are involuntary and beliefs are produced by fiat, this in essence is like a model of persuasive advertising where religious firms affect beliefs directly without explicit consideration for individuals' willingness to contribute funds to religion and the costs of fundraising.

To find the effect of religious competition, we totally differentiate the wealth condition (2) and the optimality condition (3). The determinant of the resulting system of equations is

$$\Delta_I = (c + \tau) \frac{c + \tau}{c} \sigma_u + \frac{\pi a'}{u'} \sigma_a$$

where $\Delta_I \geq 0$ assuming $\sigma_a \equiv \frac{-a''T}{a'} \geq 0$ and $\sigma_u \equiv \frac{-u''T}{u'} \geq 0$. To understand the effect of religion and religious market organization we proceed in a step-wise manner and first examine wealth effects, followed by the effect of religion and then the effect of involuntary donations.

Our comparative statics imply that changes in wealth tend to increase consumption and longevity:

$$\Delta_I \frac{dT}{dZ} = \frac{c + \tau}{c} \sigma_u \geq 0 \quad \text{and} \quad \Delta_I \frac{dc}{dZ} = \frac{\pi a'}{u'} \sigma_a \geq 0$$

Both of our comparative statics results are consistent with empirical evidence of normality of consumption and longevity when there are diminishing returns of the afterlife payoff.. Because consumption generally increases with wealth, and given that $\sigma_u \geq 0$, we we must have $a'' < 0$ or $\sigma_a > 0$. We note that if good works are interpreted as private consumption c , then we have a stark and distinctly selfish or Calvinistic interpretation of good works.

The effect of religion is a pure substitution effect, because greater religiosity as measured by a higher π increases the subjective valuation of longevity while leaving the relative price of longevity unchanged. Because the subjective valuation of longevity increases, religious certainty makes longevity appear relatively cheaper:

$$\Delta_I \frac{dT}{d\pi} = \frac{a'}{u'} T > 0 \quad \text{and} \quad \Delta_I \frac{dc}{d\pi} = -(c + \tau) \frac{a'}{u'} < 0$$

Thus, faith by itself tends to increase longevity but decrease consumption. This is consistent with findings that religiosity is associated with better health and longer lifespans. Faith reduces consumption because religious faith increases the payoff to longevity and thus increases the net marginal benefit of living longer. Thus, religious certainty acts like an exogenous reduction in the relative price of longevity and causes a substitution effect that induces individuals to substitute consumption for longevity.

Finally, involuntary donations have both wealth and substitution effects. Higher forced donations have a negative wealth effect but also strengthen religious belief. More certainty in the afterlife acts like a decrease in the relative price of longevity in comparison with the subjective valuation of longevity and tends to raise life expectancies. These influences are summarized in

$$\Delta_I \frac{dT}{d\tau} = -T \left[\frac{c + \tau}{c} \sigma_u + \left(1 - \frac{\pi a'}{Tu'} \varepsilon_a \right) \right] \quad \text{and} \quad \Delta_I \frac{dc}{d\tau} = \frac{-\pi a'}{u'} \sigma_a + (c + \tau) \left(1 - \frac{\pi a'}{Tu'} \varepsilon_a \right)$$

Intuitively, an increase in τ causes a negative wealth effect that tends to reduce both longevity and consumption. The substitution effect depends on whether τ increases the subjective valuation of longevity by more or less than it increases the relative price of longevity. If $1 > \frac{\pi' a}{u' T} \varepsilon_a$, then the increase in the relative price of longevity dominates.⁴ Under these circumstances, the net substitution effect is to decrease longevity and increase consumption, which implies that the wealth effect on longevity is reinforced, while the wealth effect on consumption is offset and depending on the relative strength of wealth and substitution effects, consumption may either increase or decrease.

The analysis for involuntary donations allows some initial conclusions about the effect of greater competitiveness among religious organizations on consumption and longevity. Greater competitiveness leads to higher donations and stronger beliefs but also may lead to lower consumption and longevity especially when substitution effects are weak and the negative wealth effects of fundraising dominate. Although, religious faith by itself increases the payoff to living longer, the cost of funds to produce religious certainty has a negative wealth effect that may dominate. According to the model in this section, one explanation for why average life expectancies in the United States lag other OECD countries, which tend to be less wealthy and less religious than the United States, would be that involuntary

⁴We note that in the next section where τ is endogenous, we will have an additional optimality condition that equates the marginal benefit of donations and the marginal cost of donations, or $\pi' a = Tu'$. Thus, assuming $1 > \frac{\pi' a}{u' T} \varepsilon_a$ amounts to assuming that donations are too high relative to their optimum.

donations funneled through the government are higher in the United States. Turning to technological progress, in the form of lower communication or fundraising costs also acts like an increase in competition. Again in this respect the United States may arguably be considered ahead of its peers.

4 A Market for Voluntary Donations

In this section we allow individuals to make voluntary donations of τ towards a purely private good that is used by religious organizations to produce faith or greater certitude about an afterlife. Voluntary donations can also be interpreted as contributing to religious capital as in Azzi and Ehrenberg (1975). Under this interpretation, religious capital is the accumulated stock of lifetime sacrifices through donations of cash, or $T\tau$. We continue with the assumption that the components of this stock have differential effects on the expected afterlife payoff with T affecting $a(T)$ and τ affecting $\pi(\tau)$, but one can easily imagine other possibilities.

When the donation of the individual τ is endogenous, we add another optimality condition to the earlier condition (3). Individuals choose donations by equating the marginal benefit of donations to the marginal cost. Rearranging this condition produces an optimality condition that equates the marginal rate of substitution of τ relative to consumption to the relative price of donations:

$$\frac{\pi'(\tau) a(T)}{u'(c)} = T \quad (6)$$

where the relative price of donations equals T because of the non-linear wealth constraint.

To complete this model we impose equilibrium in the voluntary contributions market. That is, we equate the firm's demand for funds to the individuals' supply of funds by equating the $\pi'(\tau)$ perceived by individuals in (6) to the $\pi'(\tau)$ desired by firms in (5). Thus, we have

$$\frac{\theta(\tau) X^i a(T)}{u'(c)} = T \quad (7)$$

where i indexes the market structure that religious firms are operating under with a decrease in X^i denoting a rise in competitiveness. An increase in competitiveness decreases the subjective valuation of donations relative to consumption. So, competitiveness acts like a reduction in the relative price of donations with a substitution effect of rising donations and falling consumption. However, this is not the only effect of greater competitiveness, which

also tends to increase the production of religious certainty and thus requires a higher τ . A higher τ implies a higher π and a rise of the subjective valuation of longevity relative to consumption in (3). Because the higher subjective valuation acts like a reduction in the relative price of longevity, competitiveness tends to reduce consumption and raise longevity (as before when donations were involuntary).

To analyze the comparative static properties of our model, we totally differentiate equations (2), (3), and (7) and solve for the response of the three endogenous variables c , π , and T . The determinant of the resulting system of equations is

$$\begin{aligned} \Delta_V = & (c + \tau) \frac{T}{c} \sigma_u \left\{ \frac{c + \tau}{\tau} \varepsilon_\theta + 2(1 - \varepsilon_a) \right\} \\ & + T \left\{ \sigma_a \frac{\varepsilon_a}{\varepsilon_\pi} \left(\varepsilon_\theta - \frac{\tau}{c} \sigma_u \right) + (\varepsilon_a - 1)^2 \right\} \end{aligned}$$

where $\Delta_V > 0$ assuming that the cost of producing faith is sufficiently convex or $\varepsilon_\theta \geq \frac{\tau}{c} \sigma_u$ where $\varepsilon_\theta \equiv \frac{\theta' \tau}{\theta}$. Assuming $\varepsilon_\theta \geq \frac{\tau}{c} \sigma_u$ is not as strong as it may appear at first glance, because consumption usually is far greater than donations and thus $\frac{\tau}{c}$ is fairly small. To understand the effect of religion and religious market organization we proceed as before and first examine wealth effects, followed by the effect of religion and then the effect of a more competitive religious market structure.

We find that wealth effects are positive for consumption and longevity without special restrictions. One can easily also see that voluntary donations are also a normal good when (6) is used instead of (7). By using (7), we capture the equilibrium response of donations to changes in wealth rather than the individual's supply response. The equilibrium response of donations to wealth is ambiguous without special restrictions. In particular, we find that

$$\begin{aligned} \Delta_V \frac{dT}{dZ} &= \left(\frac{T}{c} \sigma_u \right) \left[\frac{c + \tau}{\tau} \varepsilon_\theta + (1 - \varepsilon_a) \right] > 0 \\ \Delta_V \frac{dc}{dZ} &= \frac{\varepsilon_a}{\varepsilon_\pi} \sigma_a \varepsilon_\theta + (1 - \varepsilon_a)^2 > 0 \\ \Delta_V \frac{d\tau}{dZ} &= \sigma_u \left[-\frac{\varepsilon_a \sigma_a \tau}{\varepsilon_\pi c} + (1 - \varepsilon_a) \frac{c + \tau}{c} \right] \geq 0 \end{aligned}$$

Consumption and longevity respond positively to wealth, but the net effect on donations is ambiguous. The ambiguity arises because higher longevity raises the relative price of donations relative to consumption in (7) causing a substitution effect that reinforces the wealth effect for consumption and offsets the wealth effect for donations. When the induced

substitution effect dominates the original wealth effect, donations fall when wealth rises which leads to a fall in religiosity.

In other words, wealth has a secularization effect when the following secularization condition holds:

$$\frac{1 - \varepsilon_a}{\varepsilon_a} \frac{1}{\sigma_a} < \frac{1}{\varepsilon_\pi} \frac{\tau}{c + \tau}$$

If the secularization condition is violated, greater wealth produces greater religiosity. Whether wealth leads to secularization or religiosity depends on jointly on $\frac{\tau}{c+\tau}$, the curvature of the belief technology and the curvature of the afterlife payoff function. In short, the outcome is determined jointly by demand-side and supply-side parameters. To see the influence of the various terms, note first that normally $\frac{\tau}{c+\tau}$ tends to be quite small. Thus, the secularization condition is more likely when ε_π is low or when ε_a is high and σ_a is low. For constant elasticity utility functions, for example, $\sigma_a = 1 - \varepsilon_a$ so that the secularization condition is more likely when ε_a is high and the afterlife payoff is not too concave. Because the belief production function has the shape of a conventional cumulative distribution function with a flat range for high values of τ , we conclude that ε_π is low for sufficiently large values of τ . In other words, secularization is likely to accompany greater wealth when donations and religiosity are high initially and when afterlife payoffs are responsive to earthly lifespans. By contrast, religiosity is more likely a consequence in initially relatively secular societies with low levels of donations and when afterlife payoffs are perceived to be relative unresponsive to earthly lifespans.⁵

If we continue with our earlier assumption that $0 \leq \frac{\tau}{c} \sigma_u \leq \varepsilon_\theta$, then we find that an exogenous increase in religiosity has a positive effect on longevity and donations but a negative effect on consumption:

$$\begin{aligned} \Delta_V \frac{dT}{d\pi} &= \frac{a'T^2}{u'} \left[\frac{\varepsilon_\theta}{\tau} - \frac{\sigma_u}{c} \right] > 0 \\ \Delta_V \frac{dc}{d\pi} &= \frac{-a'T}{u'} \left[\left(\frac{c + \tau}{\tau} \varepsilon_\theta + (1 - \varepsilon_a) \right) \right] < 0 \\ \Delta_V \frac{d\tau}{d\pi} &= \frac{a'T}{u'} \left[\left(\frac{c + \tau}{c} \sigma_u + (1 - \varepsilon_a) \right) \right] > 0 \end{aligned}$$

Religious certainty has a positive effect on longevity because it raises the subjective valuation

⁵We note that cumulative distribution functions may also exhibit flat ranges for low values of τ . Under these circumstances, wealth increases cause secular societies to become more secular when the secularization condition holds or more religious when the secularization condition is violated.

of longevity relative to consumption. This is perceived by individuals as being equivalent to a reduction in the relative price of longevity and so there is a positive substitution effect on longevity and a negative substitution effect on consumption. Lower consumption and higher longevity increase the subjective valuation of donations in (7), with higher longevity also raising the relative price of donations. The combined effect of more efficient religious production is that donations rise.⁶

Finally, the effects of competition in the donations market (where a decrease in X means more competition) on longevity, consumption, and donations are

$$\begin{aligned}\Delta_V \frac{dT}{dX} &= \frac{T^2}{X} \left[(1 - \varepsilon_a) + \frac{c + \tau}{c} \sigma_u \right] > 0 \\ \Delta_V \frac{dc}{dX} &= \frac{T}{X} \left[-(c + \tau)(1 - \varepsilon_a) + \frac{\varepsilon_a \sigma_a}{\varepsilon_\pi} \tau \right] \geq 0 \\ \Delta_V \frac{d\tau}{dX} &= \frac{-T}{X} \left[\frac{(c + \tau)^2}{c} \sigma_u + \frac{\varepsilon_a}{\varepsilon_\pi} \tau \sigma_a \right] < 0\end{aligned}$$

Thus, competitiveness will have opposite effects on longevity and charitable giving and an uncertain effect on consumption. An increase in competitiveness increases the demand for donations by religious organizations and reduces the subjective valuation for donations in equilibrium. The increase in donations in turn increases the relative price of longevity causing a negative substitution effect for longevity. So an increase in competition undoubtedly reduces longevity and increases donations, which is consistent with our previous result that an increase in involuntary contributions τ reduces longevity. The effect on consumption is ambiguous because it depends on the balance of two substitution effects. First, lower longevity reduces the price of donations relative to consumption which causes a positive substitution effect for donations and a negative substitution effect for consumption. Second, higher donations increase the price of longevity relative to consumption which causes a positive substitution effect on consumption. Interestingly, the net effect on consumption of the two substitution effects depends on whether or not the secularization condition holds. When the secularization condition (??) holds, increased competitiveness in the market for

⁶We note that without our assumption that $0 \leq \frac{\tau}{c} \sigma_u \leq \varepsilon_\theta$, religion would have an ambiguous effect on longevity. The ambiguity arises because the relative price of longevity depends positively on the sum of consumption and donations. Falling consumption reduces the relative price, while rising donations increases the relative price. Our assumption guarantees that longevity rises or that the reduction in consumption from an increase in faith outweighs the increase in donations. The assumption is likely to hold in practise because $\frac{\tau}{c}$ usually tends to be small.

religious donations causes consumption to rise by virtue of a dominating substitution effect from a higher relative price of longevity.

How do our comparative statics results compare with Figures 1 and 2? Figure 1 shows that religious beliefs and wealth (as measured by per capita GDP) are for the most part negatively related, with the United States providing an visible exception. Figure 2 shows that longevity and wealth are positively related, with the United States a small exception. Our comparative statics results suggest that wealth shocks produce a positive longevity response and if the secularization condition is violated a positive effect on donations. Increased religious competition, by contrast, has a negative longevity effect and a positive donations effect. Thus, to explain the United States exception, one could use our model to argue that the competition effect is sufficiently strong in comparison to the wealth effect. This would be consistent with the experience in other parts of the world where lower wealth reduces longevity but lower religious competition increases longevity.

5 Do Discounting and Uncertainty Matter?

In this section we generalize our model and allow for discounting when voluntary donations are for private goods. We are interested in whether this more general specification matters for our results in the previous section. By allowing discounting we will also be able to analyze the effect of increased mortality risk on religion, as opposed to analyzing the effect of a crisis in faith or an exogenous increase religious uncertainty on longevity.

For the model with discounting we define ρ as the constant rate of time preference and define δ as the constant hazard rate of death. Then the probability of premature death is $e^{-\delta t}$ and the life expectancy of the individual is $E(T) \equiv \int_0^T \delta t e^{-\delta t} dt$. Also, we define the annuity value of a security paying one dollar every year until T as $A(T) \equiv \int_0^T e^{-(\rho+\delta)t} dt = \frac{1}{\rho+\delta} (1 - e^{-(\rho+\delta)T})$. As in Yaari (1965), the effective discount rate of $\rho + \delta$ rises with the risk of mortality and makes a person act more impatiently. We continue to focus on stationary solutions and assume that the utility discount rate is equated to the real interest rate or $\rho + \delta = r$ so that $A(T)$ enters lifetime utility as well as the lifetime wealth constraint. Finally, we assume perfect annuities markets.

The lifetime utility and wealth constraint with discounting are simply

$$A(T) u(c) + \pi(\tau) a(T) \tag{8}$$

$$A(T)(c + \tau) = Z \quad (9)$$

Individuals choose consumption, donations, and their lifespan to maximize (8) subject to (9). The individual's maximization problem yields two optimality conditions that are closely related to the conditions in the previous section without discounting, where we had $\rho + \delta \rightarrow 0$ so that $A(T) \rightarrow T$ and $A'(T) \rightarrow 1$. Specifically, the optimality conditions with discounting are

$$\frac{A'u + \pi a'}{u'} = A'(c + \tau) \quad (10)$$

$$\frac{\pi' a}{u'} = A \quad (11)$$

Imposing equilibrium in the fundraising market implies that

$$\frac{\theta X a}{u'} = A \quad (12)$$

Equations (10), and (12) are the analogs with discounting of equations (3) and (7).

We totally differentiate equations (9), (10), and (12) and solve for the response of three endogenous variables c , τ , and T to wealth, religiosity, and competition shocks. In contrast with the previous section without discounting, our derivations need to account for the fact that A is not linear. Previously, when A was linear the elasticity of A , $\epsilon_a \equiv \frac{A'T}{A}$, was unity and the elasticity of A' , $\sigma_A \equiv \frac{-A''T}{A'}$, was zero. Now we allow $\epsilon_a \leq 1$ and $\sigma_A \geq 0$. We, also, assume consistent with the previous section that $\sigma_A \leq \sigma_a$ and $\epsilon_A \geq \epsilon_a$. Intuitively, the return to longevity on earth, $A(T)$, is assumed to be less concave than the after-life return to longevity, $a(T)$. We, also, continue by assuming our previous sufficient condition that $\epsilon_\theta \geq \frac{\tau}{c}\sigma_u$, or that the unit fundraising costs are sufficiently inelastic and that the marginal utility of consumption is sufficiently elastic.

We collect these assumptions in the following

Assumption 1. (*Preferences and Technology*). Preference functions ($u(\cdot)$, $a(\cdot)$, and $A(\cdot)$) and technology functions ($\pi(\cdot)$ and $\theta(\cdot)$) satisfy

$$\epsilon_A \geq \epsilon_a, \sigma_A \leq \sigma_a, \text{ and } \epsilon_\theta \geq \frac{\tau}{c}\sigma_u \quad ((A1))$$

Under this assumptions we can show that the determinant of the system of comparative

static equations is positive, or

$$\begin{aligned} \Delta_A = & (c + \tau)A' \frac{T}{c} \sigma_u \left\{ A' \left(\frac{c + \tau}{\tau} \right) \varepsilon_\theta + \frac{A + T}{T} (\varepsilon_A - \varepsilon_a) \right\} \\ & + A \left\{ (\sigma_a - \sigma_A) \frac{\varepsilon_a}{\varepsilon_\pi} \frac{A}{T} \left(\varepsilon_\theta - \frac{\tau}{c} \sigma_u \right) + (\varepsilon_a - \varepsilon_A)^2 \right\} > 0 \end{aligned}$$

The determinant of the previous section Δ_V is a special case of Δ_A assuming $0 = \sigma_A \leq \sigma_a$ and $1 = \varepsilon_A \geq \varepsilon_a$.

We now show that for the most part discounting does not matter for our comparative static results. Specifically, under Assumption 1 the response of consumption, donations, religiosity, and longevity, to exogenous changes in wealth, religiosity, and religious market competition will be qualitatively the same as in the previous section.

We must, however, generalize our secularization condition to incorporate discounting:

Assumption 2. (*Secularization Condition*). Preference functions ($a(\cdot)$ and $A(\cdot)$) and technology functions ($\pi(\cdot)$ and $\theta(\cdot)$) satisfy

$$\frac{\varepsilon_A - \varepsilon_a}{\varepsilon_a} \frac{\varepsilon_A}{\sigma_a - \sigma_A} < \frac{1}{\varepsilon_\pi} \frac{\tau}{c + \tau} \quad ((A2))$$

The condition is assumed mainly to illustrate the force of wealth and substitution effects on donations and religiosity. In particular, it guarantees that the wealth effects on donations of a change in wealth outweighs the induced substitution effect from a change in wealth. Violation of the secularization condition results in a positive wealth effect on donations.

Given Assumptions 1 and 2, our comparative static results for exogenous wealth changes are:

$$\begin{aligned} \Delta_A \frac{dT}{dZ} &= \left(\frac{T}{c} \sigma_u \right) \left[A' \frac{c + \tau}{\tau} \varepsilon_\theta + (\varepsilon_A - \varepsilon_a) \right] > 0 \\ \Delta_A \frac{dc}{dZ} &= (\sigma_a - \sigma_A) \frac{A}{T} \frac{\varepsilon_a}{\varepsilon_\pi} \varepsilon_\theta + (\varepsilon_a - \varepsilon_A)^2 > 0 \\ \Delta_A \frac{d\tau}{dZ} &= \sigma_u \left[(\sigma_A - \sigma_a) \frac{A}{T} \frac{\varepsilon_a}{\varepsilon_\pi} \frac{\tau}{c} + (\varepsilon_A - \varepsilon_a) A' \frac{c + \tau}{c} \right] > 0 \end{aligned}$$

The effects of exogenous changes in religiosity are:

$$\begin{aligned} \Delta_A \frac{dT}{d\pi} &= \frac{a' AT}{u'} \left[\frac{\varepsilon_\theta}{\tau} - \frac{\sigma_u}{c} \right] > 0 \\ \Delta_A \frac{dc}{d\pi} &= \frac{-a' A}{u'} \left[\left(\frac{c + \tau}{\tau} \varepsilon_A \varepsilon_\theta + (\varepsilon_A - \varepsilon_a) \right) \right] < 0 \\ \Delta_A \frac{d\tau}{d\pi} &= \frac{a' A}{u'} \left[\left(\frac{c + \tau}{c} \varepsilon_A \sigma_u + (\varepsilon_A - \varepsilon_a) \right) \right] > 0 \end{aligned}$$

And the effects of competition in the donations market (where a decrease in X means more competition) are

$$\begin{aligned}\Delta_A \frac{dT}{dX} &= \frac{TA}{X} \left[(\varepsilon_A - \varepsilon_a) + A' \frac{c + \tau}{c} \sigma_u \right] > 0 \\ \Delta_A \frac{dc}{dX} &= \frac{-Tc}{X} \left[(\varepsilon_A - \varepsilon_a) \frac{c + \tau}{c} A' + (\sigma_A - \sigma_a) \frac{A}{T} \frac{\varepsilon_a}{\varepsilon_\pi} \frac{\tau}{c} \frac{A}{T} \right] \geq 0 \\ \Delta_A \frac{d\tau}{dX} &= \frac{-T}{X} \left[\frac{(c + \tau)^2}{c} \sigma_u + \frac{\varepsilon_a}{\varepsilon_\pi} \tau \sigma_a \right] < 0\end{aligned}$$

We note that contrary to the result in the previous section the response of consumption to a change in competition is ambiguous. The secularization condition now does not by itself suffice to pin down the response of consumption. However, the condition is sufficient as long as $A(T) \geq T$. That is, the secularization condition implies an increase in consumption when individuals are not especially long-lived (because $A(T)$ is concave and crosses the 45 degree line from above). Alternatively, when the secularization condition is violated we require that $A(T) \leq T$ in order to guarantee that competition causes a decrease in consumption.

We summarize our findings in the central result of this paper:

Proposition 1. (*Life Cycle Effects of Wealth, Religiosity, and Competition*) Under the conditions of Assumption 1,

- (A) Wealth has a positive effect on consumption and longevity. Wealth also has a negative effect on religious donations and faith when Assumption 2 holds (or a positive effect on religious donations and faith when Assumption 2 is violated.)
- (B) Religious faith has a positive effect on longevity and religious donations and a negative effect on consumption.
- (C) Increased competition for religious donations has a negative effect on longevity and a positive effect on religious donations and faith. Competition also has a positive effect on consumption when Assumption 2 holds and $A(T) \geq T$ (or a negative effect on religious donations and faith when Assumption 2 is violated and $A(T) \leq T$.)

The proposition indicates that increased competition has effects on longevity and religiosity that can be quite different than the effects from increased wealth. Also, the proposition can be used to explain the overall pattern in Figures 1 and 2 and why the United States is an exception. One explanation focusses on the wealth effect. According to Proposition 1(A), countries where the secularization condition holds will experience rising life expectancies and falling religiosity as their wealth increases. The United States exception would be consistent with a violation of the secularization condition. That is, the greater wealth of the United States would cause religiosity to rise and life expectancies to fall. Another explanation would accept the secularization condition for all countries and focus on the combined effects from economic development and the competitive landscape in the market for religion. According to Proposition 1(C), countries with highly competitive religious markets will experience greater religiosity but lower life expectancies. Religious markets are highly competitive in the United States and more or less uncompetitive in the rest of the OECD. Thus, competition is a force in the United States that works against economic development by encouraging religiosity and discouraging longevity. A sufficiently strong competitive force could also explain the United States exception in Figure 1 and 2. By contrast, lack of competition in the OECD peers of the United States is a force that reinforces the effects of economic development by discouraging religiosity and encouraging longevity.

6 The Effects of Increased Mortality Risk

In the previous sections we considered the response of religiosity, longevity, donations and consumption to exogenous variations in wealth, religiosity, and the competitiveness in the market for religious donations. These exogenous variations correspond to some of the main theories in the socioeconomics of religion: secularization, culture, and market competitiveness. By allowing discounting we are also able to analyze the effect of increased mortality risk on religion, as opposed to analyzing the effect of increased religious uncertainty on longevity. The effect of mortality risk was unsuccessfully analyzed by Affi and Ehrenberg (1975) and may be thought of as another aspect in the process of aging. Another interpretation for what lies behind heightened mortality risk is war and epidemics that increase the perception in one's mortality. Secularism has been on the rise in the last century which arguably has been a time of peace and health advances. If there is a connection, it is

subtle, because historically the effect of higher mortality risk has produced diverse religious responses, witness the response to the threat of plague as depicted in Bocaccio's *Decameron* or the responses to the threat of warfare throughout human history as depicted in Hesse's *Narcissus and Goldmund* (to name two of many literary examples that deal with the Middle Ages in Europe where warfare and plague were everpresent.)

We conduct our analysis of the effects of mortality risk in two stages. We first discuss how mortality risk affects consumption and longevity when donations are exogenous (as in section 3) and there is only one substitution effect through the relative price of longevity. Then we allow donations to be endogenous and introduce the second substitution effect through the relative price of donations. For consistency with previous sections, we continue with the assumption that the after-life payoff $a()$ is a function of longevity T rather than the life expectancy $E(T)$. This simplifying assumption says that good intentions matter in the sense that planned behavior over ones lifespan matters, even though realistically one may die earlier than T .

When donations are exogenous, an increase in mortality risk δ has wealth and substitution effects on consumption and longevity. To see this note that a higher δ affects the wealth constraint (9) by reducing A , or $A_\delta = \frac{-1}{(\rho+\delta)^2} (1 - (1 + (\rho + \delta)T) A') < 0$ for reasonable values for $(\rho + \delta)$ and T ., A higher δ also affects the first optimality condition (10) by reducing A' , or $A'_\delta = -TA' < 0$. The reduction of A acts like an increase in effective wealth with positive effects on consumption and longevity. Furthermore, the reduction of A' causes a substitution effect by lowering the relative price of longevity and lowering the subjective valuation of longevity. Under the assumption that $\varepsilon_u < \frac{c}{c+\tau}$ which is reasonable when donations are very small compared to consumption (as is the case in the United States), one can show that the decline in the subjective valuation is larger than the decline in the relative price. Thus, the reduction of A' effectively means that individuals face an increase in the relative price of longevity. By reducing the effective payoff to longevity, mortality risk acts like an exogenous reduction in faith π in its substitution effects. The substitution effect from the effective increase in the relative price of longevity reinforces the wealth effect on consumption and offsets the wealth effect on longevity. Thus, heightened mortality risk will encourage individuals to increase their quality of life by increasing consumption. But the overall effect on longevity is uncertain as is the effect on life expectancy.⁷ Qualitatively

⁷To see that life expectancy declines with mortality risk and increases with longevity, note that $E(T) =$

then, heightened mortality risk has wealth and substitution effects that similar to those of a reduction in involuntary contributions

Matters are more complicated when donations are endogenous because a second substitution effect comes into play. Since mortality risk reduces A' , heightened mortality risk has the effect of reducing the price of donations relative to consumption in (12). Thus, in the extended model donations and religious faith tend to increase as wealth and substitution effects are generally reinforcing. The effects on consumption and longevity remain ambiguous, since both now have offsetting wealth and substitution effects (**Finish this last Paragraph...**).

7 Conclusions

To conclude, our analysis has in a very simple model explained the religiosity and comparatively low life expectancy evidenced by the United States as compared to other countries in the world. The key mechanism is that religious competition and economic development affect an individual's degree of certainty with respect to beliefs over the afterlife, which we have termed faith though faith is assuredly something much more complex than this *reductio*. We show that competition increases faith and religious donations and decreases longevity, while economic development raises longevity but may raise or lower religiosity. Our analysis highlights the wealth and substitution effects that come about from competition and development and ultimately drive religiosity and religious donations in a life cycle model with endogenous lifespans. And though our results are somewhat suggestive, we have simplified the analysis along many dimensions by means of some very strong assumptions that could be fruitfully relaxed. Despite this, we contribute to the literature by explicitly modeling religious beliefs, evoking a market for voluntary contributions, and allowing longevity to be endogenous.

Future work might relax some of our simplifying assumptions. Religious organizations are modelled very simply without regard to the many services they provide and with very simplistic notions of what determines their competitive interactions. This is probably the weakest aspect of our model and deserving of further investigation. Our model could also be

$\delta^{-1} (1 - (1 + \delta T)e^{-\delta T})$. Taking the total derivative leads to $dE = E_\delta d\delta + E_T dT$ with $E_\delta = \delta^{-2} (e^{-\delta T} - 1) < 0$ denoting the influence of mortality risk and $E_T = \delta e^{-\delta T} > 0$ denoting the influence of longevity on life expectancy.

extended to consider the public good aspects of religious donations, perhaps along the lines of Andreoni (1989). Though the essential results should not change, the wider perspective allows one to investigate the desirability of greater competition in religious markets and the policies that influence the degree of competitiveness. If religious firms are thought of as providing purely private goods, then the tax advantages accorded too religious firms may lead to excess competition and overprovision of religion. If instead one interprets religious beliefs as a public good, then perhaps public incentives may indeed move religious firms closer to the Samuelsonian ideal, because by the Samuelson rule competitive firms tend to underprovide religion. The question of the degree of publicness of religious good provision is however an open empirical question that has received little attention but would provide tremendous benefits in designing public incentives that have some economic foundation.

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8 Appendix (Not for Publication)

For involuntary contributions we totally differentiate (2) and (3). Total differentiation implies the following system of equations:

$$\begin{pmatrix} c + \tau & T \\ \frac{\pi a'}{u'} \frac{a''}{a'} & -\frac{c+\tau}{c} \frac{u''c}{u'} \end{pmatrix} \begin{pmatrix} dT \\ dc \end{pmatrix} = \begin{pmatrix} dZ - Td\tau \\ -\frac{a'}{u'} d\pi + \left(1 - \frac{\pi a'}{u'} \frac{\pi'}{\pi}\right) d\tau \end{pmatrix}$$

For voluntary contributions to private goods, we totally differentiate (2), (3), and (7). Total differentiation implies the following system of equations:

$$\begin{pmatrix} (c + \tau) & T & T \\ -\frac{\varepsilon_a}{\varepsilon_\pi} \frac{\tau}{T} \sigma_a & \frac{c+\tau}{c} \sigma_u & (\varepsilon_a - 1) \\ (\varepsilon_a - 1) & \frac{T}{c} \sigma_u & \frac{T}{\tau} \varepsilon_\theta \end{pmatrix} \begin{pmatrix} dT \\ dc \\ d\tau \end{pmatrix} = \begin{pmatrix} dZ - A_\delta (c + \tau) d\delta \\ -\frac{a'}{u'} d\pi + A'_\delta \frac{c}{\varepsilon_u} (\varepsilon_u \frac{c+\tau}{c} - 1) d\delta \\ \frac{-T}{X} dX + A_\delta d\delta \end{pmatrix}$$

When discounting is introduced for voluntary contributions to private goods, we totally differentiate equations (9), (10), and (12). Total differentiation implies the following system of equations:

$$\begin{pmatrix} (c + \tau)A' & A & A \\ (\sigma_A - \sigma_a) \frac{1}{T} \frac{\varepsilon_a}{\varepsilon_\pi} \frac{A\tau}{T} & A' \frac{c+\tau}{c} \sigma_u & (\varepsilon_a - \varepsilon_A) \\ (\varepsilon_a - \varepsilon_A) & \frac{T}{c} \sigma_u & \frac{T}{\tau} \varepsilon_\theta \end{pmatrix} \begin{pmatrix} dT \\ dc \\ d\tau \end{pmatrix} = \begin{pmatrix} dZ \\ \frac{-a'}{u'} d\pi \\ \frac{-T}{X} dX \end{pmatrix}$$