

Education and Growth

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Introduction

It has long been recognised that knowledge or education level plays an important role for economic growth (see, for example, Lucas [1988] and Uzawa [1965]). This paper considers a 3-sector economy where knowledge expands over time through education services supplied at each moment of time. Knowledge is assumed to exert the same effect as the Hicks-neutral technological progress does on the production of the consumption-good and capital-good sectors. Knowledge here is a public input in that its use by one sector does not reduce the knowledge available to the other sector.

The economy considered here is a dynamic version of the economy analysed in Ishizawa [1988],[1991], with knowledge imparting intertemporal increasing returns to the production of the other sectors. The Central Planning Agency (CPA) allocates resources to determine the supply of education services, the output of consumption good, and that of capital good in order to maximise the intertemporal welfare of a representative consumer.

The paper shows that (i) a unique steady state exists where all real variables grow at the same rate as an exogenously given rate of growth of labour, and that (ii) a unique saddle path exists to approach to the steady state for every pair of initial values of knowledge and capital stocks around the steady state. These results obtain under the condition, among the others, that the marginal effect of knowledge on the output of the other sectors lies within a certain range determined by the production technologies in the economy.

Section I lays out the model. Section II examines a temporary equilibrium; its dynamics is analysed in section III. Section IV studies a steady state of the economy, and the following section states the result about the saddle path. A brief summary of the paper is given in Conclusion.

II. The Model

A 3-sector economy is considered in this paper, comprising the education, capital-good, and consumption-good sectors. At each moment of time t , a given amount of labour, denoted $L(t)$, and that of capital stock, denoted $K(t)$, are allocated into these sectors. The production function of each sector i , $i = 0,1,2$, is such that its output varies directly with the amounts of labour $L_i(t)$, those of capital $K_i(t)$, and knowledge level per worker, denoted $e(t) \equiv E(t)/L(t)$. They are assumed to take the following forms:

$$X_0(t) = \min\left[\frac{L_0(t)}{l_0}, \frac{K_0(t)}{k_0}\right],$$

$$X_1(t) = f^1(e(t)) \cdot \min\left[\frac{L_1(t)}{l_1}, \frac{K_1(t)}{k_1}\right],$$

$$X_2(t) = f^2(e(t)) \cdot \min\left[\frac{L_2(t)}{l_2}, \frac{K_2(t)}{k_2}\right],$$

where (l_i, k_i) are positive constants for all $i = 0,1,2$. $X_0(t)$ is the supply of the education services, $X_1(t)$ the output of capital good, and $X_2(t)$ that of consumption good, each at time t . Two comments are worth making for the production technologies specified here; first, knowledge is a public input in that its use in one sector does not reduce the level of knowledge available to the other sector. Second, it exerts the same effect as a Hicks-neutral-technological improvement does on the production of each private good $i = 1,2$. The level of knowledge is fixed at each point in time, but as shown presently, it will change over time until it reaches an equilibrium value. Thus when all variables $(e(t), L_i(t), K_i(t))$ are doubled over time, the output of the private-good sector will more than double. In this sense, intertemporal increasing returns are imbedded in the economy.

Let $(\tilde{l}^i(e), \tilde{k}^i(e))$ be the efficient use of labour and capital to produce one unit of private good i , $i = 1,2$, provided that knowledge is set at the level of e . They will be

$$(1) \quad \tilde{l}^i(e) = \frac{l_i}{f^i(e)}, \quad \tilde{k}^i(e) = \frac{k_i}{f^i(e)} \quad i = 1,2.$$

Then X_i units of output of each private good requires $L_i = \tilde{l}_i(e).X_i$ units of labour and $K_i = \tilde{k}_i(e).X_i$ units of capital. The full-employment conditions for resources will be then written as

$$(2.a) \quad L(t) = l_0.X_0(t) + \sum_{i=1}^2 \tilde{l}_i(e(t)).X_i(t),$$

$$(2.b) \quad K(t) = k_0.X_0(t) + \sum_{i=1}^2 \tilde{k}_i(e(t)).X_i(t).$$

The whole output of consumption good, $X_2(t)$, will be consumed instantaneously at each moment of time, while the education services as well as the capital good currently produced will be used to expand their stocks. The labour force is assumed to grow at an exogenously given constant rate n . These are summarised as:

$$(3) \quad C(t) = X_2(t), \quad \dot{E}(t) = X_0(t), \quad \dot{K}(t) = X_1(t), \quad \dot{L}(t) = n.L(t),$$

where $C(t)$ is the aggregate level of consumption of the economy, and \dot{x} is an abbreviation of dx/dt .

The welfare of the economy is represented with the welfare of the representative consumer; it is given by

$$(4) \quad \int_0^{\infty} \ln \left[\frac{C(t)}{L(t)} \right] . e^{-\theta.t} dt,$$

where $C(t)/L(t)$ is the representative consumer's level of consumption at time t , which of course equals the per-capita consumption of the economy.

The Central Planning Agency (CPA) maximises the welfare of the economy given in (4) subject to resource constraints (2) and the dynamic evolutions of the state variables given in (3). Let H be the Hamiltonian associated with this maximisation problem (to

avoid cluttering notations, we drop time variable t henceforth unless it causes confusion):

$$H = \ln(C/L)e^{-\theta.t} + p_0 X_0 + p_1 X_1 + p_2 (X_2 - C_2) + w.[l_0.X_0 + \sum_{i=1}^2 \tilde{l}_i(e).X_i] + r.[k_0.X_0 + \sum_{i=1}^2 \tilde{k}_i(e).X_i].$$

The CPA maximises H by controlling the levels of variables (C, X_0, X_1, X_2) over time. w and r are Lagrangian associated with resource constraints (2). p_2 is also the Lagrangian for the constraint that $C(t) = X_2(t)$ given in (3). p_0 and p_1 are the co-state variables attached with state variables E and K . Under the assumption of an interior solution for each control variable, the first-order conditions for optimality are:

$$(5.a) \quad 0 = H_c = \frac{1}{(C/L)} \cdot \frac{1}{L} \cdot e^{-\theta.t} - p_2$$

$$(5.b) \quad 0 = H_{X_0} = p_0 - w.l_0 - r.k_0 \equiv p_0 - \tilde{c}^0(w, r)$$

$$(5.c) \quad 0 = H_{X_i} = p_i - w.l_i(e) - r.k_i(e) \equiv p_i - \tilde{c}_i(e, w, r), \quad i = 1, 2,$$

where subscript of H refers to the partial derivative of H with respect to that variable. $\tilde{c}^0(\cdot)$ and $\tilde{c}^i(\cdot)$ are implicitly defined in eqts. (5.a) and (5.b), respectively.

$\tilde{c}^0(\cdot)$ depends on (w, r) alone since (l_0, k_0) are assumed constant. Similarly $\tilde{c}^i(\cdot)$ depends on (e, w, r) alone. If we know the values of (p_1, p_2, e, L) , these four equations will be solved for to determine the values of four variables (C, w, r, p_0) at each moment of time.

In addition to these optimality conditions, (p_0, p_1) is required to satisfy the following differential equations:

$$(6.a) \quad 0 = \dot{p}_0 + H_E = \dot{p}_0 - \sum_{i=1}^2 [w.\tilde{l}_e^i(e) + r.\tilde{k}_e^i(e)].\frac{X_i}{L} \equiv \dot{p}_0 - \sum_{i=1}^2 \tilde{c}_e^i(e, w, r),$$

$$(6.b) \quad 0 = \dot{p}_1 + H_K = \dot{p}_1 + r.$$

These optimality conditions will be supplemented with the familiar transversality conditions for the state variables:

$$(7) \quad p_0(t)E(t) \rightarrow 0, \text{ and } p_1(t)K(t) \rightarrow 0 \text{ as } t \rightarrow \infty,$$

which helps to exclude “bubbles” of state variables. 12 equations, eqts. (2),(3), (5), and (6), contain 12 unknowns $(C, X_0, X_1, X_2, E, K, L, p_0, p_1, p_2, w, r)$. Since knowledge is a public input, knowledge and public input will be used henceforth interchangeably. The economy satisfying these 12 equations will be called the Public Input Economy.

The aim of this paper is to find an optimal path of each of these 12 variables given the initial values of the public input $E(0)$, capital stock $K(0)$, and labour force $L(0)$ under the transversality conditions (7). For that we make drastically simplifying assumptions:

Assumption 1: $f^1(e) = f^2(e) = f(e)$ for all $e \geq 0$.

Assumption 2: $k_1 = l_2 = 0$

Assumption 1, which may be termed the Symmetry Condition, states that the public input benefits each private-good industry in the same manner. Assumption 2 implies that the consumption-good industry 2 is more capital intensive than the capital-good industry 2. In the economy with Leontief-type production technologies as considered here, a Harodian knife-edged outcome would result without this type of assumption.

II. Temporary Equilibrium

This section considers a temporary equilibrium of the economy where resources $(L(t), K(t))$, the prices of private goods (p_1, p_2) , knowledge level $e(t)$, and the current supply of education services $X_0(t)$ are given. Turn to eqts. (5b) and (5.c) first. w and r there may be interpreted as the wage and rental rate for the unit service of labour and capital, and p_i as the unit price of good i , $i=1,2$. The equations then say that each industry operating under a constant-returns-to-scale technology at a given level of knowledge earns zero profits. These are a variant of the well-known zero-profit

conditions of the standard Heckscher-Ohlin-Samuelson economy with an additional factor of knowledge. We may regard them, therefore, as the equations to determine the values of (p_0, w, r) as functions of (p_1, p_2, e) at each moment of time. Under the Assumptions 1 and 2, they produce the following solution functions for (p_0, w, r) :

$$(8.a) \quad w = f(e) \cdot \frac{p_1}{l_1}, \quad r = f(e) \cdot \frac{p_2}{k_2},$$

$$(8.b) \quad p_0 = f(e) \cdot p_2 \cdot c^0(q, 1),$$

where

$$(8.c) \quad c^0(q, 1) \equiv \frac{l_0}{l_1} \cdot q + \frac{k_0}{k_2}, \quad q \equiv \frac{p_1}{p_2}.$$

q is the relative price of capital good 1 in terms of consumption good 2. $c^0(\cdot)$ is the unit cost of the education services when $f(e) = 1$ and $p_2 = 1$.

As seen from eqts.(8), wage-rental ratio is independent of knowledge, and the unit production cost of the education services is weakly separable in the knowledge e . These are the direct consequences of the symmetry condition stated in Assumption 1. Furthermore, the symmetry condition implies that the output of private good is also weakly separable in knowledge. In fact resource constraints (2) under Assumption 2 give rises to

$$(9.a) \quad X_1 = f(e) \cdot \left(\frac{1 - l_0 \cdot \frac{X_0}{L}}{l_1} \right) L \equiv f(e) \cdot x^1(x_0) \cdot L$$

where $x^1(\cdot)$ is implicitly defined by the last equality, and $x_0 \equiv X_0/L$ is the current supply of the education services per worker. Similarly we have

$$(9.b) \quad X_2 = f(e) \cdot \left(\frac{\frac{K}{L} - k_0 \cdot \frac{X_0}{L}}{k_2} \right) \cdot L \equiv f(e) \cdot x^2(z, x_0) \cdot L,$$

where z is the capital stock per worker $z \equiv K/L$. Thus the per-capita output of private good, $x_i \equiv X_i/L$, $i=1,2$, depends on the amounts of the capital stock per worker z , the average level of knowledge level e , and the current supplies of education services x_0 ; knowledge level e affects the per-capita output of each private good in a multiplicative form:

$$(10.a) \quad x_1 = f(e)x^1(x_0)$$

$$(10.b) \quad x_2 = f(e)x^2(z, x_0)$$

III. Dynamics of the Economy

So far we have treated (p_1, p_2, z, e, x_0) as parameters. We now turn to the analysis how these variables evolve over time. First, the accumulation equations in (3) implies:

$$(11.a) \quad \dot{e} = x_0 - n.e \equiv \phi(e, x_0),$$

$$(11.b) \quad \dot{z} = f(e)x^1(x_0) - n.z \equiv \varphi(e, z, x_0).$$

Optimality condition (6.b) reduces to a simple form under Assumptions 1 and 2:

$$\dot{p}_1 = -r = f(e) \cdot \frac{p_2}{k_2} \quad (\text{see eqt. (8.a)})$$

This will be re-written as follows by means of relative price q introduced above:

$$(12) \quad \dot{q} = q \cdot \left[-\frac{\dot{p}_2}{p_2} - \frac{f(e)}{k_2 \cdot q} \right] \equiv q \cdot \left[\pi - \frac{f(e)}{k_2 \cdot q} \right] \\ \equiv \tilde{\rho}(e, q, \pi)$$

where π is the rate of deflation of the price of consumption good 2:

$$\pi \equiv -\frac{\dot{p}_2}{p_2}.$$

A temporary equilibrium condition (8.b) must hold at every moment of time: it gives rise to the following differential equation that:

$$(13.a) \quad \frac{\dot{p}_0}{p_0} = \varepsilon(e) \cdot \frac{\dot{e}}{e} + \theta_L^0(q, 1) \cdot \frac{\dot{q}}{q} - \pi,$$

where $\varepsilon(e) \equiv f'(e) \cdot e / f(e)$ is the elasticity of the output of private goods with respect to knowledge, and $\theta_L^0(q, 1) \equiv c_q^0(q, 1) \cdot q / c^0(q, 1)$ is the share of labour cost in the education sector. This rate of change in the price of the public input must be compatible with the rate of change of the price given by optimality condition (6.a). Dividing the equation by given p_0 in (8.b) yields under Assumption 1 that

$$(13.b) \quad \frac{\dot{p}_0}{p_0} = -\frac{\varepsilon(e)[q \cdot x^1(x_0) + x^2(z, x_0)]}{e \cdot c^0(q, 1)} \quad (\text{see eqts. (10)}) \\ \equiv \xi(e, z, q, x_0)$$

Since the price change given in eqts.(13) must equal each other, we have:

$$(14.a) \quad \pi = \xi(e, z, q, x_0) + \varepsilon(e) \cdot \frac{\dot{e}}{e} + \theta_L^0(q, 1) \cdot \frac{\dot{q}}{q}$$

The equation reduces to the following due to eqts. (11.a) and (12):

$$(14.b) \quad \pi = \frac{1}{\theta_K^0(q,1)} \left[\xi(e, z, q, x_0) + \varepsilon(e) \cdot \frac{\phi(e, x_0)}{e} + \theta_K^0(q,1) \cdot \frac{f(e)}{k_2 \cdot q} \right]$$

$$\equiv \pi(e, z, q, x_0)$$

where $\theta_K^0(q,1) \equiv 1 - \theta_L^0(q,1)$ is the share of capital cost in the education sector.

We now turn to eqt. (5.a), which determines the optimal level of consumption at each moment of time. Due to eqts. (3) and (9.b) which entail that $C/L = f(e) \cdot x^2(z, x_0)$, we have

$$\frac{1}{L(0)} \cdot e^{-(n+\theta)t} = p_2 \cdot f(e) \cdot x^2(z, x_0)$$

This gives rise to the differential equation that:

$$(15.a) \quad -(n+\theta) = -\pi + \varepsilon(e) \cdot \frac{\dot{e}}{e} + \delta_z(z, x_0) \cdot \dot{z} + \delta_0(z, x_0) \cdot \dot{x}_0,$$

where $\delta_z(z, x_0) \equiv x_z^2(z, x_0)/x^2(z, x_0) > 0$, and $\delta_0(z, x_0) \equiv x_0^2(z, x_0)/x^2(z, x_0) < 0$.

Substituting eqts.(10) gives

$$(15.b) \quad \dot{x}_0 = \frac{1}{-\delta_0(z, x_0)} \left[(n+\theta) - \pi + \frac{\phi(e, x_0)}{e} + \delta_z(z, x_0) \cdot \varphi(e, z, x_0) \right]$$

$$\equiv \tilde{\chi}^0(e, z, x_0, \pi).$$

We now have 4 differential equations to describe the dynamics of 4 variables (e, z, q, x_0) . They are:

$$(11.a) \quad \dot{e} = \phi(e, x_0),$$

$$(11.b) \quad \dot{z} = \varphi(e, z, x_0),$$

$$(12) \quad \dot{q} = \tilde{\rho}(e, q, \pi),$$

$$(15.b) \quad \dot{x}_0 = \tilde{\chi}^0(e, z, x_0, \pi),$$

We also have a side condition for π :

$$(14.b) \quad \pi = \pi(e, z, q, x_0)$$

In the next section we examine a steady state of the economy by studying a steady state of this system of differential equations.

IV Steady State of the Public Input Economy

This section searches for a steady state at which $\dot{e} = \dot{z} = \dot{q} = \dot{x}_0 = 0$, then considers a steady state of the economy. The search will be made in 2 steps:

Step 1: Search a pair of state variables (e, z) which bring about $\dot{e} = \dot{z} = 0$.

Step 2: Search a pair of state variables (e, z) which bring about $\dot{e} = \dot{q} = 0$,
and $\pi = (\theta + n)$.

Although no consideration is taken for $\dot{x}_0 = 0$ directly in these steps, eqt.(17.a) verifies that any pair (e^*, z^*) satisfying the both steps simultaneously will produce $\dot{x}_0 = 0$; hence (e^*, z^*) is a steady state.

Step 1. It follows from eqts.(11) and $\dot{e} = \dot{z} = 0$ that

$$(16) \quad f(e)x^1(n.e) = n.z.$$

We make a following assumption to facilitate our understandings of a steady state through a picture in the space of state variables:

Assumption 3. $f(e) = \alpha.e$ for some positive constant α

Given the specific form of $x^1(\cdot)$ in eqt.(9.a), Fig.1 draws the locus of (e, z) satisfying the eqt.(16). This figure will be combined with a figure drawing (e, z) obtained in Step 2.

Fig.1 here

Step 2. $\dot{q} = 0$ and eqt.(12) entail that

$$(17) \quad q = \frac{f(e)}{k_2 \cdot \pi} \equiv q(e, \pi).$$

Substituting this and $x_0 = n \cdot e$ obtained from $\dot{e} = 0$ into (14.a) with $\dot{e} = \dot{q} = 0$ gives

$$\pi = \xi(e, z, q(e, \pi), n \cdot e) \equiv \tilde{\xi}(e, z, \pi).$$

The pair of (e, z) sought for in Step 2 derives from the simultaneous equations that

$\pi = \tilde{\xi}(e, z, \pi)$ and $\pi = (\theta + n)$ to be solved for (e, z) . Under Assumption 3, the pair must satisfy the following quadratic equation:

$$(18.a) \quad 0 = a \cdot e^2 + b \cdot e + c \cdot z$$

where,

$$(18.b) \quad \begin{cases} a \equiv \alpha \cdot l_0 \cdot (\theta + n) \\ b \equiv (\theta + n)(\theta + 2n)l_1 \cdot k_0 - \alpha \\ c \equiv -l_1 \cdot (\theta + n) \end{cases}$$

Obviously $a > 0$ and $c < 0$, but the sign of b is unknown *a priori*. Fig.2.a illustrates (e, z) satisfying eqt.(18.a) when $b < 0$, and Fig.2.b when $b > 0$. Since α represents the marginal effect of knowledge on the production of each private good, negative (positive) b shows that its effect is large (small) relative to other parameters. We state, without giving a proof, that $(-b/a) < 1/(2nl_0)$ if $b < 0$.

Fig.2.a and Fig.2.b here

Thus when $b < 0$, the loci depicted in Fig.1 and Fig.2.a intersects at a point A (e^*, z^*) as illustrated in Fig.3.a. Evidently the intersecting point exists and is unique.

Proposition 1. Assume that the parameters of the Public Input Economy are such that b is negative in (18.b). Then under Assumptions 1-3, there exists a unique steady state of the economy where (i) the output of every good as well as aggregate knowledge and capital stock grows at the same rate of labour, but (ii) the price of every good as well as factor prices declines at the rate of $(\theta + n)$.

Proof: As shown above, a unique pair of positive constants (e^*, z^*) exists at which $\dot{e} = \dot{z} = \dot{q} = \dot{x}_0 = 0$ and $\pi = (\theta + n)$. Eq.(11.a) then says that $x_0 (= n.e^*)$ is constant over time, and hence $X_0 (= x_0.L)$ grows at the same rate as L . Furthermore, eqts. (9) show that so does $X_i, i = 1, 2$, as well. Constancy of (e^*, z^*) means that E and K grow at the same rate of L . This proves the first part. Since $p_1 = q.p_2$, constant of $q (= q(e^*, \theta + n)$ from (17)) implies p_1 declines at the same rate as p_2 . Constant q and e then entail via eqt.(8) that (w, r, p_0) decline at the rate of p_1 and p_2 , which is $(\theta + n)$. This proves the second part. Q.E.D.

It is shown that under Assumptions 1-3, no qualification is necessary to establish a steady state of the Public Input Economy when $b < 0$. In the case where $b > 0$, however, we need a qualification to establish a unique steady state as drawn in Fig.3.b. A sufficient condition for that is that the tangency line of (e, z) satisfying eqt.(18.a), de/dz , is steeper than that satisfying eqt.(16).

This condition and its implications are put in the form of assumption and remark (again without giving proof).

Assumption 4. The parameters of the economy are such that the following holds:

$$(\theta + n)(\theta + 2n).k_0 l_1 > \alpha > n.(\theta + n).k_0 l_1$$

The first inequality shows that b is negative so that the marginal effect of knowledge on the production of private good is relatively small, but the second inequality requires that its effect be not too small.

Remark 1. Under Assumption 4, eqts.(16) and (18.a) have a unique positive solution (e^*, z^*) as drawn in Fig.3.b.

The remark readily establishes the following proposition (the proof is the same as the

previous proposition).

Proposition 2. Assume that the parameters of the Public Input Economy are such that b is positive in (18.b). Then under Assumptions 1-4, there exists a unique steady state of the economy where (i) the output of every good as well as the stocks of public input and capital grows at the same rate of labour, but (ii) the price of every good as well as factor prices declines at the rate of $(\theta + n)$.

V. Saddle Path

This section simply states the results, which together imply that there exists a unique path converging to (e^*, z^*) in Figs.3, but without presenting a proof for each result. Now consider the following 4 differential equations containing 4 variables (e, z, q, x_0) by substituting $\pi(e, z, q, x_0)$ into eqts.(12) and (15.b)

$$(11.a) \quad \dot{e} = \phi(e, x_0),$$

$$(11.b) \quad \dot{z} = \varphi(e, z, x_0),$$

$$(12) \quad \dot{q} = \tilde{\rho}(e, q, \pi(e, z, q, x_0)) \equiv \rho(e, z, q, x_0),$$

$$(15.b) \quad \dot{x}_0 = \tilde{\chi}^0(e, z, x_0, \pi(e, z, q, x_0)) \equiv \chi(e, z, q, x_0)$$

Linear approximation of the functions on the right of the equations generates the following linear differential equations where the partial derivatives are evaluated at the values in the steady state of the economy:

$$\begin{bmatrix} \dot{e} \\ \dot{z} \\ \dot{q} \\ \dot{x}_0 \end{bmatrix} = \begin{bmatrix} \phi_e & 0 & 0 & \phi_0 \\ \varphi_e & \varphi_z & 0 & \varphi_0 \\ \rho_e & \rho_z & \rho_q & \rho_0 \\ \chi_e & \chi_z & \chi_q & \chi_0 \end{bmatrix} \begin{bmatrix} e \\ z \\ q \\ x_0 \end{bmatrix}$$

Let D denote the matrix on the right of the equations, $|D|$ its determinant, and $g(\lambda)$

the characteristic equation for the matrix: $g(\lambda) = |\lambda I - D|$.

Remark 3. Under Assumptions 1-3, $g(0) > 0$ and $g(-n) < 0$.

The following assumption is made to ensure that the matrix has 2 positive and 2 negative (real) eigen values.

Assumption 5. The marginal effect of the public input, α , is not so strong as to exceed an upper bound given by parameters. To be more specific, α satisfies the inequality that

$$\alpha < (\theta + 2n)^2 k_0 l_1$$

Remark 4. Under Assumptions 1-3 and 5, $g(\theta + n) < 0$.

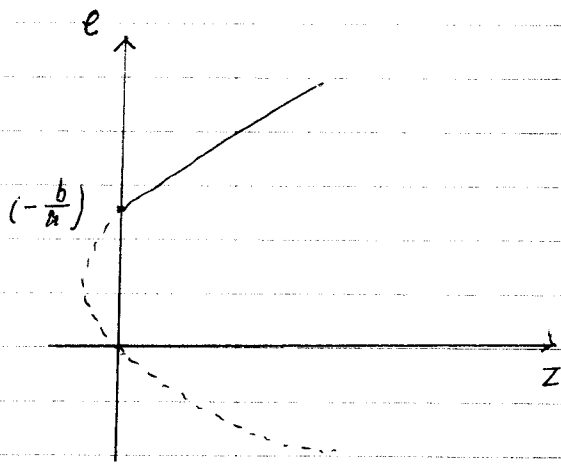
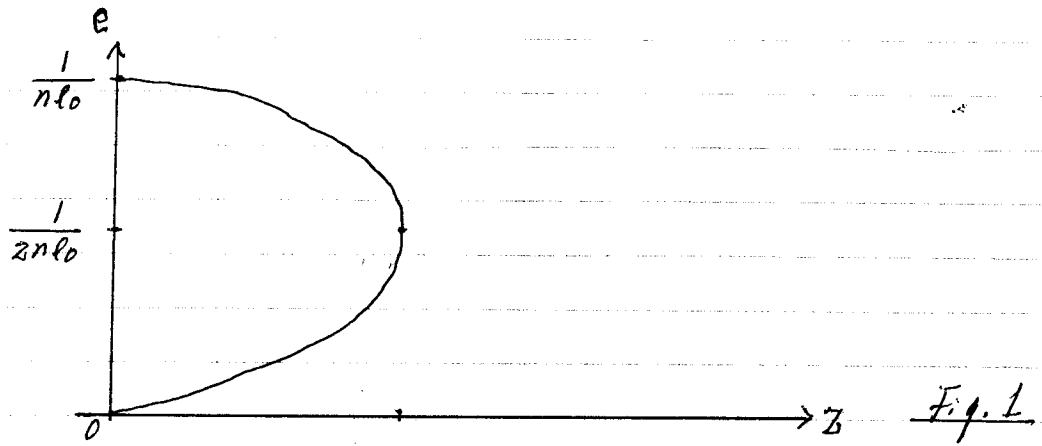
Since $g(\lambda)$ is the fourth-order polynomial, Remarks 3 and 4 ensure that D has 4 distinct real Eigen values: two of them are positive and the other 2 negative. We can choose, therefore, appropriate initial values $(q^*(0), x_0^*(0))$ for control variables (q, x_0) for any given initial values of state variables $(e(0), z(0))$ around the steady state. This shows the existence of a saddle path toward (e^*, z^*) when a small disturbance is made around the steady state.

Proposition 3. Assume that a steady state exists in the Public Input Economy. Then for a small displacement of state variables, there exists a unique saddle path starting with appropriate initial values of control variables (q, x_0) .

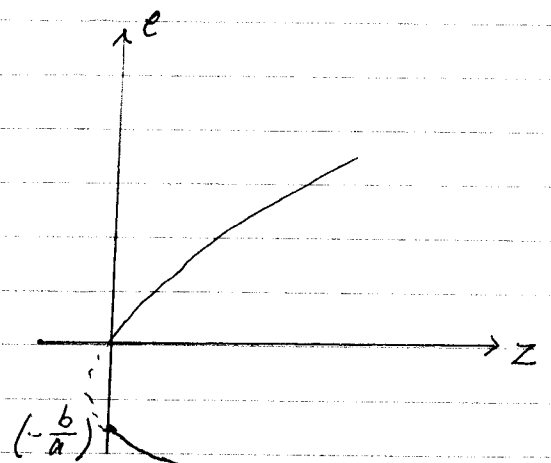
Conclusion. This paper has considered a 3-sector growth economy where knowledge brings about a Hicks neutral technological progress to the production of the consumption and capital goods. It is shown that the economy has a unique steady state and a saddle path exists for any small distance of capital stock and average level of knowledge around the steady state.

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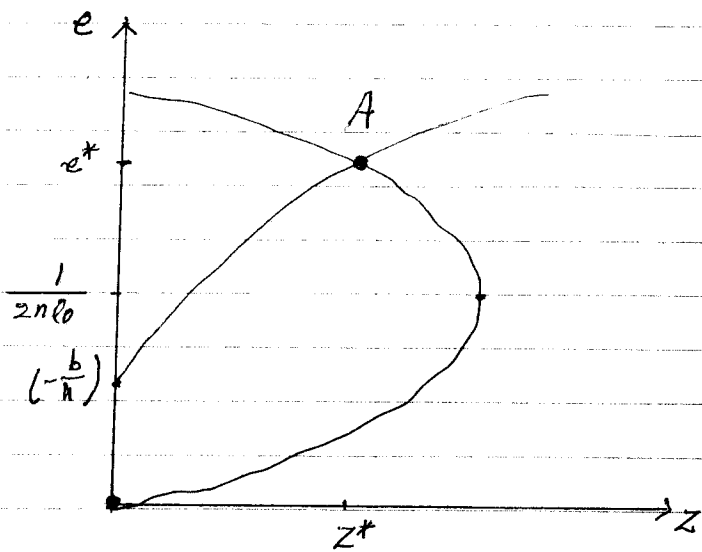
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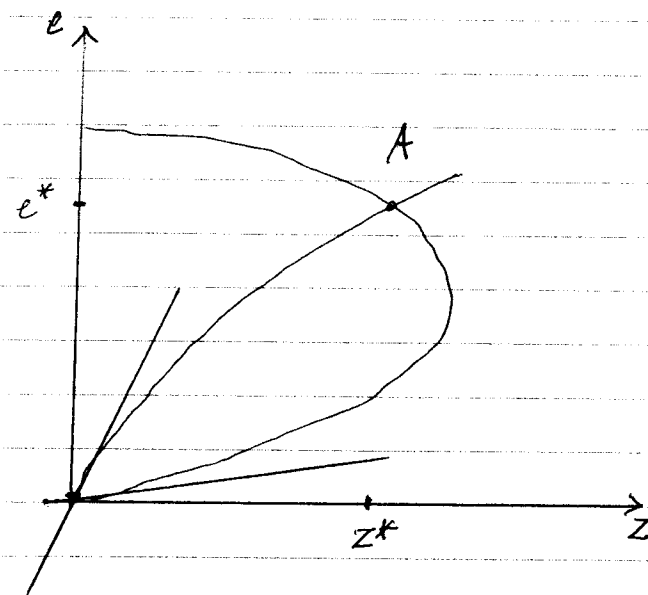
$b < 0$



$b > 0$



$b < 0$



$b > 0$