Optimal Sharing Strategies in Dynamic Games of Research and Development[1]

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September 2007

[1]We have benefited from conversations with John Conlon, Eser Kandogan, Suzanne Scotchmer, Ben Shneiderman, and members of the Research Division of IBM. Nisvan Erkal thanks the Faculty of Economics and Commerce, University of Melbourne for its financial support.

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Abstract

This paper analyses the dynamic aspects of knowledge sharing in R&D rivalry. In a model where research projects consist of multiple sequential stages, our goal is to explore how the innovators' incentives to share intermediate research outcomes change with progress and with their relative positions in an R&D race. We consider an uncertain research process, where progress implies a decrease in the level of uncertainty that a firm faces. We assume that firms are informed about the progress of their rivals and make joint sharing decisions either before or after each success. Changes in the firms' absolute and relative positions affect their incentives to stay in the race and the expected duration of monopoly profits if they finish the race first. We show that firms always prefer to have sharing between their independent research units if they are allowed to collude in the product market. However, competing firms may have either decreasing or increasing incentives to share intermediate research outcomes throughout the race. If the lagging firm never drops out, the incentives to share always decrease over time as the research project nears completion. The incentives to share are higher earlier on because sharing has a smaller impact on each firm's chance of being a monopolist at the end of the race. If the lagging firm is expected to drop out, the incentives to share may increase over time. These results have important implications for the design of optimal innovation policy.

1 Introduction

The ability to create knowledge-based assets plays an increasingly important role in determining firms' competitiveness in the market place. The goal of this paper is to analyze dynamic aspects of knowledge sharing in research and development (R&D) rivalry. Knowledge sharing is an important way in which firms can acquire the technological knowledge they need during their innovation process. Firms are likely to benefit from sharing knowledge with competitors. However, such alliances pose especially difficult challenges. This leaves us with the following question. When would we expect cooperation to emerge between competitors?

In the economics literature, knowledge sharing between rival firms has been the focus of many papers. These papers have mainly studied firms' incentives to share research outcomes at one point in time, either before the start of research, as in the case of research joint ventures, or after the development of a technology, as in the case of licensing.[1] In reality, the decision to share intermediate steps with rivals may be an integral part of a dynamic research process. Hence, the aim of this paper is to ask not whether but when firms prefer to share their research outcomes during a research process and what the emerging patterns of sharing activities are.

While sharing may cause researchers to benefit from each other's expertise and generate better ideas, it may also result in a reduction in the commercial value of their ideas. From a social welfare perspective, sharing of research outcomes is desirable because it results in less duplication. Hence, it is important to determine how close profit-driven firms come to maximizing welfare. In the economics literature, knowledge spillovers are stated as one of the most important reasons for rival firms to agree to share knowledge. However, from a dynamic perspective, another important aspect is uncertainty. The process of research is generally characterized by a high level of uncertainty in the beginning. Progress in research can be described as a decrease in the level of uncertainty that researchers face. Hence, one of the novel aspects of this project is to focus on the role uncertainty plays in the decisions to share knowledge and to analyze how firms' incentives to share research outcomes change during a

[1] See, for example, Kamien (1989) on licensing, and Katz (1986), D'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992) on research joint ventures. Patenting and informal sharing between employees of firms are two other methods through which knowledge may be disseminated between firms. See, for example, Scotchmer and Green (1990) on early innovators' incentives to patent and Severinov (2001) on informal sharing between employees.

research process as the level of uncertainty they face decreases.

We assume that research projects consist of several sequential steps. Researchers cannot proceed to the next step before successfully completing the prior step. Moreover, they cannot earn any profits before completing all steps of the project. In a dynamic R&D process, firms' incentives to share change as their positions in the race change for two reasons. First, the expected duration of monopoly profits for the leading firm depends on the progress the firms make during the research process. Second, the probability that any two firms will be rivals in the product market changes with progress.

An important feature of the model is that we assume the different steps of research are symmetric in all respects except in regards to how far away they are from the end of the project. In other words, the options and technology available to the firms are the same in all steps of the research process. We deliberately assume that there are no spillovers during the research process. It has been stressed in the literature that firms may have higher spillover rates and bigger appropriability problems in earlier stages of research than in later stages of research.[2] Although the rate of spillovers may shape the incentives to share, we show that it is not the only relevant factor. Assuming that there are no spillovers between the research efforts of different firms allows us to focus on the role uncertainty plays in knowledge sharing.

We assume that firms are informed about the progress of their rivals and make joint sharing decisions either before or after each success. While sharing may cause researchers to benefit from each other's expertise and help them avoid wasteful duplication of R&D, it may also result in a reduction in the commercial value of their ideas. Because sharing decreases the lead of one firm, it reduces the expected profits that the leader derives from finishing the race first and being a monopolist for some period of time. This cost is even greater if, but for the sharing, the lagging firm would drop out of the race.

Hence, the decision to share and the pattern of sharing activities critically depend on the lagging firm's incentives to stay in the race in case of no sharing. The results reveal that firms always prefer to have sharing between their independent research units if they are allowed to collude in the product market. Under rivalry, the incentives to share intermediate research outcomes decreases monotonically with progress if the lagging firm is expected never to drop

[2] See, for example, Katz (1986), Katz and Ordover (1990), and Vonortas (1994).

out. The incentives to share are higher earlier on because there is more uncertainty earlier on. Sharing has a smaller impact on each firm's chance of being a monopolist at the end of the race.

If the lagging firm is expected to drop out, the incentives to share may increase with progress. This is because earlier in the research process the lagging firm may have a higher incentive to drop out and, hence, the leading firm may have a higher chance of eliminating rivalry by not sharing. We also illustrate that the incentives to share increase as the gap between the firms decreases.

We next use our framework to analyze the impact of patent policy on firms' sharing decisions. The strength of patent policy can have an important impact on firms' sharing decisions because it determines the costs of inventing around patented technologies. We show that if a strengthening in patent policy causes a change in the investment decision of the lagging firm at any of the asymmetric histories, then sharing incentives in general get weaker. Otherwise, they generally get stronger.

In addition to contributing to the literature on knowledge sharing, this paper is also related to the literature on the management of innovation (Aghion and Tirole, 1994a and 1994b). The design of an optimal R&D strategy is a multi-faced problem. Two aspects of this problem, regarding the intensity of the R&D effort and the riskiness of the R&D projects chosen by firms, have been dealt with extensively in the literature.[3] In this paper we are interested in analyzing how the optimal strategies of firms change with progress. Other papers that have studied how firms' optimal strategies change over time in a dynamic model of R&D are Grossman and Shapiro (1986 and 1987), Cabral (2003) and Judd (2003). Grossman and Shapiro (1986 and 1987) analyze how firms vary their efforts over the course of a research project. In an infinite-period race, Cabral (2003) allow firms to choose between two research paths with different levels of riskiness. He shows that the leader chooses a safe technology and the laggard chooses a risky one. Judd (2003) shows that there is excessive risk-taking by innovators. Our paper differs from these papers because we analyze how firms' incentives to share and diversify change over the course of a research project.

[3]See Reinganum (1989) for a survey of the papers that focus on the intensity of firms' R&D efforts. Bhattacharya and Mookherjee (1986), Klette and de Meza (1986) and Dasgupta and Maskin (1987) analyze the riskiness of the research projects chosen by firms. Cardon and Sasaki (1998) analyze whether firms prefer to work on similar or different R&D paths.

The paper proceeds as follows. In the next section, we describe our set-up. In Section 3, we explore what happens if the firms are allowed to collude in the product market. In Section 4, we analyze the effect of competition on the dynamic sharing incentives of firms in a model with ex-post sharing contracts and 2 research steps. We consider the case of N research steps in Section 5. In Sections 6 and 7, we discuss extensions of our basic model with ex-ante sharing contracts and asymmetric firms respectively. After discussing the impact of patent policy on sharing incentives in Section 8, we conclude in Section 9.

2 Model

2.1 Research Environment

Since we are interested in the effect of competition on firms' incentives to share, we consider an environment with two firms, $i = 1, 2$, that each invest in a research project. On completion of the project, a firm can produce output in a product market. We assume the firms produce goods that may be either homogeneous or differentiated, and that they compete as duopolists in the product market.[4]

To capture the idea of progress, we assume that a research project has N distinct steps of equal difficulty. Hence, we assume that the firms divide the research project into different steps and that each firm defines the steps in the same way. A firm cannot start to work on the next step before completing the prior step, and all steps of the project need to be completed successfully before a firm can produce output. There is no difference between the steps in terms of the technology or the options available to the firms.

In the literature on multi-stage research, the phases of research are often thought of as qualitatively different. For example, there may be two steps identified as "research" and "development." We do not make this distinction, but rather we seek to derive endogenous differences in the research phases that result from dynamics in the decisions made by the firms. A basic intuition is that as firms approach the end of the research process, their decisions might increasingly reflect the impending rivalry.

We assume that each firm operates an independent research facility. We model research

[4] We assume that the firms conduct the research to solve the same technical problem. However, unmodelled differences in production technologies can still lead them to produce differentiated products.

activity using a Poisson discovery process. Time is continuous, and the firms share a common discount rate r. To conduct research, a firm must incur a flow cost c per unit of time.[5] Investment provides a stochastic time of success that is exponentially distributed with hazard rate α. This implies that at each instant of time, the probability that the firm completes a step is α. After completing a step, a firm can immediately begin research on the next step. The successes of the two firms are statistically independent. To represent the progress made by the firms, we use the notation of a research history (k_1, k_2), where k_i stands for the number of successes of firm i.

At each point in time prior to completing the project, a firm decides whether or not to invest. A decision not to invest is assumed to be irreversible and equivalent to dropping out of the game.[6] Each firm is risk neutral and makes decisions to maximize its discounted expected continuation payoff given the strategy of the other firm. The payoff structures are more fully described below. A firm that drops out earns a continuation payoff of zero. Given the memorylessness nature of the Poisson process, if a firm is conducting research, it will not stop unless there is a change in its relative position in the research process. If the rival completes one of the steps successfully, the firm may decide to drop out of the game at this point. We implicitly assume that when one firm develops a step successfully, it does not result in any technological spillovers. The successful firm can either keep the innovation a secret or patent it. Patenting does not prevent the rival from developing a non-infringing technology that serves the same purpose.

We will also allow firms to share their research. If one firm has completed one (or more) steps that the other firm has not, the leading firm can share its research with the lagging firm. After sharing, both firms can proceed to the next research step.[7] The timing of sharing decisions and the contracts that govern the sharing process are described below.

[5] We do not allow the firms to choose continuous levels of research effort in our basic model. This assumption can be motivated by presuming a fixed amount of effort that each firm can exert, which is determined by the capacity of its R&D division. As an example, Khanna and Iansiti (1997) explain that given the highly specialized nature of the R&D involved in designing state-of-the-art mainframe computers, firms in this industry find it very expensive to increase their number of researchers available to them. We relax this assumption later on and consider the case of continuous effort levels.

[6] Later, we may relax this assumption so that the decision not to invest can be reversed. We do not think that the qualitative nature of our results will change.

[7] Sharing in our model has the same impact as patenting in Scotchmer and Green (1990). In both models, the lagging firm can proceed to the next step after disclosure.

Regarding the information structure, we make the following assumptions. The firms will be able to share their research successes, but one firm cannot acquire the rival's innovation without such sharing. For example, a firm cannot observe the technical content of the rival's research without explicit sharing.[8] Everything else in the game is common knowledge. In particular, firms observe whether their rival is conducting research as well as whether the rival has a success. Third parties such as courts also observe this information.

We next consider the product market competition and the sharing process before explaining how the firms' payoffs are represented.

2.2 Product Market Competition

After a firm completes all stages of the research process, it can participate in the product market. The firms produce goods that may either be homogeneous or differentiated to some degree by unmodelled differences in the production technologies. We represent the product market competition in the following reduced form way.

If both firms have completed the research project, they compete as duopolists and each earns a flow profit of $\pi^D \geq 0$ forever. If only one firm has completed the research project, the firm earns a monopoly flow profit of $\pi^M > 0$ as long as the other firm does not produce output. Here, $\pi^M > \pi^D$. As a benchmark, we will consider the case that the firms make production decisions to maximize their joint profits in the product market. This results in a joint flow profit of π^J where $\pi^J \geq 2\pi^D$ and $\pi^J \geq \pi^M$. We use the notation $\widetilde{\pi}^D = \frac{\pi^D}{r}$, $\widetilde{\pi}^M = \frac{\pi^M}{r}$, and $\widetilde{\pi}^J = \frac{\pi^J}{r}$.

These payoffs are sufficiently flexible to capture various models of product competition. For example, if the firms produce homogeneous products and compete as Bertrand or Cournot competitors, then $\pi^J = \pi^M > 2\pi^D$. If the firms produce differentiated products, then $\pi^J > \pi^M$ and the relationship between π^M and $2\pi^D$ will depend on the degree of product differentiation that exists between the products. For low levels of product differentiation, $\pi^M > 2\pi^D$; for high levels of product differentiation, $\pi^M \leq 2\pi^D$.[9]

[8] Alternatively, we could assume that successful firms win immediate patents. A leading firm could then prevent a lagging firm from copying its research by enforcing its patent. If both firms complete the same step, they win non-infringing patents.

[9] The magnitudes of each of the profits π^D, π^J and π^M do not depend on the decisions taken during the research phase. In future research, we may relax this assumption.

As an example, consider a demand function of the type $q_i = (a(1-\gamma) - p_i + \gamma p_j) / (1-\gamma^2)$, where $0 < \gamma < 1$ so that the products are substitutes.[10] The goods are more differentiated the higher is γ. It is possible to show that $\pi^M \leq 2\pi^D$ if and only if γ is sufficiently large.[11]

From now on, we consider the case that there are $N = 2$ steps in the innovation process. In Section 5, we consider how our results extend to the case of an N-step innovation process.

2.3 Sharing of Research Outcomes

There are potential efficiencies in our model for firms to cooperate in the research stage. Suppose that one firm successfully completes a stage of research before the other firm does. We assume that the successful firm can costlessly share this knowledge with the other firm, thereby saving the lagging firm from having to continue to invest to complete the stage. From the point of view of social efficiency, such sharing will always be efficient because it prevents resources being spent to duplicate research results.

Because of the efficiencies of sharing, regulators in many countries encourage sharing arrangements, especially in the early stages of research. Firms may use a variety of contractual arrangements to govern the sharing process. There may be some legal restrictions, however, that prohibit sharing contracts that would inhibit competition in the output market. We want to consider firms incentives to share research using legal contracts. To this end, we want to classify contracts as either legal or illegal and limit our attention to legal contracts. However, even in our relatively simple dynamic framework, there are many contracts that might be written and it is not always obvious which ones we might want to classify as anti-competitive. Our first approach will be to consider two types of sharing contracts that are commonly observed in practice and have also been analyzed elsewhere in the literature. Later, we may consider a wider family of contracts.

The main sharing contract we consider is *ex post sharing* or licensing, where the leading firm shares its results with the lagging firm in exchange for a fixed fee. Sharing will occur whenever the joint profits of the two firms are higher with sharing than without sharing. We do not place any restrictions on the fee, but we assume that the successful firm (the leader)

[10] Singh and Vives (1984) show how these demand functions derive from particular consumer preferences.

[11] The Hotelling models provide other examples of differentiated duopoly that can correspond to these profits.

makes a take-it-or-leave-it offer to the other firm (the follower).[12] The leader, therefore, has all of the bargaining power in the negotiation and will offer a fee that leaves the follower just indifferent between accepting and rejecting.[13]

Because the research project has 2 steps, there are six histories at which one firm has more knowledge than the other. These are the histories $(1,0)$, $(0,1)$, $(2,0)$, $(0,2)$, $(2,1)$ and $(1,2)$. We assume that if sharing occurs, it physically occurs instantly once one of these histories is reached. Given the memoryless nature of the Poisson process, this assumption is not very restrictive. We also assume that when a leading firm is more than one step ahead of the lagging firm, all the additional steps are shared, so that the lagging firm catches up to the leading firm. This is a simplifying assumption.

We also briefly consider a second type of sharing contract, *ex ante sharing.* We assume that at the histories $(0,0)$ and $(1,1)$, the firms can make a joint decision about investing in the next research step and agree that once the step is completed, both firms will have access to the knowledge.[14] The sharing agreement allows for contingent payments between the firms when the step is completed and the physical sharing of knowledge occurs. Knowledge sharing arrangements of this nature are often referred to as research joint ventures (RJV). Formally, we assume that the research technologies are not affected by the agreement. This means there are no synergies between the firms in the research process.[15] Rather, the RJV is an agreement that allows both firms to have access to a success achieved by either one. Hence, it creates the opportunity for the firms to avoid wasteful duplication of R&D results. Alternatively, it allows the firms to agree to have one of the two facilities shut down altogether.[16]

[12] Below, we demonstrate explicitly how sharing works.

[13] The division of bargaining power here is appealing because it insures that each firm earns the full return to its research effort. Other divisions of bargaining power might also be considered. An existing literature considers how other divisions of bargaining power in licensing arrangements may affect the research incentives. See, for example, Katz and Shapiro (1985 and 1986), Shapiro (1985), Green and Scotchmer (1990 and 1995), Aghion and Tirole (1994), and D'Aspremont, Bhattacharya and Gerard-Varet (2000).

[14] In histories where the firms do not have the same number of successes, they can make a sharing agreement which involves both ex post and ex ante sharing. This is a common occurrence in RJVs, where the firms share their existing knowledge in an area in order to be able to work on the same research question together.

[15] Analyzing the collaborative R&D agreements of alliances and consortia registered under the National Cooperative Research Act in the US, Majewski (2004) shows that when the participants are direct competitors, they are likely to avoid spillovers.

[16] Such an asymmetric shut-down decision will never be optimal in this model, whether or not the shut-down decision is reversible.

2.4 Histories and Payoff Structures

To describe the game at any point in time, we need to specify how many firms are still active in the game, how many successes each active firm has, and whether there has been sharing. We use the following notation. Let (k_1, k_2) denote a research history where k_i is the number of steps that firm i has completed. The histories can be partially ordered so that (k_1, k_2) is *earlier* than (k_1', k_2') if and only if $k_i \leq k_i'$ for $i = 1, 2$, with strict inequality for at least one firm. In the following analysis, we refer to histories where $k_1 = k_2$ as symmetric histories and to those where $k_1 \neq k_2$ as asymmetric histories.

If a firm has dropped out of the game, we use X to denote this in the history. For example, to represent the history where firm 1 is working on the second step and firm 2 has dropped out of the game, we use $(1, X)$.[17]

Finally, to complete the description of the histories, we need to incorporate the sharing decisions into our notation for the history of the game. At symmetric histories, where there is no possibility of ex post sharing, we continue to use the notation (k, k) to denote that each firm has k successes. At asymmetric histories, we need to indicate whether the firms have made a sharing decision. At the instant that a firm achieves a success, we denote the history as (k_1, k_2) with $k_1 \neq k_2$. At this point, the firms make a sharing decision.[18] If the firms share, the history becomes (k, k) where $k = \max\{k_1, k_2\}$. If the firms do not share, the history becomes (k_1, k_2, NS). For example, consider the history $(2, 1)$. If the firms share, then the history becomes $(2, 2)$. If the firms do not share, then the history becomes $(2, 1, NS)$. In a continuation game at (k_1, k_2, NS), the firms do not get another chance to share until the next innovation is achieved.

At any point during the research process, we denote the discounted expected continuation payoff of firm i starting at the history (k_1, k_2) by $V_i(k_1, k_2)$. This payoff is developed recursively from future continuation payoffs. Consider, for example, the continuation payoff of firm 1 at the history $(1, 0, NS)$, $V_1(1, 0, NS)$. Suppose there will not be any sharing between the firms

[17] We do not extend the partial ordering to histories where a firm has dropped out. This is because we will only need to refer to the ordering at histories where both firms are still active in the game.

[18] We assume that the sharing decision takes place in the same instant of time as the success, but we are using separate notation to capture the history before and after the sharing decision. The history (k_1, k_2) precedes the history (k_1, k_2, NS) in the partial ordering of histories. The two histories have the same ordering relative to all other histories in the game.

at any future history and the lagging firm will always choose to invest. If firm 1 develops the second step and firm 2 continues to invest after firm 1 develops the second step, then firm 1's continuation payoff is $V_1(2,0,NS)$. If firm 2 develops the first step before firm 1 develops the second step and both firms stay in the game, then firm 1's continuation payoff is $V_1(1,1)$. Hence, we have

$$\begin{aligned} V_1(1,0,NS) &= \int_0^{\infty} e^{-(2\alpha+r)} \left(\alpha V_1(2,0,NS) + \alpha V_1(1,1) - c\right) dt \\ &= \frac{\alpha V_1(2,0,NS) + \alpha V_1(1,1) - c}{2\alpha + r} \end{aligned}$$

where the payoffs $V_1(2,0,NS)$ and $V_1(1,1)$ are similarly developed from future continuation payoffs.

After a firm has finished the research process, it earns continuation profits in the output market. To see how the payoffs are constructed, suppose firm 1 is the leading firm and firm 2 continues to research the second step. If there is no possibility of sharing, then we are at the history $(2,1,NS)$. Firm 1 can produce output as a monopolist. In each instant of time, firm 2 has a probability α of success. As soon as firm 2 is successful, firm 1 starts to earn duopoly profits forever after. Hence, we have

$$\begin{aligned} V_1(2,1,NS) &= \int_0^{\infty} e^{-(\alpha+r)} \left(\pi^M + \alpha \frac{\pi^D}{r}\right) dt \\ &= \frac{\pi^M + \alpha \frac{\pi^D}{r}}{\alpha + r}. \end{aligned}$$

Next, consider an example involving sharing. At the history $(2,1)$, the firms could either share to reach $(2,2)$ or not share to reach $(2,1,NS)$. The firms share if and only if their joint profits rise. That is, they share iff $V_J(2,2) - V_J(2,1,NS) > 0$. If they share, then the leading firm has all of the bargaining power. The firms arrive at a deal that leaves the lagging firm indifferent between sharing and no sharing. That is:

$$\begin{aligned} V_2(2,1,S) &= V_2(2,1,NS) \\ V_1(2,1,S) &= V_J(2,2) - V_2(2,1,NS) \end{aligned}$$

To achieve these payoffs, the lagging firm pays a fee F to the leading firm equal to his private

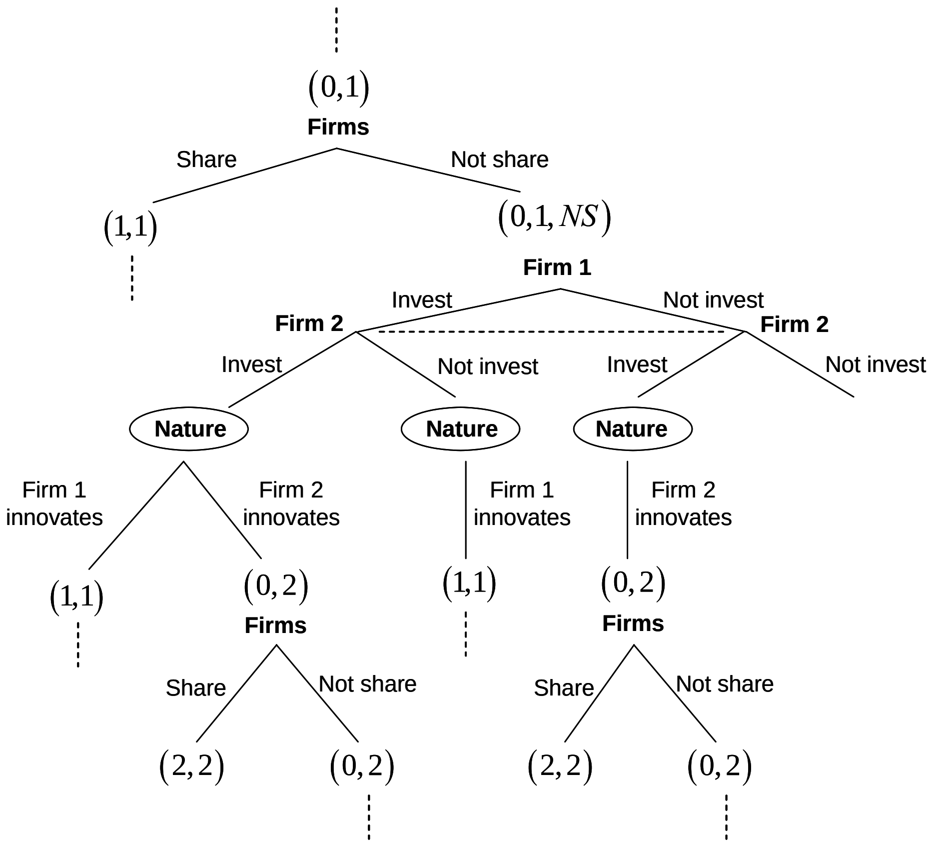

Figure 1: Game Tree following History (0,1)

benefit of sharing:

$$F = V_2(2,2) - V_2(2,1,NS)$$

The explicit payoffs and the fee F are easily computed.[19]

The payoffs at other histories are developed similarly using recursion.

2.5 Example of Game Structure

To clarify the timing of decisions in the research phases of the game, consider the following illustration. As in Scotchmer and Green (1990), we use a discrete game tree as a stylized representation of the underlying continuous time model. We assume that firms share using the

[19] The lagging firm earns $V_2(2,1,S) = V_2(2,1,NS) = \frac{\alpha\frac{\pi^D}{r} - c}{\alpha + r}$ where we derive $V_2(2,1,NS)$ in a similar fashion to how we derived $V_1(2,1,NS)$ above. It is easy to see that $V_J(2,2) = 2V_2(2,2) = 2\frac{\pi^D}{r}$. Using these, we can compute F and $V_1(2,1,S)$.

ex post licensing arrangements discussed above.[20]

At the beginning of the game, both firms simultaneously decide whether to invest or not. As shown in the subgame depicted in Figure 1, after one of the firms has made a discovery, the firms can jointly decide whether to share the winner's discovery. As mentioned before, sharing brings them to a symmetric position in the R&D phase. If they decide not to share, the laggard decides whether to continue to invest in order to develop the first innovation and the leader decides whether to invest to develop the second innovation. If they decide to share, they both simultaneously decide whether to invest in order to develop the second innovation.

If the laggard (re)develops the first innovation before the leader develops the second innovation, both firms start to invest to develop the second innovation. If the leader develops the second innovation before the laggard develops the first innovation, the firms again decide whether to share, this time both of the innovations. If they agree to share, they start to compete in the product market. If they agree not to share, the leader starts to produce while the laggard decides whether to continue to invest.

3 Benchmark Analysis

Before solving the game, we consider a benchmark case where the firms cooperate to maximize their joint profits. We assume that the firms make all investment, sharing, and product market decisions jointly.

At each history prior to the product market, the firms jointly decide whether to share the results of their research (if one firm is ahead) and whether one or both firms will invest.[21] Once one firm completes the research project, the firms make joint decisions about the product market. We do not make any assumptions about how the firms divide the joint profits, but we simply assume that each decision is made to maximize the sum of the continuation profits of the two firms.[22] We have the following result.[23]

[20] Scotchmer and Green (1990) include a rigorous justification for this representation based on a dominant strategy argument. That argument needs to be modified for our model in part because unlike them, we model the investment decision as irreversible. However, the basic idea is the same.

[21] Throughout the paper, we assume that if a firm stops researching before completion of the project, it cannot reenter the game at a later date. This means that investment decisions are also participation decisions. We make the assumption for simplicity, but consider the consequences of relaxing it later.

[22] We do not consider the possibility that the firms can commit to decisions prior to making them, but there is no dynamic inconsistency in the joint profit maximization problem.

[23] The proposition is proved in the appendix below.

Proposition 1 *Suppose that the firms maximize their joint continuation profits. Then, at any history such that one firm has more research successes than the other, the optimal sharing decision is for the leading firm to share its research with the lagging firm. Given this sharing pattern, the firms make investment decisions only at the symmetric histories* $(0,0)$ *and* $(1,1)$*. At the history* $(2,2)$*, the firms cooperate in the product market and earn joint continuation profits* $\widetilde{\pi}^J = \frac{\pi^J}{r}$*.*

(i) If $\pi^J \geq \frac{2cr}{\alpha} + \frac{cr^2}{2\alpha^2}$*, the optimal investment decision is for both firms to invest at the histories* $(0,0)$ *and* $(1,1)$*. At* $(0,0)$*, the joint continuation profits are* $\frac{4\alpha^2}{r(2\alpha+r)^2}\pi^J - \frac{2(4\alpha+r)}{(2\alpha+r)^2}c$*.*

(ii) If $\pi^J < \frac{2cr}{\alpha} + \frac{cr^2}{2\alpha^2}$*, neither firm invests at* $(0,0)$ *regardless of whether the firms would subsequently invest at* $(1,1)$*. At* $(0,0)$*, the joint continuation profits are* 0*.*

Proposition 1 reveals several features of our model. First, it is jointly optimal for the firms to share a success as soon as it is developed by either one of them and to move on to the next stage of the R&D process. This reflects the traditional justification for sharing arrangements as a way for firms to avoid wasteful duplication. Second, if it is optimal for one firm to invest, it is optimal for both firms to invest. This is a feature of the Poisson discovery process that we are using. Indeed, with flow costs of investment, if there were N identical research facilities, then it would be optimal for all of them to conduct research simultaneously until one of the facilities achieves a success. This speeds up the time to innovation, and the benefits of the time savings outweigh the costs of running simultaneous facilities. Later, we discuss how our results might change if we used a model of the research process that does not have this feature.

The proposition illustrates how the cost and benefit parameters affect payoffs. In the region where firms invest, their joint continuation profits at the beginning of the game is increasing in α, the hazard rate for the Poisson discovery process. A higher α means that the research is likely to be successful sooner. The joint profits are also decreasing in the discount rate r and the flow cost of research c. Similar comparative statics results obtain for the continuation profits at other points in the game.

4 Ex-post Sharing

The first type of sharing contract we consider is *ex post sharing.* Suppose that one firm completes a research step ahead of the other firm. We consider a sharing contract where the leading firm shares its results with the lagging firm in exchange for a fixed fee. Such sharing takes place as long as it results in higher industry profits. Although a range of fees would typically be acceptable to both firms, as discussed in Section 2.3, we will assume that the leader has all the bargaining power and sets the licensing fee by making a take-it-or-leave-it offer to the follower.

For each specification of the basic parameters of the game, we use backwards induction to find the subgame perfect equilibria in pure strategies. We consider generic values of parameters.[24] In this section, we first present the general properties of the equilibrium. Then, we present a full characterization for all possible values of π^M and π^D using a diagram and discuss how the equilibrium outcome changes as the profit levels change.

The primary benefit of sharing is that it avoids the wasteful duplication of R&D. The primary cost is the effect on output market competition. Because sharing erodes the lead of one firm, it reduces the expected profits that the leader derives from finishing the race first and being a monopolist for some period of time. This cost is even greater if, but for the sharing, the lagging firm would drop out of the race.[25] The trade-off between monopoly and duopoly profits, has not been addressed at great length in the patent race literature because patent models usually assume a winner-take-all payoff structure.[26]

A central question is how the incentives to share change over time. Because each of the research steps is identical from a technology standpoint, a conclusion that sharing incentives must change over time is not obvious. Certainly, if one firm is ahead of another, that may impact each firm's individual choices. However, if we consider the histories $(1, 0)$ and $(2, 1)$, it is not obvious that the sharing incentives should be any different. In both cases, the leader is one step ahead of the follower. Sharing is socially efficient in both cases and generates the same savings in terms of the elimination of wasteful R&D. The history $(1, 0)$ is, however, earlier than the history $(2, 1)$. At the earlier history, there is more uncertainty to be resolved before the firms enter the product market. We will consider how this uncertainty affects firms decisions.

[24] In our proof, we divide the space of parameters into regions such that the equilibrium set is constant on each region. We do not consider parameters on the boundaries of the regions. For these parameters, there can be multiple equilibria that exist only on the boundary and not for a generic set of parameters.

[25] Scotchmer and Green (1990) examine the effect of secrecy on the drop-out decision of the firm.

[26] Katz (1986) considers a model with one stage of research such that firms first engage in cooperative and independent R&D and then compete in an oligopolist output market. Also, see Cardon and Sasaki (1998) and Severnov (2001) for two more recent models of innovation where the firms compete as differentiated duopolists.

The number of steps that the lagging firm is behind is also a factor in firms' sharing decisions. We control for this, however, by comparing histories such that the lagging firm is a fixed number of steps behind the leading firm. This implies that in an N-step research process, we compare sharing incentives at all histories $(k+g, k)$ where g is a fixed gap between the leading firm and the lagging firm. The size of the gap can be as small as 1 or as large as $N-1$. When $N=2$, we compare the sharing decisions at $(2,1)$ and $(1,0)$ where the leading firm is one step ahead of the lagging firm.

From a dynamic perspective, what matters is whether one history precedes another. To consider dynamics, we compare histories $(k+g, k)$ and $(k'+g, k')$. If $k < k'$, then the history $(k+g, k)$ precedes the history $(k'+g, k')$. For example, when $N=2$, the history $(1,0)$ precedes the history $(2,1)$.

The next definition states a formal monotonicity property for the general N-step model. When the property holds, sharing incentives may be said to decline over time as the firms approach the end of the game. We define the property for histories such that firm 1 is the leader. Because the equilibria in our game are symmetric, when the property holds, it also holds for histories such that firm 2 is the leader.

Definition 1 *An equilibrium satisfies the monotonicity property if whenever the firms share at the history $(k+g, k)$, then they also share at the earlier history $(k'+g, k')$ where $k' < k$. Here k and k' range from 0 to $N-g$ and $g = 1, ..., N-1$.*

We are now in a position to analyze the model when $N=2$. Our central question is whether the monotonicity property holds. When $N=2$, the monotonicity property states that whenever firms share at the history $(2,1)$, they also share at the earlier history $(1,0)$. There are three sharing patterns that satisfy this property. In the first pattern, the firms share at both $(1,0)$ and $(2,1)$. In the second pattern, the firms do not share at either $(1,0)$ or $(2,1)$. These patterns are weakly monotonic. In the third pattern, the firms share at $(1,0)$, but they do not share at $(2,1)$. This pattern is strongly monotonic. The monotonicity property fails if the firms do not share at $(1,0)$, but do share at $(2,1)$.

There are two principle motivations for firms to decide against sharing. First, if the lagging firm continues to research, it will take longer to finish the project allowing a longer expected period of monopoly profits for the leading firm. Second, if the lagging firm exits the game, the leader can expect to earn monopoly profits forever upon finishing. It turns out that these two motivations can lead to different dynamics over time. To see this, we first consider environments such that exit does not occur.

Definition 2 *Region A consists of those parameter values such that in every subgame perfect equilibrium of the game, firms do not exit at any history either on the equilibrium path or off the equilibrium path. Region B consists of all other parameter values.*

Region A is given as follows:

Lemma 1 *Region A consists of all parameters such that $\pi^D \geq \frac{cr}{\alpha}(2 + \frac{r}{\alpha})$.*

Lemma 1 focuses on a firm that is as far behind the leader as possible when the leader has not shared its research. Because the lagging firm does not have any bargaining power, its payoff at $(2, 0, NS)$ or $(0, 2, NS)$ is the payoff it would get by conducting the two steps of research on its own and then producing in the output market as a duopolist. Intuitively, this is the worst possible position for a firm. We show that the lagging firm stays in at these histories if and only if the inequality $\pi^D \geq \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right)$ holds. We also show that when the inequality holds, the firms stay in the game at all other histories.

The next proposition records our main monotonicity result.

Proposition 2 *The monotonicity property holds for every subgame perfect equilibrium in Region A.*

The proof of the proposition requires us to consider detailed equilibrium conditions.[27] However, there is an underlying intuition for the result that we explain here. The benefit of sharing is the savings of duplicated R&D costs for one step of research. This benefit does not change over time. In contrast, the cost of sharing changes over time. The cost of sharing is measured in terms of the effect of sharing on future profits in the product market. Because of the resolution of uncertainty, sharing is more costly at $(2, 1)$ than it is at $(1, 0)$. To see

[27] The formal derivation of all the equilibria is in a companion Appendix that is available on request. In the Appendix below, we derive the equilibrium in one region to illustrate the backward induction.

this, note that in Region A, since the lagging firm never exits the game, the firms will be competing as duopolists in the product market eventually. At $(2,1)$, the leading firm is done with the project and is assured of monopoly profits until the lagging firm catches up. If the firms share, then the leading firm foregoes these monopoly profits, as both firms immediately begin competing as duopolists. In contrast, consider the earlier history $(1,0)$. After sharing, the new history is $(1,1)$. The leader has not foregone all of his chance to earn monopoly profits, because he can still finish the game first. In addition, at $(1,0)$,the leader had no guarantee of monopoly profits anyway. The expectation of future monopoly profits is not so much lower after sharing than if the firms had not shared. Thus, sharing is less costly at $(1,0)$ than at $(2,1)$. In contrast, the benefits of sharing in terms of R&D cost savings do not change over time. The net effect is that firms are more likely to share earlier in the game. As we prove in Proposition 2, the incentive to share is always stronger at $(1,0)$ than at $(2,1)$. The intuition seems robust, so that we expect the monotonicity result to hold more generally for games with N-step research projects. We develop a result along these lines in Section 5.

We next consider region B. In this region, a lagging firm may exit the game if the leader does not share at some history. This introduces an important strategic motive for a leading firm to refuse to share. Our question is whether, in light of this, the pattern of sharing continues to satisfy the monotonicity property. We find that this is not the case. A lagging firm may be more likely to drop out earlier in the game, when it has more research left to complete. Given this, a leading firm may be less likely to share earlier in the game.

Proposition 3 *When $\frac{r}{\alpha} < 2$, there exist parameter values such that a subgame perfect equilibrium does not satisfy the monotonicity property. When $\frac{r}{\alpha} > 2$, the monotonicity property holds for every subgame perfect equilibrium in Region B.*

The monotonicity result of Proposition 2 extends to Region B, provided that $\frac{r}{\alpha} > 2$. A no sharing decision is never followed by a sharing decision. However, the monotonicity result cannot be further extended.

As we demonstrate in the proof of the proposition,[28] when $\frac{r}{\alpha} < 2$, there exist values of π^D

[28] The formal derivation of all the equilibria is in a companion appendix that is available on request. In the appendix below, we derive the equilibrium in one region to illustrate the backward induction. This example demonstrates a non-monotonicity on the equilibrium path.

and π^M such that in equilibrium the firms do not share step 1 at $(1,0)$, but do share step 2 at $(2,1)$. The lagging firm drops out at the history $(1,0,NS)$, but stays in at the later history $(2,1,NS)$. The leader does not share at $(1,0)$ because this would maintain a rival that is otherwise eliminated. Since the rival drops out at $(1,0,NS)$, the history $(2,1)$ is not reached in equilibrium.[29] Hence, the non-monotonicity pattern is not observed along the equilibrium path. The parameter restriction means that the firms must be relatively patient and good at research for this type of equilibrium to exist. Sharing at $(1,0)$ allows both firms to work on step 2 and hastens the end of the research phase. The firms are willing to forego this benefit when $\frac{r}{\alpha}$ is not too large.

We also demonstrate an equilibrium in which a non-monotonic sharing pattern arises on the equilibrium path. In equilibrium, the lagging firm drops out at the history $(2,0,NS)$, but stays in at $(2,1,NS)$ and $(1,0,NS)$. The firms do not share step 1 at $(1,0)$, because this way they can reach the history $(2,0,NS)$, at which point the lagging firm drops out. A non-monotonic sharing pattern arises because after $(1,0,NS)$, the firms sometimes reach the history $(1,1)$. Both firms invest in step 2. The game may then proceed to the history $(2,1)$, at which point the leading firm shares step 2 with the lagging firm.

For this type of equilibrium to exist, we need to impose a stronger restriction on $\frac{r}{\alpha}$. As the proof shows, for $\frac{r}{\alpha} > \frac{1}{2}(\sqrt{5}-1)$, there are no non-monotonic sharing patterns along the equilibrium path. For $\frac{r}{\alpha} < \frac{1}{2}(\sqrt{5}-1)$, there exist values of π^D and π^M such that the non-monotonic sharing pattern arises on the equilibrium path.

Figure 2 depicts the equilibrium outcomes in the case when we see non-monotonic sharing patterns both on and off the equilibrium path.[30] This is the case that $\frac{r}{\alpha} < \frac{1}{2}(\sqrt{5}-1)$. The diagram lists the sharing pattern for each region. For example, at the top left of the diagram, the sharing pattern S,NS,NS describes the following sequence of decisions: i) at $(1,0)$, the leader shares (S) step 1; ii) at $(2,0)$, the leader does not share (NS) step 1; iii) at $(2,1)$, the leader does not share (NS) step 2. The diagram shows that there are two regions with the non-monotonic sharing pattern NS,NS,S. This pattern is non-monotonic because the leader does

[29] By symmetry, the follower also drops out at the history $(0,1,NS)$. Thus, there is no path to $(2,1)$.

[30] For these parameters, there are multiple equilibria at $(0,0)$ in some of the regions. In the regions, both firms can be in or both firms can be out at $(0,0)$. In the diagram, we selected the equilibrium such that both firms invest at $(0,0)$. Otherwise, the region such that neither firm invests at $(0,0)$ would be larger. A full description of all the equilibria including all multiplicities is available in the companion appendix to the paper.

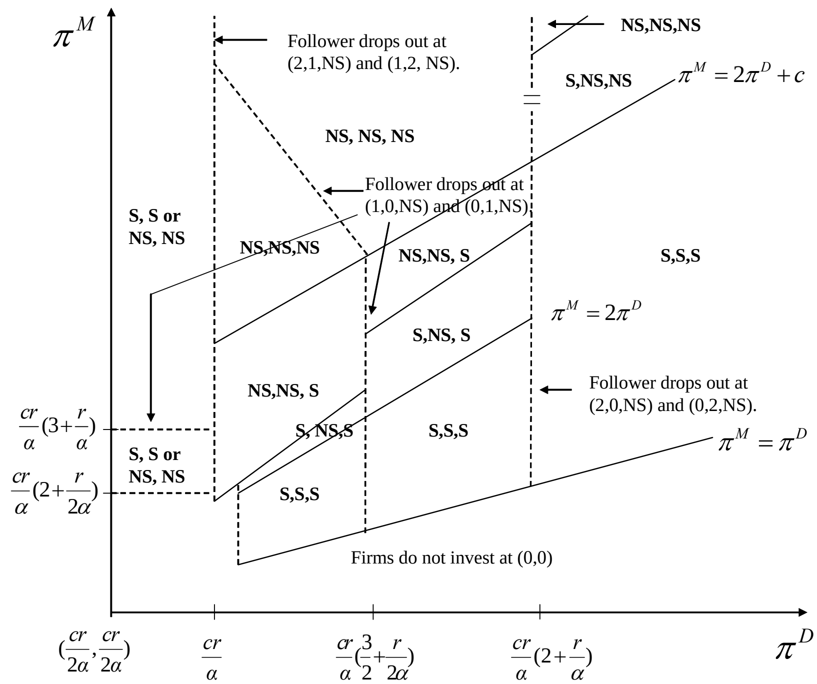

Figure 2: Equilibrium Outcomes: $\alpha = .5, r = .2, c = .5$

not share step 1 at $(1,0)$, but does share step 2 at $(2,1)$. The two regions with this sharing pattern are separated by a vertical line. In the region to the left of the line, the follower drops out at the history $(1,0,NS)$. Because of this, the history $(2,1)$ is not reached in the equilibrium.[31] Thus, an observer of the game would not observe a non-monotonicity. In the region to the right of the line, the follower stays in the game at the history $(1,0,NS)$. Because of this, the history $(2,1)$ is reached along the equilibrium path.

The diagram also shows the sharing patterns in other regions. Consider region A. Here, we have that $\pi^D \geq \frac{cr}{\alpha}(2+\frac{r}{\alpha})$. In Region A, the lagging firm never drops out of the game and the sharing patterns are monotonic. Of course, by Proposition 2, we already knew that this result must hold. Consider how the sharing pattern changes as π^M increases, but with the value of π^D held fixed. For small values of π^M, the sharing pattern is S,S,S. As monopoly profits increase, sharing breaks down at the history $(2,1)$.[32] As monopoly profits are increased further, sharing eventually breaks down at the earlier history $(1,0)$ as well. This is a monotonicity result for the comparative static analysis. As monopoly profits π^M increases, sharing breaks down, but it breaks down at later histories first.

Two regions in the diagram have a sharing pattern of S,NS,S. Here, the leading firm does not share step 1 at $(2,0)$, even though it does share step 1 at the earlier history $(1,0)$. We do not interpret this as a non-monotonicity result, because we only compare sharing decisions at histories where the gap between the leader and the follower is the same.

On the far left of the diagram, for parameters $\pi^D < \frac{cr}{\alpha}$, the leading firm is indifferent between sharing or not sharing at $(2,0)$. Either way, the lagging firm drops out of the race and the decision does not affect payoffs on the equilibrium path.

5 N-step Research Process

In this section, we discuss some results obtained in a model with N research steps of equal difficulty.

As a starting point, we consider our benchmark model where the firms cooperate to maximize their joint profits. The firms make all investment, sharing, and product market decisions

[31] By symmetry, the follower also drops out at the history $(0,1,NS)$. Thus, there is no path to $(2,1)$.

[32] Sharing also breaks down at the histories $(2,0)$ and $(0,2)$, but we do not have any other histories to compare these to with the same gap of 2 steps between the leader and the follower.

jointly. Proposition 1 extends in a straightforward way. At asymmetric histories, the optimal sharing decision is for the leading firm to share its research with the lagging firm. At the final history (N, N), the firms cooperate in the product market to earn joint continuation profits $\widetilde{\pi}^J$. If the joint continuation profits are above a critical threshold, then both firms invest at all earlier symmetric histories. Otherwise, at the start of the game, neither firm invests.

We next turn to our monotonicity result that sharing declines over time. Proposition 2 extends to a model with 3 research steps. We have

Proposition 4 *When $N = 3$, the monotonicity property holds for every subgame perfect equilibrium in Region A. Region A consists of all parameters such that $\pi^D \geq \frac{cr}{\alpha}(3 + 3\frac{r}{\alpha} + \frac{r^2}{\alpha^2})$.*

To prove the proposition, we derive the equilibria as we did for the case of $N = 2$.[33] Region A is the set of parameters such that the lagging firm would stay in the game at the history $(3, 0, NS)$. At this history, the lagging firm is as far behind as possible and has no hope of ever earning monopoly profits. Because the lagging firm does not have any bargaining power, its payoff at $(3, 0, NS)$ is the payoff it would get by conducting all three steps of research on its own and then producing in the output market as a duopolist. Region A is all parameters such that this payoff is positive.

The monotonicity property implies that if the firms share at the histories $(k + 1, k)$, then they share at $(k, k - 1)$ for $k = 1, 2$. At these histories, the leading firm is one step ahead of the lagging firm. The monotonicity property also implies that at the history $(3, 1)$ when the leading firm is two steps ahead, they share at the earlier history $(2, 0)$.

The monotonicity property cannot be strengthened to comparisons between histories such that the leading firm is ahead by a differing number of steps. For example, we find an equilibrium such that the firms share at $(2, 1)$, but do not share at the earlier history $(2, 0)$. The reason is that at $(2, 0)$, the leading firm is further ahead and has more to give up in terms of forgone monopoly profits.[34]

We expect that Proposition 2 could be extended further to a model with N research steps.

[33] The proof is available from the authors on request. The calculations are straightforward, but long. In the appendix below, we derive the parameter condition that defines Region A.

[34] When $N = 2$, we also found equilibria in Region B such that the firms share at $(2, 1)$, but not at $(2, 0)$. The sharing pattern arises because the lagging firm exits after the decision not to share. When $N = 3$, we find this sharing pattern in Region A, even though the lagging firm does not ever exit the game.

The intuition behind the proposition is general and does not depend on the assumption that $N = 2$. The benefit of sharing is the savings on R&D costs. These cost savings do not change over time. However, the cost of sharing, measured in terms of foregone monopoly profits, do change over time. The advantage to the leading firm to being a fixed number of steps ahead of the lagging firm increases over time as uncertainty is resolved. The net effect is that the firms have decreasing incentives to share as the game progresses.

We have not proved Proposition 2 for the general case of N research steps because the equilibrium calculations become too cumbersome. Instead, we analyzed a related problem that we interpret as a partial generalization of our monotonicity result. Consider any starting history $(k+1, k)$ in the N-step model such that the leading firm is one step ahead of the lagging firm. If the firms share at this history, then the new history becomes $(k+1, k+1)$. If they do not share, then the new history is $(k+1, k, NS)$. Assume that at all histories after $(k+1, k, NS)$ and $(k+1, k+1)$ the firms do not share and they also do not exit the game. Under this assumption, we can derive formulas representing the firms' joint continuation payoffs. We can compare the continuation payoffs from sharing and not sharing at $(k+1, k)$. The benefit of sharing (which is equal to the cost savings by the lagging firm) is an increasing (linear) function of the flow cost of research c. Because of this, there is a threshold cost $c(k+1, k)$ such that the firms decide to share if and only if $c \geq c(k+1, k)$. Using numerical analysis, we can show that the threshold cost $c(k+1, k)$ is increasing in k for $N \leq 20$.[35]

The finding suggests that sharing is more likely to occur at earlier histories. The cost parameter c is more likely to be above the sharing threshold cost $c(k+1, k)$ at earlier histories than at later ones. The result is different from Proposition 2 because the assumptions about firms' behavior after $(k+1, k)$ may not be consistent with any equilibrium. However, the result is consistent with the intuition that the incentives to share decline over time when firms never exit.

Our other result, Proposition 3, is that there are parameter values in Region B such that a subgame perfect equilibrium does not satisfy the monotonicity property. These non-monotonic equilibria continue to exist as subgames of the N-step model. This is because the subgame

[35] We compared the payoffs by evaluating them on a discrete grid of parameter values. The formulas appear to be sufficiently continuous that we do not expect we missed any singularities in our simulations. The computations are available on request.

that begins at the history $(N-2, N-2)$ is a two-stage game. Parameters that support the equilibrium are in Region B of the 2 step game. The parameters are also in Region B of the N-step game because Region B grows as N increases.[36] Thus, Proposition 3 continues to hold.[37]

6 Ex-ante Sharing

In this section, we consider a second type of sharing contract. We assume that at any (symmetric) history, the firms can make a joint decision about investing in the next research step and agree that once the step is completed, both firms will have access to the knowledge as in a RJV.

Consider a sharing contract that is signed at the beginning of the game. The history is $(0, 0)$. The firms both agree to conduct research on the first step. The firms also agree that when one firm has a success, it will share the success with the lagging firm. In exchange, the lagging firm will pay a fee to the leading firm. At the time the contract is signed, the fee is contingent - it is paid by the lagging firm to the leading firm at the instant of innovation. We assume that the fee is set so that the lagging firm is indifferent between paying the fee to get the result, and not paying the fee and not getting the result. This means that the leading firm extracts the full value of its success. Therefore, as was the case under our ex post sharing contracts, both firms have efficient incentives to invest ex ante.[38]

We consider a similar sharing contract at the history $(1, 1)$. We then analyze the sharing pattern to find whether the firms are more likely to share at $(0, 0)$ than at $(1, 1)$. Our analysis shows that there is no difference in terms of sharing incentives between the ex ante sharing contracts and the ex post sharing contracts. Because of this, the dynamics of sharing over time are essentially the same as before.

[36] Region A shrinks as N increases, because a lagging firm has a lower payoff from staying in the game at $(N, 0)$ than at $(N-1, 0)$. Region B grows as Region A shrinks, since they are complementary sets.

[37] If a firm drops out of the N-stage game prior to the history $(N-2, N-2)$, the continuation equilibrium would still exist but would represent off-the-equilibrium path behavior.

[38] The contract need not provide any direct incentives for investment and would induce efficient effort even if the effort levels were non-contractible.

7 Asymmetric Firms

So far we have assumed that the firms are symmetric in their research capabilities. Although this is not necessarily a realistic assumption, it helped us to focus on and isolate the impact of progress and uncertainty on the firms' sharing decisions. In this section, we relax the symmetry assumption by allowing the firms to have different research costs. We explore when the monotonicity property continues to hold in Region A, where the lagging firm invests at all histories.

Let the research costs of firm 1 and firm 2 be c_1 and $c_2 > c_1$, respectively. To start with, we assume that each firm has the same cost of research in both stages. Hence, there is symmetry between the research costs of the same firm across the different stages of research, but asymmetry between the research costs of different firms. This scenario allows us to explore the sharing dynamics between efficient and inefficient firms.

Cost asymmetry introduces potentially new type of dynamics since it implies that we can have cases when there is sharing at $(2,1)$ but not at $(1,2)$. This is because the sharing condition at $(2,1)$ is $2\pi^D + c_2 > \pi^M$ while the sharing condition at $(1,2)$ is $2\pi^D + c_1 > \pi^M$. Hence, the sharing condition in the last stage depends on the research cost of the lagging firm. Since $c_2 > c_1$, in cases when $2\pi^D + c_2 > \pi^M > 2\pi^D + c_1$, there will be sharing at $(2,1)$ but not at $(1,2)$.

Consider first the case when $2\pi^D + c_1 > \pi^M$, which implies that the firms decide to share at both $(2,1)$ and $(1,2)$. In this case, the result stated in Proposition 2 continues to hold.

In the case when $2\pi^D + c_2 > \pi^M > 2\pi^D + c_1$, one question of interest is whether the monotonicity property holds for the more efficient (inefficient) firm separately. That is, is it the case that if the more efficient (inefficient) firm shares at $(2,1)$ ($(1,2)$), it also shares at $(1,0)$ ($(0,1)$)? Interestingly, we find that if there will be sharing at $(2,1)$ but not at $(1,2)$, the firms share at $(1,0)$ but not necessarily at $(0,1)$. This is a natural extension of the monotonicity property stated in Definition 1 to the case of asymmetric firms.

Consider now how the results would change if the research costs of a firm were changing over time. Suppose, as above, that firm 1 has a cost of c_1 in both stages, but firm 2 faces a cost of c_2^1 in the first stage and a cost of c_2^2 in the second stage. Assuming the firms decide to share both at $(2,1)$ and at $(1,2)$, it is straightforward to show that the sharing condition at

(1, 0) is

$$2\alpha\pi^D > \alpha\left[c_2^2 - c_2^1 - c_1\right] + r\left[c_2^2 - c_2^1\right]. \quad (1)$$

Clearly, this condition holds if $c_2^2 < c_2^1$. Hence, if firm 2's costs are decreasing over time, the monotonicity result holds. This is not surprising since it implies that firm 2 becomes more efficient over time. If the firms prefer to share when firm 2 is more efficient, it makes sense for them to share when firm 2 is less efficient since the benefits from sharing are increasing in the research cost of the lagging firm.

If $c_2^2 > c_2^1$, there may not be sharing at $(1, 0)$ even though there is sharing at $(2, 1)$ depending on the difference between c_2^2 and c_2^1. Hence, the monotonicity result may be violated in this case. It is interesting to note that in this case, the sharing condition at $(1, 0)$ is less likely to hold as c_1 decreases.

8 Impact of Patent Policy

So far we have assumed that once a firm successfully develops a research step, it can either keep the technology secret or patent it. If there is patenting, the follower can develop a noninfringing technology that serves the same purpose, and continues to face the same research cost or hazard rate. In this section, we discuss the impact of patent policy on firms' sharing incentives assuming that patenting affects the follower's costs of doing research.

Since patenting forces the follower, if it decides to invest, to work around the patent of the leader, we assume that it causes an increase in the research cost of the follower. The increase in the rival's research cost may depend on how broad a patent protection the leader gets. Stronger patent policy (i.e., broader patent protection) may make it harder for rival firms to invent around (Gallini, 1992). Hence, it is feasible to think that patenting affects the rival's costs of doing research adversely and that the rival's research cost will be increasing in the breadth of the leader's patent.[39]

We assume that both steps of the research process are patentable (i.e., satisfy the patentability requirement). Hence, as soon as a firm successfully completes a stage, it gets a patent. We assume that the firms have symmetric costs c at the histories $(0, 0)$ and $(1, 1)$ when they are

[39] In reality, patenting of a technology can potentially affect rivals' costs in either direction. The patent may cause the rivals' costs to decrease if it makes it easier for them to develop an alternative technology.

working towards the same research step. After one of the firms completes the next stage, the follower cannot continue to work on the same research path because doing so would imply infringement. Hence, if it decides to stay in the game, it has two options. It can either make a licensing deal with the leader or switch to a more expensive research path and invest to complete the research process in a noninfringing way.[40]

This implies that the firms face asymmetric research costs at asymmetric histories. While the leader has a cost of c, the follower has a cost of c^P. Hence, while in Section 7 we have considered different types of firms, in this section we assume the firms are symmetric to start with, but they become asymmetric as the game progresses and the firms successfully develop the different research steps. We assume that patenting in both stages affects the research cost of the follower in the same way by increasing it from c to c^P.

We know from our analysis in Section 7 that the sharing condition at the histories $(2,1)$ and $(1,2)$ (as well as at the histories $(2,0)$ and $(0,2)$) is given by $2\pi^D + c^P > \pi^M$. Clearly, if $c^P > c$, a strengthening in patent policy (which can be interpreted as an increase in c^P), as long as it does not result in drop out, increases the benefits from sharing. This is because the policy affects the follower's outside option. If it has to incur higher research costs following the patenting decision of the leader, it will be willing to pay a higher licensing fee to the leader in exchange for the technology. Hence, with broader patent protection, the benefits from sharing will be higher because the firms can save on higher costs of research.

If the sharing condition at $(2,1)$ and $(1,2)$ is satisfied, the sharing condition at $(1,0)$ and $(0,1)$ is given by

$$2\alpha\pi^D > cr - c^P\left(2\alpha + r\right),$$

which clearly holds for $c^P \geqslant c$. Hence, the monotonicity result in Region A, where the follower always decides to continue to invest, is not violated if patenting affects the follower's research cost adversely.

If the sharing condition at $(2,1)$ and $(1,2)$ is not satisfied, the sharing condition at $(1,0)$ and $(0,1)$ is given by

$$2\alpha^2\left(3\alpha + 2r\right)\pi^D + \pi^M\alpha\left(r^2 - 2\alpha^2\right) > cr\left(\alpha + r\right)^2 - c^P\left(4\alpha^3 + 5\alpha^2 r + 3\alpha r^2 + r^3\right),$$

[40] Hence, we assume that there are different research paths the firms can take to achieve the same research outcome and the different research paths correspond to different research costs. If one of the firms gets a patent, the follower has to switch to another research path to avoid infringement.

which implies that sharing becomes more attractive as c^P increases.

If the parameters are such that the increase in the follower's research cost causes it drop out, we are in Region B and the firms may not find it beneficial to share at $(1, 0)$ because the follower will drop out when the game reaches $(2, 1)$ or $(2, 0)$. Hence, the incentives to share at $(1, 0)$ and $(0, 1)$ may decrease as c^P increases.

9 Conclusion

The paper considers the optimal pattern of knowledge sharing in the context of technological competition. Developing a theoretical foundation for optimal sharing strategies has important implications for the design of optimal as well as efficient research environments.

We have analyzed how the incentives to share change over time as a research project reaches maturity. The decision to share and the pattern of sharing activities critically depend on the lagging firm's incentives to stay in the race in case of no sharing. The results reveal under rivalry, the incentives to share intermediate research outcomes decreases monotonically with progress if the lagging firm is expected never to drop out. The incentives to share are higher earlier on because there is more uncertainty earlier on. Sharing has a smaller impact on each firm's chance of being a monopolist at the end of the race.

In many models of R&D, there is an assumption that firms share at an early research stage but not at a later one. This result shows that this sharing pattern can be derived from the optimizing behavior of firms in a dynamic game where the research technology does not change over time.

If the lagging firm is expected to drop out, the incentives to share may increase with progress. This is because earlier in the research process the lagging firm may have a higher incentive to drop out and, hence, the leading firm may have a higher chance of eliminating rivalry by not sharing.

Our results have important implications in terms of policy-making in innovative environments. Since 1980s, government both in the US and in Europe actively promote joint R&D projects through subsidies, tolerant antitrust treatment, or government-industry partnerships.[41] Our framework can be used to illustrate under what circumstances such policy is

[41] For example, in the US, the National Cooperative Research and Production Act (NCRPA) of 1993 provides

necessary and whether it should be directed towards early stage or later stage research. The monotonicity result stated in Proposition 2 implies that in industries where duopoly profits are relatively high, it may be more important to emphasize sharing in later stages of research. Proposition 3 implies that in industries where duopoly profits are relatively low, it may be important to encourage sharing in early stages of research. Considering at which stage of the research process the firms may have lower incentives to share is important especially if it is necessary or more effective to use different policy approaches to encourage joint research at different stages.

that research and production joint ventures be subject to a rule of reason analysis instead of a per se prohibition in antitrust litigation. In the EU, the Commission Regulation (EC) No 2659/2000 (the EU Regulation) provides for a block exemption from antitrust laws for RJVs, provided that they satisfy certain market share restrictions and allow all joint venture participants to access the outcomes of the research.

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Appendix

A Proof of Proposition 1

We solve the model under the assumption that the two firms maximize their joint payoffs. We derive continuation profits at each history working backwards through the decision nodes.

At $(2,2)$, the firms cooperate in the output market to earn the joint flow profit π^J forever. Recall that $\pi^J \geq \max\{2\pi^D, \pi^M\}$ so that profits in the output market are greatest when the firms produce cooperatively. The joint continuation profits are

$$V_J(2,2) = \frac{\pi^J}{r} = \widetilde{\pi}^J.$$

At the histories[42] $(2,1)$ and $(2,0)$, the leading firm shares all available research with the lagging firm. This prevents the wasteful duplication of R&D. The firms then cooperate in the product market to earn joint continuation profits of $\widetilde{\pi}^J$. Thus, we have that

$$V_J(2,1) = V_J(2,0) = V_J(2,2) = \widetilde{\pi}^J.$$

At the history $(1,1)$, if neither firm invests, the joint continuation profits are 0. If one firm invests (either firm), then the firm invests a flow cost of c and in each instant the probability of success is α. When the success arrives, the firms share the research and cooperate in the product market to earn flow profits of π^J. At $(1,1)$, the joint continuation profits are

$$V_J(1,1) = \int_0^\infty e^{-(\alpha+r)}\left(\alpha\widetilde{\pi}^J - c\right)dt = \frac{\alpha\widetilde{\pi}^J - c}{\alpha + r}.$$

If both firms invest, then each firm incurs a flow cost of c and the flow probability that at least one firm succeeds is 2α.The joint continuation profits are

$$V_J(1,1) = \int_0^\infty e^{-(2\alpha+r)}\left(2\alpha\widetilde{\pi}^J - 2c\right)dt = \frac{2\alpha\widetilde{\pi}^J - 2c}{2\alpha + r}. \quad (2)$$

Given these payoffs, the firms will either both invest or both not invest. The firms invest if and only if $V^J(1,1) \geq 0$. This occurs[43] if and only if $\widetilde{\pi}^J \geq \frac{c}{\alpha}$.

[42] The histories $(1,2)$ and $(0,2)$ are analyzed in the same way as $(2,1)$ and $(2,0)$. We do not repeat the analysis here.

[43] For simplicity (but with some abuse of notation), we ignore non-generic parameters such that some firm is indifferent between two actions, as would be the case here if $\widetilde{\pi}^J = \frac{c}{\alpha}$.

Working backwards, we reach the history $(1,0)$. As at $(2,0)$ and $(2,1)$, sharing eliminates wasteful duplication of R&D. Because the firms make decisions cooperatively, there is no cost to them to sharing. Sharing either strictly increases their joint continuation profits or has no effect on the profits because the firms are in any event exiting the race. Without loss of generality, we will assume that the firms share at $(1,0)$.

Finally, we consider the history $(0,0)$. If neither firm invests, their joint continuation profits are 0. If both firms invest, then their joint continuation profits are

$$V_J(0,0) = \int_0^\infty e^{-(2\alpha+r)} (2\alpha V_J(1,1) - 2c)\, dt = \frac{2\alpha V_J(1,1) - 2c}{2\alpha + r}.$$

The continuation profits depend on whether the firms invest at $(1,1)$. If the firms do not invest at $(1,1)$, then they clearly will not invest at $(0,0)$. If the firms invest at $(1,1)$, then they invest at $(0,0)$ if and only if $V_J(0,0) \geq 0$. This is the case if and only if $V_J(1,1) \geq \frac{c}{\alpha}$. Using the expression for $V_J(1,1)$ above, we find that the firms invest at $(0,0)$ if and only if $\pi^J \geq \frac{cr}{\alpha}$ and

$$\pi^J \geq \frac{2cr}{\alpha} + \frac{cr^2}{2\alpha^2}.$$

The last inequality above implies the inequality $\pi^J \geq \frac{cr}{\alpha}$. Thus, this inequality is a necessary and sufficient condition for both firms to invest at $(0,0)$. If the firms invest, then their joint continuation profits are

$$V_J(0,0) = \frac{2\alpha V_J(1,1) - 2c}{2\alpha + r} = \frac{4\alpha^2 \pi^J}{r(2\alpha+r)^2} - \frac{2(4\alpha+r)c}{(2\alpha+r)^2}.$$

B Proof of Lemma 1

To prove the lemma, we must show that Region A consists of all parameters such that $\pi^D \geq \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right)$. In a companion Appendix that is available on request, we analyze all the equilibria of the game. That analysis also proves the lemma. Here we take a different approach. We focus on the payoff that a firm would earn by conducting two steps of research on its own and then producing in the output market as a duopolist. This payoff is necessarily a lower bound on any firm's payoff at any history and in any equilibrium. This is because a firm always has an option to complete two steps of research on its own to earn duopoly profits or greater in the output market. We claim that in Region A, the payoff of the lagging firm at $(2,0,NS)$ equals this payoff. To see this, consider the decisions of the lagging firm beginning at $(2,0,NS)$. By the definition of Region A, the firm does not drop out of the game at $(2,0,NS)$. Instead, it completes the first step of research *on its own* to arrive at $(2,1)$. The firms may or may not share step 2 at $(2,1)$. Either way, because the lagging firm has no bargaining power its payoff is the same as its payoff at $(2,1,NS)$. In Region A, the lagging firm does not drop out at $(2,1,NS)$. Instead, it completes the second step of research *on its own* to arrive at $(2,2)$. At $(2,2)$,the firm is a duopolist. This shows that the payoff of the lagging at $(2,0,NS)$ equals the payoff to a firm of conducting two steps of research on its own and then producing in the output market as a duopolist.

We finish the lemma by computing the payoff to a firm of conducting two steps of research and then producing in the output market as a duopolist. We work backwards through time to compute the payoff.

After completing the two steps of research, the firm produces output as a duopolist to earn $\widetilde{\pi}^D = \frac{\pi^D}{r}$. Working backwards, suppose the firm has completed one step of research. To complete the second step of research, the firm invests a flow cost of c and in each instant the probability of success is α. The firm's expected payoff is

$$\int_0^\infty e^{-(\alpha+r)}\left(\alpha\widetilde{\pi}^D - c\right)dt = \frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}.$$

Again, working backwards, consider the first step of research. The firm again invests a flow cost of c and in each instant the probability of success is α. The firm's expected payoff is

$$\int_0^\infty e^{-(\alpha+r)}\left[\alpha\left(\frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}\right) - c\right]dt = \frac{\alpha\left(\frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}\right) - c}{\alpha+r}.$$

This payoff is strictly positive if and only if

$$\pi^D > \frac{cr}{\alpha}\left(2+\frac{r}{\alpha}\right), \tag{3}$$

which is the inequality that defines Region A.

C Proof of Proposition 2

To prove that the monotonicity property holds in Region A, we consider the cases when there is sharing at $(2,1)$ and $(1,2)$, and analyze whether this implies sharing at $(1,0)$ and $(0,1)$. For symmetric histories such as $(2,1)$ and $(1,2)$, we analyze only one of the histories as the analysis is the same for both.

At $(2,2)$, each firm produces output and earns discounted duopoly profits of

$$V_1(2,2) = V_2(2,2) = \frac{\pi^D}{r} = \widetilde{\pi}^D. \tag{4}$$

The firms share at $(2,1)$ iff this maximizes their joint profits. Their joint profits under sharing are

$$V_J(2,2) = V_1(2,2) + V_2(2,2) = 2\widetilde{\pi}^D$$

since when the firms share, the game reaches the history $(2,2)$. Joint profits under no sharing are

$$V_J(2,1,NS) = V_1(2,1,NS) + V_2(2,1,NS) = \frac{\pi^M + 2\alpha\widetilde{\pi}^D - c}{\alpha + r}, \tag{5}$$

where

$$V_1(2,1,NS) = \frac{\pi^M + \alpha V_1(2,2)}{\alpha + r} = \frac{\pi^M + \alpha\widetilde{\pi}^D}{\alpha + r}$$

since firm 1 earns monopoly profits until firm 2 completes the second step and

$$V_2(2,1,NS) = \frac{\alpha V_2(2,2) - c}{\alpha + r} = \frac{\alpha\widetilde{\pi}^D - c}{\alpha + r} \tag{6}$$

since the lagging firm by definition does not drop out in Region A.

We get $S \succ NS \iff$

$$2\widetilde{\pi}^D(\alpha + r) > \pi^M + 2\alpha\widetilde{\pi}^D - c$$

or

$$2\pi^D + c > \pi^M. \tag{7}$$

This condition may or may not hold in Region A. From now on, we assume that it holds and that the firms share step 2 at $(2,1)$.[44] When the firms share, the lagging firm pays the leading firm a licensing fee equal to its entire benefit from sharing. This fee, $F(2,1)$, can be easily computed using (4) and (6).

$$F(2,1) = V_2(2,2) - V_2(2,1,NS) = \frac{\pi^D + c}{\alpha + r}. \tag{8}$$

[44] In the companion Appendix to the paper, we solve the game region by region. Region A contains subregions that we label as Region 1,2, and 3. Region 1 is the subregion where the firms share at $(2,1)$.

Before considering the sharing decision at $(1,0)$, we need to see whether the firms share step 1 at $(2,0)$. At $(2,0)$, joint profits under sharing are $V_J(2,2) = 2\widetilde{\pi}^D$ since if the firms share, the game reaches the history $(2,1)$ and we know from condition (7) that at this history the firms share. Joint profits under no sharing are

$$\begin{aligned} V_J(2,0,NS) &= V_1(2,0,NS) + V_2(2,0,NS) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c}{\alpha + r} \\ &= \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}. \end{aligned}$$

Since the firms share at $(2,1)$, $V_J(2,1) = 2\widetilde{\pi}^D$. Substituting we get $S \succ NS \iff$

$$\begin{aligned} 2\widetilde{\pi}^D(\alpha + r) &> \pi^M + 2\alpha\widetilde{\pi}^D - c \\ 2\pi^D + c &> \pi^M. \end{aligned}$$

This is condition (7), which we have assumed to hold. Hence, the firms share step 1 at $(2,0)$.

At $(1,0)$, if the firms share, the game reaches the history $(1,1)$. Hence, joint profits under sharing are $V_J(1,1)$. Joint profits under no sharing are

$$V_J(1,0,NS) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D + \alpha V_J(1,1) - 2c}{2\alpha + r}.$$

We have $S \succ NS \iff$

$$(2\alpha + r) V_J(1,1) > 2\alpha\widetilde{\pi}^D + \alpha V_J(1,1) - 2c. \tag{9}$$

Note that $V_J(1,1) = 2V_1(1,1) = 2V_2(1,1)$ and

$$V_2(1,1) = \frac{\alpha V_2(1,2) + \alpha V_2(2,1) - c}{2\alpha + r} = \frac{\alpha V_J(2,1) - c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D - c}{2\alpha + r}, \tag{10}$$

where the last equality follows from the fact that the firms share at $(2,1)$. Substituting for $V_J(1,1)$ in (9) we get

$$\begin{aligned} (\alpha + r)\left(4\alpha\widetilde{\pi}^D - 2c\right) &> (2\alpha + r)\left(2\alpha\widetilde{\pi}^D - 2c\right) \\ \pi^D &> -c, \end{aligned}$$

which is trivially true. Hence, the firms share step 1 at $(1,0)$. Since the firms share both at $(1,0)$ and at $(2,1)$, the monotonicity property holds and we are done.

When the firms share, the lagging firm pays the leading firm a license fee equal to its entire benefit from sharing. That is, $F(1,0) = V_2(1,1) - V_2(1,0,NS)$. We can substitute for $V_2(1,1)$ from (10). $V_2(1,0,NS)$ is given by

$$V_2(1,0,NS) = \frac{\alpha V_2(1,1) + \alpha V_2(2,0) - c}{2\alpha + r}. \tag{11}$$

Since the lagging firm is assumed to have no bargaining power, its profit at $(2,0)$ is equal to

$$V_2(2,0) = V_2(2,0,NS) = \frac{\alpha^2 \tilde{\pi}^D - c(2\alpha + r)}{(\alpha + r)^2}$$

even though the firms share at $(2,0)$. Hence, $F(1,0)$ simplifies to

$$F(1,0) = \left(\frac{\pi^D + c}{\alpha + r}\right)\left(\frac{5 + 6\frac{r}{\alpha} + 2(\frac{r}{\alpha})^2}{4 + 8\frac{r}{\alpha} + 5(\frac{r}{\alpha})^2 + (\frac{r}{\alpha})^3}\right).$$[45]

In the Appendix that is a companion to this paper, we complete the analysis for Region A by showing that the investment conditions at $(2,1,NS)$, $(2,0,NS)$, $(1,0,NS)$, $(1,1)$, $(1,X)$, $(0,0)$, and $(0,X)$ hold as long as condition (3), which defines Region A, holds. We also derive the equilibria in Region B (i.e., when condition (3) fails).

D Proof of Proposition 3

We solve for the equilibria of the game for all parameter values in the companion Appendix to this paper that is available on request. That analysis proves Proposition 3. Figure 2 illustrates the equilibria for an example with non-monotonicities in some regions. For readers who do not wish to read the companion Appendix, we show how to derive one of the equilibria below. The equilibria for other parameter values are solved similarly.

E Derivation of a Non-Monotonic Equilibrium

We solve the game in the following region of parameters: $\frac{cr}{\alpha}\left(\frac{3}{2} + \frac{r}{2\alpha}\right) < \pi^D < \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right)$ and $2\pi^D\left(\frac{2\alpha+2r}{2\alpha+r}\right) + c\left(\frac{2r}{2\alpha+r}\right) < \pi^M < 2\pi^D + c$. This is a subregion of Region B. A straightforward calculation shows that the region is non-empty if and only if $\frac{r}{\alpha} < \frac{1}{2}\left(\sqrt{5} - 1\right)$ where

[45] Comparing the fees $F(1,0)$ and $F(2,1)$, we find that $F(2,1) > F(1,0)$ iff $\frac{r}{\alpha}$ is above a cut-off of approximately $\frac{r}{\alpha} \cong 0.325$.

$\frac{1}{2}\left(\sqrt{5}-1\right) \simeq 0.62$. The equilibrium is also derived in the companion Appendix to this paper, where the region is labeled as Region 6.

To find an equilibrium, we work backwards from the end of the game. We derive the continuation profits at each history and solve for the equilibrium actions. For symmetric histories such as $(2,1)$ and $(1,2)$, we analyze only one of the histories as the analysis is the same for both.

The last history is the history $(2,2)$. At this history, the firms have two successes each and are done with the research. They produce output and each earns discounted duopoly profits of $V_i(2,2) = \frac{\pi^D}{r} = \widetilde{\pi}^D$.

Working backwards, the next history is $(2,1,NS)$, where firm 1 is finished with its research and produces output. Firm 2 has 1 success and the firms have decided not to share. Firm 2 decides whether or not to invest in step 2. If firm 2 invests, its continuation profit is

$$V_2\left(2,1,NS\right) = \frac{\alpha\widetilde{\pi}^D - c}{\alpha + r}. \tag{12}$$

This payoff is positive because by assumption $\pi^D > \frac{cr}{\alpha}$. Hence, firm 2 invests at $(2,1,NS)$.

To see whether the firms share step 2 at $(2,1)$, we compare joint profits under sharing with joint profits under no sharing. Since the analysis is the same as the one in Section (C), we do not repeat here. The firms share at $(2,1)$ iff (7) holds. This condition holds in this region and the firms share step 2 at $(2,1)$.

At the history $(1,1)$, each firm has one success. There is no sharing decision to be made. The firms must, however, decide whether to invest to develop the second step. Assuming firm 1 invests, firm 2 will also invest if

$$V_2\left(1,1\right) = \frac{\alpha V_2\left(2,1\right) + \alpha V_2\left(1,2\right) - c}{2\alpha + r} = \frac{\alpha V_J\left(2,1\right) - c}{2\alpha + r} > 0.$$

Since the firms share at $(2,1)$, $V_J\left(2,1\right) = 2\widetilde{\pi}^D$. Substituting we get

$$V_2\left(1,1\right) = \frac{2\alpha\widetilde{\pi}^D - c}{2\alpha + r} > 0, \tag{13}$$

which simplifies to $\pi^D > \frac{cr}{2\alpha}$. Since this condition holds in the region, firm 2 invests. Hence, each firm invests at $(1,1)$ if the other does.

If firm 1 does not invest at $(1,1)$, the new history is $(X,1)$. Firm 2 invests if

$$V_2\left(X,1\right) = \frac{\alpha V_2\left(X,2\right) - c}{\alpha + r} = \frac{\alpha\widetilde{\pi}^M - c}{\alpha + r} > 0, \tag{14}$$

where $V_2(X,2) = \widetilde{\pi}^M$ because at $(X,2)$, firm 2 produces output as a monopolist. The condition simplifies to $\pi^M > \frac{cr}{\alpha}$, which holds because $\pi^M > \pi^D$and in this region $\pi^D > \frac{cr}{\alpha}$. Hence, firm 2 invests at $(X,1)$. It follows that both firms invest at $(1,1)$.

At the history $(2,0,NS)$, firm 1 produces output. The firms have decided not to share. Firm 2 invests iff

$$V_2(2,0,NS) = \frac{\alpha V_2(2,1) - c}{\alpha + r} > 0.$$

Since the lagging firm has no bargaining power, its earnings under sharing are the same as its earnings under no sharing at the history $(2,1)$. The earnings under no sharing, $V_2(2,1,NS)$,are given in (12). Substituting and rearranging gives us

$$V_2(2,0,NS) = \frac{\alpha^2 \widetilde{\pi}^D - c(2\alpha + r)}{(\alpha + r)^2} > 0$$

or

$$\pi^D > \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right).$$

This condition fails in the region, so firm 2 drops out at $(2,0,NS)$.[46]

To see whether the firms share step 1 at $(2,0)$, we compare the joint profits under sharing with joint profits under no sharing. Joint profits under sharing are $V_J(2,1) = 2\widetilde{\pi}^D$ since if the firms share, the game reaches the history $(2,1)$ and the firms share step 2. Joint profits under no sharing are $V_J(2,0,NS) = V_1(2,X) = \widetilde{\pi}^M$ since firm 2 drops out of the game if the firms do not share. In this region, we have that $\pi^M > 2\pi^D$. Hence, the firms do not share at $(2,0)$. The lagging firm then drops out of the game.

Working backwards from either $(2,0)$ or $(1,1)$, we next consider the history $(1,0,NS)$. At this history, firm 1 has one success and firm 2 has no successes and the firms have decided not to share. Each firm must decide whether to invest. If firm 1 invests, firm 2 also invests if

$$V_2(1,0,NS) = \frac{\alpha V_2(1,1) + \alpha V_2(2,0) - c}{2\alpha + r} > 0 \qquad (15)$$

We can substitute for $V_2(1,1)$ from (13). Moreover, $V_2(2,0) = 0$ since the firms do not share at $(2,0)$ and the lagging firm drops out. After substituting and simplifying, (15) becomes

$$\pi^D > \frac{cr}{\alpha}\left(\frac{3}{2} + \frac{r}{2\alpha}\right).$$

[46] This result also follows from Lemma 1.

This holds in the region, so the lagging firm 2 invests at $(1,0,NS)$ if firm 1 does. It is straightforward to show that the leading firm 1 invests at $(1,0,NS)$ if firm 2 invests. If firm 2 does not invest, the history becomes $(1,X)$ and the leading firm invests as shown above. It follows that the leading firm invests at $(1,0,NS)$ whether or not the lagging firm invests. Thus, both firms invest at $(1,0,NS)$.

To see whether the firms share step 1 at $(1,0)$, we compare joint profits under sharing with joint profits under no sharing. If the firms share, the game reaches the history $(1,1)$. Hence, joint profits are $V_J(1,1)$. Joint profits under no sharing are

$$V_J\left(1,0,NS\right)=\frac{\alpha V_J\left(2,0\right)+\alpha V_J\left(1,1\right)-2c}{2\alpha+r}=\frac{\alpha\widetilde{\pi}^M+\alpha V_J\left(1,1\right)-2c}{2\alpha+r}. \tag{16}$$

We have $NS\succ S\Longleftrightarrow$

$$\alpha\widetilde{\pi}^M+\alpha V_J\left(1,1\right)-2c>\left(2\alpha+r\right)V_J\left(1,1\right)$$

Substituting for $V_J(1,1)=2V_2\left(1,1\right)$ from (13) and simplifying, we have

$$\pi^M>2\pi^D\left(\frac{2\alpha+2r}{2\alpha+r}\right)+c\left(\frac{2r}{2\alpha+r}\right).$$

This inequality holds in the region, so the firms do not share at $(1,0)$.

At the history $(0,0)$, assuming firm 2 invests, firm 1 will also invest if

$$V_1\left(0,0\right)=\frac{\alpha V_1\left(1,0,NS\right)+\alpha V_1\left(0,1,NS\right)-c}{2\alpha+r}=\frac{\alpha V_J\left(1,0,NS\right)-c}{2\alpha+r}>0$$

Substituting from (16) and (13), we get

$$4\alpha\pi^D+\left(2\alpha+r\right)\pi^M>\frac{cr}{\alpha^2}\left(4\alpha+r\right)\left(2\alpha+r\right)+2cr.$$

Since $\pi^M>2\pi^D$ in this region, the condition holds if

$$\left(8\alpha+2r\right)\pi^D>\left(4\alpha+r\right)\left(2\alpha+r\right)\frac{cr}{\alpha^2}+2cr.$$

Since $\pi^D>\frac{cr}{\alpha}\left(\frac{3}{2}+\frac{r}{2\alpha}\right)$ in this region, the condition holds if

$$\left(8\alpha+2r\right)\left(\frac{3}{2}+\frac{r}{2\alpha}\right)\frac{cr}{\alpha}>\left(4\alpha+r\right)\left(2\alpha+r\right)\frac{cr}{\alpha^2}+2cr.$$

This simplifies to $2\alpha\left(2\alpha+r\right)>0$, which always holds. Hence, firm 1 invests at $(0,0)$ if firm 2 invests.

Assuming firm 2 does not invest, the history becomes $(0, X)$. Firm 1 invests if

$$V_2(0, X) = \frac{\alpha V_2(1, X) - c}{\alpha + r} = \frac{\alpha\left(\frac{\alpha\tilde{\pi}^M - c}{\alpha + r}\right) - c}{\alpha + r} > 0,$$

where we substituted for $V_2(1, X)$ from (14). Simplifying we get

$$\pi^M > \frac{cr}{\alpha}(2 + \frac{r}{\alpha}). \tag{17}$$

In this region, we have that

$$\pi^M > 2\pi^D \text{ and } \pi^D > \frac{cr}{\alpha}(\frac{3}{2} + \frac{r}{2\alpha}).$$

These two conditions together imply that (17) holds. Hence, firm 1 invests at $(0, X)$. It follows that both firms invest at $(0, 0)$.

This completes the derivation of the equilibrium. The equilibrium is unique. The equilibrium is non-monotonic because the firms share at $(2, 1)$ but not at $(1, 0)$. The histories $(1, 0)$ and $(2, 1)$ are both reached on the equilibrium path, so the non-monotonicity arises on the equilibrium path.

F Proof of Proposition 4

A derivation of all the equilibria of the game in Region A for the case of $N = 3$ is available on request. That analysis proves the proposition. Here we derive the condition given in the proposition that defines Region A. As in Lemma 1, we focus on the payoff that a firm would earn by conducting three steps of research on its own and then producing in the output market as a duopolist. By the same reasoning as in Lemma 1, this payoff equals the payoff of the lagging firm at $(3, 0, NS)$ and is a lower bound on any firm's payoff at any history and in any equilibrium of the game. Therefore, Region A is defined as all those parameters for which this payoff is positive.

Suppose that the firm has completed the first step of research on its own. As derived in Lemma 1, the firm's expected payoff from computing the two remaining steps of research on its own is

$$\int_0^\infty e^{-(\alpha+r)}\left[\alpha\left(\frac{\alpha\tilde{\pi}^D - c}{\alpha + r}\right) - c\right] dt = \frac{\alpha\left(\frac{\alpha\tilde{\pi}^D - c}{\alpha + r}\right) - c}{\alpha + r}.$$

Working backwards, consider the first step of research. The firm invests a flow cost of c and has a hazard rate of α. The firm's expected payoff is

$$\int_0^{\infty} e^{-(\alpha+r)} \left[\alpha \left(\frac{\alpha \left(\frac{\alpha \tilde{\pi}^D - c}{\alpha + r} \right) - c}{\alpha + r} \right) - c \right] dt = \frac{\alpha \left(\frac{\alpha \left(\frac{\alpha \tilde{\pi}^D - c}{\alpha + r} \right) - c}{\alpha + r} \right) - c}{\alpha + r}.$$

This payoff is strictly positive if and only if

$$\pi^D \geq \frac{cr}{\alpha} \left(3 + 3\frac{r}{\alpha} + \frac{r^2}{\alpha^2} \right),$$

which is the inequality that defines Region A.