Tough Love and Intergenerational Altruism*

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Abstract

This paper develops and studies a tough love model of intergenerational altruism. We model tough love by modifying the Barro-Becker standard altruism model in two ways. First, the child's discount factor is endogenously determined, so that low consumption at young age leads to a higher discount factor later in his life. Second, the parent evaluates the child's life time utility with a constant high discount factor. In contrast to the predictions of the standard altruism model that transfers from parents are independent of exogenous changes in the child's discount factor, our tough love model predicts that transfers will fall.

I Introduction

How different generations are connected is an important economic issue that has implications for individual economic behavior like savings, investment in human and physical capital, and bequests which in turn affect aggregate savings and growth. It also has nontrivial policy implications as in Barro (1994), who has found that there will be no net wealth effect of a change in government debt in the standard altruism model. The infinite horizon dynamic macro models are typically based on the standard altruism model

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proposed by Barro (1974) and Becker (1974) in which the current generation derives utility form its own consumption and the utility level attainable by its descendant.

A striking implication of the standard altruism model is that when the child becomes impatient, transfers from the parent to the child do not change when the child is borrowing constrained [*see section II below*]. This implication of the model is not consistent with recent empirical evidence on pecuniary and non-pecuniary parental punishments (see Weinberg (2001), Hao, Hotz, and Jin (2008) for empirical evidence). For example, imagine that a child befriends a group of impatient children and suddenly becomes impatient because of their influences. Then the child starts to spend more time playing with the new friends and less time studying. In worse cases, the child starts to smoke, drink, or consume illegal drugs. At least some parents are likely to respond by pecuniary punishments such as lowering allowances or non-pecuniary punishments such as grounding. Bhatt(2008) used National Longitudinal Survey of Youth 1997 data on a cross section of children who were 12-16 years in 1997 and found that transfers received by children are negatively affected by the substance use by children.

This paper modifies the standard model so that it implies that the parent lowers transfers to the child when the child exogenously becomes impatient under a wide range of reasonable parameters. For this purpose, this paper develops and studies a *tough love* model of intergenerational altruism, in which the parent is purely altruistic to the child, but exhibits tough love: he cares about the long run welfare of the child and hence may allow the child to suffer in the short run.

We model parental tough love by combining the two ideas that have been studied in the literature in various contexts. First, the child's discount factor is endogenously determined, so that low consumption at young age leads to a higher discount factor later in his life. This is based on the endogenous discount factor models of Uzawa (1968) except that the change in the discount factor is immediate in Uzawa's formulation whereas spoilt child with high consumption grows to become impatient in our formulation. Second, the parent evaluates the child's life time utility function with a constant discount factor that is higher than that of the child. Since the parent is the social planner in our simple model, this feature is related to recent models (see Caplin and Leahy 2004; Sleet and Yeltekin 2005, 2007; Phelan 2006, and Farhi and Werning 2007) in which the discount factor of the social planner is higher than thats of the agents.

An argument for plausibility of endogenous discounting can be found in Becker and Mulligan (1997). They model an individual whose discount factor depends on the remoteness or vividness of their imagination of future pleasures. [1]

It is necessary to be careful in evaluating the empirical evidence for endogenous discounting because of two problems. First, we have the endogeneity problem in that patient people with high discounting factors tend to accumulate financial and human wealth. Thus we may find that rich people have higher discount factors than poor people even when the discount factor of an individual is decreasing in wealth as in Uzawa's (1968) model. Second, endogenous discounting and wealth-varying intertemporal elasticities of substitution (IES) (see Atkeson and Ogaki 1996) can have similar implications in growing economies, and may be hard to distinguish from one another.

The endogeneity problem mentioned above is addressed in Ikeda, Ohtake, and Tsutsui (2005). In this paper they found that without accounting for the possible endogeneity between discount factors and wealth the discount factor appears to be an increasing function of income/wealth. After they take the endogeneity problem in consideration, they find evidence in favor of the view that the discount factor is decreasing in wealth. [2]

Another way to control the endogeneity problem is to give different levels of consumption to the subjects before an experiment to see which subjects are more patient. Implementing this idea is very difficult in experiments with human subjects. Rats were used to implement this in experiments. The results were in favor of the view that the discount factor is decreasing in wealth as reported in Kagel, Battalio, Green (1995, Chapter 7, Section 3).

Using the Panel Study of Income Dynamics (PSID), Lawrance (1991) employed the Euler equation approach to estimate the endogenous discount factor model. In principle, her instrumental variable method should take care of the endogeneity problem. Lawrance found evidence in favor of the discount factor that is increasing in wealth. However, Ogaki and Atkeson (1997) point out that Lawrance did not allow the intertemporal elasticity of substitution (IES) to vary with wealth. Ogaki and Atkeson allow both the IES and the discount factor to vary with wealth for a panel data of households

[1] Becker and Mulligan's model involves investment to increase vividness of the imagination. We adopt Uzawa's (1968) model and do not study this investment aspect of endogenous discounting in this paper.

[2] They control the endogeneity problem by analyzing how the discount factor changes with the size of a prize obtained in another experiment.

in Indian villages. They find evidence in favor of the view that the discount factor is constant and that the IES is increasing in wealth. It is possible that the discounting factor is decreasing in wealth for richer households, but Lawrance found the opposite result because she did not allow the IES to change.

Turning to the plausibility of the parent using a higher discount factor than the child, an extreme case is a parent with a new born baby. When the baby is born, it is very impatient and it cries for food all the time but the parent does not give in to this persistent demand of the baby. This is likely because the parent evaluates the baby's utility over its life time with a higher discount factor as compared to the baby's very low discount factor.

We think that it is likely that many parents continue to evaluate their children's life time utility when they are no longer babies. Parents may continue to do this until children learn to be as patient as their parents.

In our model, these two features (endogenous discount factor of the child and the parent's evaluation with a high discount factor) lead the parent to show tough love behavior in which the parent takes into account of the influence of the amount of transfer of income to the child on the discount factor of the child.

A question to ask in the context of present discussion is that is there any empirical evidence for parents' behavior influencing their children's discount factors and other economic preferences and attitudes. While the economic literature on the effect of parenting style on child's economic attitudes and behavior is sparse, there is substantial work that addresses this issue in the Psychology literature. Diana Baumrind(1966) identifies three modes of parental control. The first mode is *permissive* where the parent acts as a resource to the child and does not actively involve himself in shaping the current as well as the future behavior of the child. The second mode is *Authoritarian* where the parent uses a set standard of conduct which is theologically or religiously motivated and tries to shape and control the child's behavior with overt use of power. The third mode is *Authoritative* where the parent actively involves himself in shaping up the child's behavior and attitudes and uses reasoning and discipline to ensure a well rounded long run development of the child. He affirms child's current behavior, separating right from wrong, and also set standards for child's future behavior.

There is substantial evidence in the Psychology literature in favor of the influence of parents in the development of children's willingness to delay rewards. Mischel (1961) studied children in West Indian islands of Grenada and

Trinidad. He found that the children of Grenada showed greater preference for a higher reward later than a smaller immediate reward when compared with the children of Trinidad. He also found that this difference is driven mainly by the critical role fathers played in handing down cultural values of thrift to the children of Grenada and those of immediate gratification to the children of Trinidad.

Bandura and Mischel (1965) conducted an experiment on 250 school going children to explore the effects of an adult's discounting preferences of these children. They found that children who previously were more keen to obtain immediate rewards now displayed increased willingness to wait for a more valued reward at a later date following their exposure to an adult exhibiting patience.

Carlson and Grossbart(1988) used survey data on the mothers of school going children(Kindergarten through sixth grade) and divided them into groups based on the parenting style starting from neglecting through to rigid controlling. They found evidence for authoritative parents granting less consumption autonomy to the child, greater communication with the child about consumption related issues, higher consumer socialization goals and greater monitoring of children's consumption vis-a-vis both permissive and authoritarian parents.

More recently Webley and Nyhus (2006) used DNB household survey data and found evidence for the hypothesis that parental orientations have an effect on the economic behavior of the children as well as economic behavior in adulthood. In their analysis they observed high degrees of association between children's savings and parental savings, household income and economic socialization of parents.

Using the terminology used in the Psychology literature, the parent in the standard altruism model acts like a permissive parent while the parent in the tough love altruism model acts like an authoritative parent.

Our model is closely related to Akabayashi's (2006) model in which the child had endogenous discounting and the parent evaluates the child's life time utility with a fixed discount factor. The main difference is that Akabayashi adopted Beckar-Mulligan (1997) type formulation of endogenous discounting so that the child becomes more patient when his human capital is higher. Together with asymmetric information about the child's ability, Akabayashi's model can explain abusive repeated punishments by parents under certain parameter configurations. In contrast, we adopted a Uzawa-type formulation for our model, which can explain parents' tough love behavior. As a model of parental punishments, our model is related to Weinberg's (2001) model. Weinberg's model is a static, incentive model based on asymmetric information. Our model is a dynamic model without any uncertainty.

The remainder of the paper is organized as follows. Section II explains the structure and main findings of the tough love model with only consumption good[3] and contrast the implications of the model with those of the standard altruism model. Section III proposes two alternative models of altruism aimed toward testing the robustness of the tough love model. Section IV discusses whether child is indeed better off with tough love. Section VI introduces leisure in the tough love model and section VII concludes.

II A consumption good economy

Imagine a three-period model economy with two agents, parent and child. For simplicity we consider the case of a single parent and a single child. The model has six features. First, the parent cares about his own consumption but is also altruistic toward the child. He assigns a weight of η to his own utility where $0 < \eta < 1$. Child on the other hand is non-altruist and derives utility from only his own consumption stream $\{C_t\}_{t=1}^{3}$. Second, the life of the parent and the child overlap only in period 1. Third, transfers, T, are made only in period 1 [4]. Fourth, incomes of both the parent and the child are given exogenously. Fifth, the child is borrowing constrained in period 1. Lastly, there is no uncertainty in the economy.

Standard Altruism

In this model, both parent and thw child use the same constant discount factor while evaluating the child's future utility. We call this model the *standard altruism* model.

[3]Introducing leisure as the second good along with the assumption of perfectly observable child's effort level does not change the main results of the paper. The results for the model with leisure as the second good are available upon request.

[4]We assume that transfers are made from the parent to the child and there are no reverse transfers.

The parent's problem is,

$$\max_{T} \left[\eta\ v(y_p - T) + (1-\eta)\Big[u(C_1^*) + \beta_2 u(C_2^*) + \beta_2\beta_3 u(R^2(y_1 + T + \frac{y_2}{R} - C_1^* - \frac{C_2^*}{R}))\Big]\right] \tag{1}$$

subject to

$$C_1 = y_1 + T \tag{2}$$

$$\{C_1^*, C_2^*\} \equiv \arg\max_{C_1, C_2}\Big[u(C_1) + \beta_2 u(C_2) + \beta_2\beta_3 u(R^2(y_1 + T + \frac{y_2}{R} - C_1 - \frac{C_2}{R}))\Big] \tag{3}$$

The following notations will be used. $u(C)$ and $v(C)$ are the standard concave utility functions of the parent and the child resepectively. $\beta_{t,p}$ is the discount factor used by the parent to evaluate the child's future utility and $\beta_{t,k}$ is the discount factor used by the child in period t [5]. We denote the parent's income in period 1 by y_p. y_1 and y_2 represents child's period 1 and period 2 income levels[6]. R is the gross nominal interest rate.

We substitute out the borrowing constraint faced by the child in period 1 and rewrite the parent's optimization problem as

$$\max_{T} \left[\eta\ v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_2 u(C_2^*) + \beta_2\beta_3 u(R(y_2 - C_2^*))\Big]\right] \tag{4}$$

subject to

$$\{C_2^*\} \equiv \arg\max_{C_2}\ \Big[u(C_2) + \beta_3 u(R(y_2 - C_2))\Big] \tag{5}$$

[5] In this model we have $\beta_{t,p} = \beta_{t,k} = \beta_t$

[6] For simplicity we assume child gets no income in last period of his life and simply consume his savings from past periods

Let us focus on the child's optimization program. From the first order condition for the child's problem described in equation (5), we get

$$u_{C_2}(C_2) - \beta_3 R u_{C_2}(R(y_2 - C_2)) = 0 \tag{6}$$

where,

$$u_x(x) \equiv \frac{\partial u(x)}{\partial x}$$

Assuming that the utility function satisfies conditions for the implicit function theorem[7], we can in principle solve equation (6) for C_2 as a function of the model parameters and the state variables.

$$C_2^* = C_2(y_2, \beta_3, R) \tag{7}$$

The optimal period 2 consumption for the child is independent of the period 1 transfers of the parent and hence can be dropped from the parent's optimization program. Hence we can rewrite the parent's problem described by equations (4) and (5) as

$$\max_T \left[\eta v(y_p - T) + (1-\eta)u(y_1 + T)\right] \tag{8}$$

The first order condition for the above problem is given by,

$$-\eta v_T(y_P - T) + (1-\eta)u_T(y_1 + T) = 0 \tag{9}$$

Again, using the implicit function theorem, we get,

$$T^* = T(y_P, y_1, \eta) \tag{10}$$

Comparative Statics

Using the results from previous section, we derive the theoretical predictions of the standard altruism model. For this purpose we consider two experiments. First, we consider an exogenous change in child's discount factor. Specifically we decrease the child's discount factor β_3 and observe how this rise in the child's impatience is accomodated by the parent in terms through a

[7] u(.) is continuously differentiable with non zero *Jacobian*

change in period 1 transfers. From equation (10) optimum period 1 transfers by the parent in the standard altruism model are in fact independent of the child's discount factor implying that an exogenous change in the child's discount factor will have no effect on the period 1 transfers made by the parent. Hence, parents with standard altruism motive will not respond to increasingly impatient behavior of the child. This implication of the model is not consistent with data where we find that both pecuniary and non-pecuniary punishments are used by parents to influence their children's behavior and outcomes.

Second, we consider an exogenous wealth redistribution between the child and the parent and observe the effect of any such redistribution on the optimum tranfsers made by the parent in period 1 [8]. Imagine a dollar increase in the parent's period 1 income, y_p, followed by a dollar decrease in the child's period 1 income, y_1. To measure the effect of this redistribution on period 1 transfers by the parent we rewrite equation (9) as

$$\frac{v_T(y_p - T)}{u_T(y_1 + T)} = \frac{1-\eta}{\eta} \tag{11}$$

For a fixed value η, the ratio of marginal utilities of consumption for the parent and the child is constant in the standard altruism model. Given a standard continuously differentiable concave utility function equation, (11) implies that a dollar increase in y_p and a dollar decrease in y_1 lowers the numerator and increases the denominator by the same amount and hence T must increase by exactly one dollar to keep the ratio of marginal utilities constant. This is the *redistributive neutrality* property of the standard altruism model–a dollar increase in the parent's income, y_p, followed by a dollar decrease in the child's period 1 income, y_1 increases the period 1 transfers T by exactly one dollar.

Tough Love Altruism

One of the main implications of the standard altruism model is that the parent will not reduce transfers in response to a rise in impatience of the child as captured by the fall in the child's discount factor. This prediction is inconsistent with the recent empirical evidence on the use of pecuniary punishment

[8]The purpose behind this experiment is to illustrate the *observational equivalence* between the standard altruism model and the tough love altruism model

mechanisms by parents to influence child's behavior. Weinberg(2001) for example used the Child Development Survey data and established that some parents may use pecuniary incentives and this ability is increasing in parents' income. We propose a tough love altruism model that provides for a channel through which parents can influence child's economic behavior. We introduce the tough love motive of the parent via asymmetric discounting preferences between generations. The main difference between this model and the standard altruism model is that in this model the parent uses a constant and high discount factor to evaluate the child's life time utility while the child himself uses a discount factor which is endogenously determined as a decreasing function of his period 1 consumption.

$$\beta_{t,k}(C_1) \quad ; \quad \frac{\partial \beta_{t,k}}{\partial C_1} < 0$$

With the borrowing constraint faced by the child in period 1, his discount factor is given by $\beta_{t,k}(y1+T)$.

The underlying motivation for this type of endogeneity of the child's discount factor is the belief that the effect of the parent's transfers on the child's consumption habits is strongest in period 1. This in turn is motivated by evidence from the child psychology literature. Maital (1991) provides an excellent analysis of the evolution of time preferences for an economic agent. His main finding is that learning to wait begins in the childhood and to a large extent is learnt from parents and friends.
Now, the parent optimizes by solving the following optimization problem,

$$\max_{T} \left[\eta \; v(y_p - T) + (1-\eta)\Big[u(y_1+T) + \beta_{2,p}u(C_2^*) + \beta_{2,p}\beta_{3,p}u(R(y_2 - C_2^*))\Big]\right] \tag{12}$$

subject to

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1+T)u(R(y_2 - C_2))\Big] \tag{13}$$

From the first order condition for the child's problem described in equation (13) we get

$$u_{C_2}(C_2) - \beta_{3,k}(y1+T)Ru_{C_2}(R(y_2 - C_2)) = 0 \tag{14}$$

In principle solve (14) for C_2 as a function of the model parameters and the state variables.

$$C_2^* = C_2(y_2, \beta_{3,k}(y1 + T), R) \tag{15}$$

Unlike the standard altruism model, now the optimal period 2 consumption for the child is not independent of period 1 transfers of the parent and hence cannot be dropped from the parent's optimization program. As a result we cannot use the methodology used for solving the parent's problem in the standard altruism model. In our tough love model there is no closed form solution to the parent's problem for any functional form for the utility function. Hence, we solve the problem described in equations (12) and (13) numerically as a non linear root finding problem . For this purpose we impose the following parametrization [9]

$$u(C) = v(C) = \frac{C^{1-\sigma}}{1-\sigma} \tag{16}$$

The discount factor is given by,

$$\beta(y1 + T) = \beta_0 + \frac{1}{1 + a(y1 + T)} \quad where \quad a > 0 \; and \; \beta_0 <= 0 \tag{17}$$

Hence as the parameter β_0 decreases, at any given level $y1$ and T the discount factor falls implying more impatient behavior on the part of the child.

Comparative Statics

In this part of the paper we compare our tough love altruism model the standard altruism case. For this purpose we first solve our tough love model for the parametric specification given in (16) and (17) and a given set of model parameter values. This gives us the benchmark optimum transfers and consumption stream $\{T^*, C_1^*, C_2^*, C_3^*\}$. Next, we carry out two kinds of comparative statics exercises, namely an exogenous change in the child's discount factor and an exogenous wealth redistribution and derive the implications of these changes for period 1 transfers in the tough love model.

[9] Our simulations results are robust to alternative parametric specifications of the utility function and also to a wide range of model parameter values

The results we obtain from this exercise are compared with the comparative static results of the standard altruism model.

First, consider an exogenous decrease in the child's discount factor. Formally this is achieved by decreasing the preference parameter β_0. The evidence for the tough love motive is derived by asking how the parent alter his period 1 transfers when the child becomes more impatient as measured by the lower discount factor. Our hypothesis is that a parent who has a tough love motive will try to correct the child's impatience by reducing his transfers. The results for a given set of model parameter values are summarized in Table 1 below. The main finding of the simulation exercise is that there is a monotonic decline in period 1 transfers by parents to the child with a rise in the child's impatience as captured by the falling value of the parameter β_0. As we observe from Table 1, period 1 transfers fall monotonically from 0.9989 to 0.7075 as we decrease the parameter β_0 from 0.0 to -0.8. This is in sharp contrast to the comparative statics results for the standard altruism model wherein the optimal period 1 transfers are independent of child's discounting preference.

Table 1. Tough Love Altruism Model

Global Parameters
$\eta = 0.5$; $\sigma = 1.5$; $R = 1.2$;
$\beta_p = 1$; $y_1 = y_2 = 3$; $y_p = 5$; $a = 0.01$

Optimum	$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
T^*	0.9989	0.9736	0.9273	0.7075
C_1^*	3.9989	3.9736	3.9273	3.7075
C_2^*	1.5651	1.8285	2.0295	2.3397
C_3^*	1.7218	1.4058	1.1646	0.7924
$\beta(C_1^*)$	0.9615	0.5618	0.3622	0.1643

Table 2 presents the simulations with $\sigma < 1$. Again we find transfers decline monotonically as we lower child's discount factor by decreasing β_0.

An important result of our paper is that the tough love model and the standard altruism model are often observationally equivalent. By observational equivalence we mean the comparative static result from a change in an exogenous variable in the tough love model can be obtained by a standard

Table 2. Tough Love Altruism Model

Global Parameters				
$\eta = 0.5$; $\sigma = 0.7$; $R = 1.2$;				
$\beta_p = 1$; $y_1 = y_2 = 3$; $y_p = 5$; $a = 0.01$				
Optimum	$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
T^*	0.9976	0.9449	0.8729	0.6829
C_1^*	3.9976	3.9449	3.8729	3.6829
C_2^*	1.4834	2.0342	2.3924	2.7725
C_3^*	1.8199	1.1589	0.7291	0.2730
$\beta(C_1^*)$	0.9616	0.5620	0.3627	0.1645

altruism model for a given specification of the preference parameters [10]. To illustrate this equivalence between the two models, we consider an exogenous wealth redistribution of wealth between the child and the parent. We find that a dollar increase in the parent's income followed by a dollar decrease in the child's income would increase the parental transfers to the child by exactly one dollar.

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} \;=\; 1 \; (Redistributive\ Neutrality) \tag{18}$$

$\|$ *For proof see Appendix A* $\|$

To summarize the main results so far, the tough love model is often observationally equivalent to the standard altruism model. However, this observational equivalence is broken when child is borrowing constrained and experiences an exogenous change in his discount factor : *with tough love, an exogenous decrease in the child's discount factor is followed by a decline in optimal transfers. In the standard altruism model parental transfers are not affected by child's rising impatience.*

[10] This in principle should allow for preference parameters that can change across time periods. However, for tractability, we assume time invariant preference parameters while carrying out our simulations.

III How important is Tough Love ?

The main theoretical implication of our tough love altruism model is that the parent will use pecuniary punishments in response to impatient behavior on the part of the child. This is also the only difference between the standard standard altruism models used in most of the macroeconomic literature and our tough love altruism model.In this section we show that in order to break the observational equivalence between the standard altruism model and the tough love model both the elements that constitute tough love in our model namely higher parental discount factor and endogenous discount factor for the child are required. For this purpose, we propose two altrenaticve models of altruism. First is the paternalistic altruism model and the second is the endogenous altruism model. We then carry out the same comparative statics experiments that were used to differentiate between the tough love altruism and the standard altruism models. We show that the paternalistic altruism model is observationally equivalent to the standard altruism model and that the endogenous altruism altruism model generates predictions that are very sensitive to parametric specifications of the model.

Paternalistic Altruism Model

In this model both the parent and the child use constant discount factors to evaluate future utility. However, unlike the standard altruism model, in this model the discount factor used by the parent is higher than the child's discount factor, i.e. $\beta_{t,p} > \beta_{t,k}$ where $\beta_{t,p}$ is the discount factor used by the parent to evaluate the child's future utility and $\beta_{t,k}$ is the discount factor used by the child in period t. The parent's problem is

$$\max_{T} \left[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p} u(C_2^*) + \beta_{2,p}\beta_{3,p} u(R(y_2 - C_2^*))\Big]\right] \tag{19}$$

subject to

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k} u(R(y_2 - C_2))\Big] \tag{20}$$

As before, we solve the child's optimization problem first which gives us optimal period 2 consumption of the child

$$C_2^* = C_2(y_2, \beta_{3,k}, R) \tag{21}$$

The optimal period 2 consumption for the child is independent of the period 1 transfers of the parent and so it can be dropped from the parent's optimization program.

We rewrite the parent's problem described by equations (19) and (20) as

$$Max_T \left[\eta v(y_p - T) + (1-\eta)u(y_1 + T)\right] \tag{22}$$

The first order condition for the above problem is given by,

$$-\eta v_T(y_P - T) + (1-\eta)u_T(y_1 + T) = 0 \tag{23}$$

The above equation in principle can be solved for optimum period 1 transfers,

$$T^* = T(y_P, y_1, \eta) \tag{24}$$

Comparative Statics

As discussed before, we bring out the differences between the alternative models of altruism using two kinds of comparative statics experiments. First, we conisder an exogenous decrease in the child's discount factor $\beta_{3,k}$. From equation (25) optimum period 1 transfers by the parent is independent of the discount factor of the child. Therefore, just like standard altruism model, in this model also there is no effect of decrease in discount factor on the period 1 transfers T.
Second, we consider a dollar increase in the parent's income followed by a dollar decrease in the child's period 1 income. Rearranging (23)

$$\frac{v_T(y_p - T)}{u_T(y_1 + T)} = \frac{1-\eta}{\eta} \tag{25}$$

Hence, for a given η the ratio of marginal utilities of consumption for the parent and the child is constant. For this ratio to remain constant a dollar increase in y_p and a dollar decrease in $y1$ must be followed by a dollar increase in the transfers T. This is the redistributive neutrality result that is shared by both the standard altruism and the tough love altruism model.

To summarize we find that if we only introduce a higher parental discount factor then the comparative static results of the paternalistic model for both exogenous changes in the discount factor of the child and exogenous wealth redistributions are observationally equivalent to those obtained under the standard altruism model.

Endogenous Altruism Model

In this model as assumed in the tough love altruism model, the discount factor used by the child is endogenously determined as a decreasing function of his period 1 consumption.

$$\beta_{t,k}(c_1) \quad ; \quad \frac{\partial \beta_{t,k}}{\partial C_1} < 0$$

With the borrowing constraint faced by the child in period 1, the discount factor is given by $\beta_{t,k}(y1+T)$. However, unlike the tough love altruism model now the parent also uses the above discount factor for evaluating the child's future utility. So the key difference is the assumption

$$\beta_{t,p}(x) = \beta_{t,k}(x)$$

The parent's problem in this model is

$$\max_{T} \Bigg[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p}(y1+T)u(C_2^*) + \beta_{2,p}(y1+T)\beta_{3,p}(y1+T)u(R(y_2 - C_2^* \tag{26}$$

subject to

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1+T)u(R(y_2 - C_2))\Big] \tag{27}$$

From the first order condition for the child's problem we get

$$u_{C_2}(C_2) - \beta_{3,k}(y1+T)Ru_{C_2}(R(y_2 - C_2)) = 0 \tag{28}$$

The above equation yields the optimal period 2 consumption of the child

$$C_2^* = C_2(y_2, \beta_{3,k}(y1+T), R) \tag{29}$$

The optimal period 2 consumption for the child is not independent of period 1 transfers of the parent and hence cannot be dropped from the parent's optimization program. We solve the problem described in (27), (28) numerically as a non linear root finding problem. The solution method and the parametrizatiob adopted is identical to the one we used for the tough love altruism model.

Comparative Statics

To compare the theoretical implications of this model with other models of altruism discussed in this paper we follow the same strategy. We consider two experiments, namely an exogoneous decrease in the discount factor of the child and an exogenous wealth distribution. For the first comparative static exercise we make the child more impatient by decreasing the preference parameter β_0 and then trace out the effect of this change on the period 1 transfers T. The results for the assumed set of model parameter values are summarized in Table 3. Again we find that as β_0 is reduced monotnically, parents in endogenous altruism model will decrease transfers.

Table 3. Endogenous Altruism Model

Global Parameters $\eta = 0.5$; $\sigma = 1.5$; $R = 1.2$; $y_1 = y_2 = 3$; $y_p = 5$; $a = 0.01$				
Optimum	$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
T^*	1.4343	1.3265	1.2667	1.1988
C_1^*	4.4343	4.3265	4.2667	4.1988
C_2^*	1.5672	1.8313	2.0333	2.3493
C_3^*	1.7193	1.4025	1.1600	0.7809
$\beta(C_1^*)$	0.9575	0.5585	0.3591	0.1597

The results summarized in Table 3 above seem to suggest that the endogenous altruism model is identical to the tough love model. However, unlike the results of the tough love model, this result is very sensitive to the assumption about σ. Table 4 below presents the simulation results with $\sigma < 1$. Now we find that as β_0 falls, transfers increase monotonically. Hence, with the endogenous altruism model, depending on the assumption about model parameters we may get counterintuitive result of parents rewarding impatience of the child.

Table 4. Endogenous Altruism Model

Global Parameters $\eta = 0.5$; $\sigma = 0.7$; $R = 1.2$; $y_1 = y_2 = 3$; $y_p = 5$; $a = 0.01$				
Optimum	$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
T^*	0.2111	0.4393	0.5438	0.6323
C_1^*	3.2111	3.4393	3.5438	3.6323
C_2^*	1.4753	2.0264	2.3866	2.7716
C_3^*	1.8297	1.1683	0.7361	0.2740
$\beta(C_1^*)$	0.9689	0.5668	0.3658	0.1650

For the second comparative statics exercise we find that this model also predics redistributive neutrality. This result is observationally equivalent to the response obtained with the standard altruism, the tough love altruism and the paternalistic altruism models. Formally,

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} \;=\; 1\;(Redistributive\ Neutrality) \tag{30}$$

$\|$ *For proof see Appendix B* $\|$

IV Are parents loving in the Tough Love Altruism Model ?

In this section we present some simulation results with the objective of illustrating that the outcome of the tough love model in terms of the child's lifetime utility is preferred to the one that was obtained with the endogenous altruism model. For this purpose we solve both the models for a given value of β_0. However, to make the child's lifetime utility comparison, we evaluate the child's lifetime utility in both the models at the discount factor obtained under the tough love model, $\beta(C^*_{TL})$. Next, we consider a change in β_0 (a decline) and again evaluate the child's lifetime utility in both the models at the discount factor obtained under the tough love model. The idea is to measure the difference in the effect in terms of lifetime utility, of an exogenous decrease in the discount factor of the child across the tough love and endogenous altruism models.

The results of this exercise for a particular set of model parameter values are provided in Table 5. We find that in the tough love model a decrease in the child's discount factor generates lower transfers but higher lifetime utility when compared to the endogenous altruism model. We interpret these results as implying that the parent in the tough love model behave in accordance with the long run welfare of the child.

Table 5. Child's Utility Comparison

Global Parameters			
$\eta = 0.5$; $\sigma = 1.5$; $R = 1.2$; a=0.01			
$\beta_p = 1$; $y_1 = 0.1$, $y_2 = 0.4$, $y_p = 0.4$			
$\tau_1 = 18$, $\tau_2 = 40$, $\tau_3 = 20$			
$\beta_0 = 0$	$\beta_0 = -0.4$	$\beta_0 = -0.6$	$\beta_0 = -0.8$
β_0 (1)	$\beta(C^*_{TL})$ (2)	$V_{TL}(\beta(C^*_{TL}))$ (3)	$V_{END}(\beta(C^*_{TL}))$ (4)
0	0.9975	-131.6684	-278.4951
-0.05	0.9476	-51.3465	-122.0620

V Tough Love Altruism Model with Leisure

Till now we have considered an economy where agents derive utility only from consumption. In this section we generalize our setup in an important dimension by allowing for leisure as a choice variable for the child. We continue to assume perfect information. This in our set up means that the parent can fully observe the child's effort level. The rest of the model assumptions are retained with transfers being made only in period 1 and the child being borrowing constrained in period 1. The following notations are used. L_1 and L_2 denote the amount of leisure consumed by the child in period 1 and period respectively. w_1 and w_2 denote the wage income of the child in the two periods. For simplicity we assume the child earns no wage income in period 3 and simply consume his past savings.The parent's problem is

$$\max_{T} \ \eta\, v(y_p - T) + (1-\eta)\Bigg[u(w_1(1-L_1^*)+T, L_1^*)\beta_{2,p}u(C_2^*, L_2^*) + \beta_{2,p}\beta_{3,p}u(R(w_2(1-L_2^*)-C_2^*))\Bigg] \quad (31)$$

subject to

$$\{C_2^*, L_1^*, L_2^*\} \equiv \arg\max_{C_2,L_1,L_2}\Bigg[u(w_1(1-L_1)+T, L_1) + \beta_{2,k}(w_1(1-L_1)+T)u(C_2, L_2) + \beta_{2,k}(w_1(1-L_1)+T)\beta_{3,k}(w_1(1-L_1)+T)u(R(w_2(1-L_2)-C_2))\Bigg] \quad (32)$$

From the first order conditions for the child's problem we get :

$$u_{C_2}(C_2, L_2) - \beta_{3,k}(w_1(1-L_1)+T)Ru_{C_2}(R(w_2(1-L_2)-C_2)) = 0 \quad (33)$$

$$u_{L_2}(C_2, L_2) - \beta_{3,k}(w_1(1-L_1)+T)Rw_2u_{L_2}(R(w_2(1-L_2)-C_2)) = 0 \quad (34)$$

$$\Bigg[u_{L_1}(C_1, L_1) - w_1 u_{C_1}(C_1, L_1) - w_1 \frac{\partial \beta_{2,k}(C_1}{\partial L_1}\big[u(C_2, L_2)$$

$$+ \frac{\partial \beta_{2,k}(C_1)}{\partial L_1} \beta_{3,k}(C_1) u(R(w_2(1-L_2) - C_2))\big]$$

$$-w_1 \frac{\partial \beta_{3,k}(C_1}{\partial L_1} \beta_{2,k}(C_1) u(R(w_2(1-L_2) - C_2))\Bigg] = 0 \qquad (35)$$

As we observe from equations (34), (35) and (36) , first period transfers enter as a parameter in all of the choice variables of the child. We solve the above problem numerically as a non linear root finding problem since there is no closed form solution to the child's problem for any functional form for the utility function. For this purpose we impose the following parametric specification:

$$u(C, L) = Log(C) + d\frac{L^{1-\gamma}}{1-\gamma} \ : \ (Child's\ Utility\ Function) \qquad (36)$$

$$v(C) = Log(C) \ : \ (Parent's\ Utility\ Function) \qquad (37)$$

The child's discount function is given by,

$$\beta(w_1(1-L_1)+T) = \beta_0 + \frac{1}{1+a(w_1(1-L_1)+T)} \quad where \quad a > 0 \ and \ \beta_0 < 0 \qquad (38)$$

Table 6 summarizes the results of the simulations for two alternative scenarios identified by a decrease in the paramter β_0. We observe that as β_0 falls from 0 to -0.01, the parent with a tough love motivelower transfers to the child. At the same time there is also a fall in the child's lifetime income corresponding to the fall in β_0. This result suggests that even though the tough love model implies redistributive neutrality it may be consistent with Altonji, Hayashi and Kotlikoff;s 1997) empirical findings that in data there is no evidence for redistributive neutrality .

VI Conclusion

In the simple setting of a three period economy with a single parent and a single child with perfect information and borrowing constraints, we develop a

Table 6. Tough Love Altruism Model with Leisure

Optimum	$\beta_0 = 0$	$\beta_0 = -0.01$
Global Parameters		
$\eta = 0.5$; $\gamma = 0.7$; $r = 1.03$; $a = 0.01$		
$w_1 = 1$; $w_2 = 2$; $y_p = 2$; $\tau_1 = 18$; $\tau_2 = 40$; $\tau_3 = 20$;		
T^*	0.7085	0.6769
Child's First Period Income	9.3802	9.1318

model of intergenerational transfers wherein the tough love motive for parents is a driving force behind the parent's behavior. The simulation results for the tough love model for a reasonable range of parameter values show that as the child becomes more impatient, the parent react by cutting down transfers in an attempt to inculcate a more patient consumption behavior. This is consistent with our intuition of tough love parenting. On the other hand with standard altruism, the parent seems to be oblivious to the child's increasingly impatient behavior by not changing transfers with rising impatience of the child.

Since exogenous changes in the child's discount factor to make him impatient is likely to cause a behavior that calls for the parent's corrective actions, the tough love model is more consistent with empirical evidence on parental punishments than the standard altruism model.

In the version of the tough love model with leisure, an exogenous change in the discount factor to make the child more impatient can cause both lower income and lower transfer from the parent. This feature of the model may be consistent with empirical findings by Cox (1987) and Altonji, Hayashi and Kotlikoff 1997). Cox (1987) ran a horse race between altruism and exchange motive by studying the relationship between transfers received and income of the recipient. Using President's Commission on Pension Policy (PCPP) data he found support for exchange motive being the key factor behind inter vivos transfers. Another testable implication of the standard altruism model is the *redistributive neutrality* also known as the *transfer derivative restriction*: a dollar decrease in parent's income coupled with a dollar increase in child's income will lead to a dollar decrease in transfer from parent to the child. Altonji, Hayashi and Kotlikoff 1997) used PSID data and found in fact that transfers only decrease by 13 cent and hence strongly rejected the transfer derivative restriction implied by the standard altruism model. Even though the tough love model implies the redistributive neutrality, if cross-sectional data contains both exogenous income changes and endogenous income changes caused by exogenous changes in the discount factors, then the tough love model with leisure can be consistent with results by Cox and Altonji, Hayashi, and Kotlikoff. [11]

Preliminary empirical results in Horioka, Ogaki, and Ohtake (2008) suggest that some parents show tough love while others do not. They analyze Osaka University Center of Excellence Survey data for Japan and U.S. The survey contains two questions that are arguably useful in identifying who exhibit tough love. One of these two questions is "Imagine that you have a 2-year old child that has a high fever and is in pain. The child's doctor tells you that both the fever and pain are harmless. He can give you a medicine that cures the sickness but slightly weakens the child's immune system when the child becomes 50 years old. "In the future, it will be interesting to analyze characteristics of parents who show tough love to their children.

[11] This explanation can potentially reconcile apparent inconsistency between empirical results against the redistributive neutrality and Laitner and Thomas' (1996) results in favor of parents' altruism for children. They used TIAA-CREFF retirees data and focused on bequest as the channel for parental altruism. They found that for the subsample of respondents characterized by willingness to leave a bequest, the projected amount of the bequest is largest for the households with lowest assessments of their children's likely earnings in future.

Appendix A

Consider the parent's maximization program in the tough love model:

$$\max_{T} \Bigg[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p} u(C_2^*) \\ + \beta_{2,p}\beta_{3,p} u(R(y_2 - C_2^*))\Big]\Bigg] \tag{A-1}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1 + T) u(R(y_2 - C_2))\Big] \tag{A-2}$$

Now from the first order condition of the child's maximization problem,

$$u_{C_2}(C_2) - \beta_{3,k}(y1 + T) R u_{C_2}(R(y_2 - C_2)) = 0 \tag{A-3}$$

Then using the implicit function theorem we get,

$$C_2^* = C2(y_2, y1 + T, R) \tag{A-4}$$

In this wealth redistribution experiment we only change y_p and y_1. Hence we can treat R and y_2 as constants. Also note that from child's first period borrowing constraint,

$$C_1 = y_1 + T \tag{A-5}$$

Using these facts, we can rewrite child's optimal period 2 consumption as,

$$C_2^* = C2(C_1) \tag{A-6}$$

Substituting child's optimal second period consumption in the parent's problem we get,

$$Max_T \;\; \eta v(y_p - T) + (1-\eta)\Bigg[u(y_1 + T) + \beta_{2,p} u(C2(C_1)) \\ + \beta_{2,p}\beta_{3,p} u(R(y_2 - C2(C_1)))\Bigg] \tag{A-7}$$

Define $C_p = y_p - T$ for notational simplicity. Then the first order condition for the parent's problem is,

$$-\eta v'(C_p) + (1-\eta)\Big[u'(C_1) + \beta_{2,p}u'(C2(C_1))C2'(C_1) - \beta_{2,p} \\ \beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2'(C_1)\Big] = 0 \tag{A-8}$$

Now we totally differentiate equation (A-8) assuming R, y_2, $\beta_{2,p}$ and $\beta_{3,p}$ are constants. We get,

$$-\eta v''(C_p)dy_p + \eta v''(C_p)dT + (1-\eta)\Big[u''(C_1)dy_1 + u''(C_1)dT + \beta_{2,p}u''(C2(C_1))C2'(C_1)^2 dy_1 \\ +\beta_{2,p}u''(C2(C_1))C2'(C_1)^2 dT + \beta_{2,p}u'(C2(C_1))C2''(C_1)dy_1 + \beta_{2,p}u'(C2(C_1))C2''(C_1)dT \\ +\beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 dy_1 + \beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 dT \\ +\beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)dy_1 + \beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)dT\Big] = 0 \tag{A-9}$$

From equation (A-9) it is straightforward to show that,

$$\frac{\partial T^*}{\partial y_p} = \frac{\eta v''(C_p)}{A1} \tag{A-10}$$

$$\frac{\partial T^*}{\partial y_1} = -\frac{A2}{A1} \tag{A-11}$$

where

$$A1 \equiv \eta v''(C_p) + (1-\eta)\Big[u''(C_1) + \beta_{2,p}u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}u'(C2(C_1))C2''(C_1) \\ +\beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 + \beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)\Big] \tag{A-12}$$

and

$$A2 \equiv (1-\eta)\Bigg[u''(C_1) + \beta_{2,p}u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}u'(C2(C_1))C2''(C_1) +$$

$$\beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 + \beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)\Bigg] \tag{A-13}$$

Hence,

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} = \frac{\eta v''(C_p) + A2}{A1} = \frac{A1}{A1} = 1 \tag{A-14}$$
$$\Leftrightarrow Redistributive \quad Neutrality$$

Appendix B

Consider the parent's maximization program in the endogenous altruism model:

$$\max_{T} \left[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p}(y1 + T)u(C_2^*) + \beta_{2,p}(y1+T)\beta_{3,p}(y1+T)u(R(y_2 - C_2^*))\Big]\right] \tag{B-1}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1+T)u(R(y_2 - C_2))\Big] \tag{B-2}$$

From the first order condition for the child's problem we get :

$$u_{C_2}(C_2) - \beta_{3,k}(y1+T)Ru_{C_2}(R(y_2 - C_2)) = 0 \tag{B-3}$$

Using the implicit function theorem we get,

$$C_2^* = C_2(y_2, \beta_{3,k}(y1+T), R) \tag{B-4}$$

In this wealth redistribution experiment we only change y_p and y_1. Hence we can treat R and y_2 as constants. Also note that from child's first period borrowing constraint,

$$C_1 = y_1 + T \tag{B-5}$$

Using these facts, we can rewrite child's optimal period 2 consumption as,

$$C_2^* = C2(C_1) \tag{B-6}$$

Substituting child's optimal second period consumption in the parent's problem we get,

$$\max_{T}\left[\eta v(C_p) + (1-\eta)\Big[u(C_1) + \beta_{2,p}(C_1)u(C_2(C_1)) + \beta_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1)))\Big]\right] \tag{B-7}$$

where,

$$C_p = y_p - T$$
$$C_1 = y_1 + T \tag{B-8}$$

F.O.C for the parent's problem,

$$\Big[-\eta v'(C_p) + (1-\eta)\Big[u'(C_1) + \beta'_{2,p}(C_1)u(C_2(C_1)) + \beta_{2,p}(C_1)u'(C_2(C_1))C'_2(C_1)$$
$$\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1))) + \beta_{2,p}(C_1)\beta_{3,p}(C_1)'u(R(y_2 - C_2(C_1)))$$
$$-R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Big]\Big] = 0 \tag{B-9}$$

Now we totally differentiate equation (B-9) assuming R and y_2 are constants. We get,

$$\Big[-\eta v''(C_p)dy_p + \eta v''(C_p)dT + (1-\eta)\Big[u''(C_1) + \beta''_{2,p}(C_1)u(C_2(C_1)) + 2\beta'_{2,p}(C_1)u'(C_2(C_1))C'_2(C_1)$$
$$+\beta_{2,p}(C_1)u''(C_2(C_1))C'_2(C_1)^2 + \beta_{2,p}(C_1)u'(C_2(C_1))C''_2(C_1) + \beta''_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$+2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1))) - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$
$$+\beta_{2,p}(C_1)\beta''_{3,p}(C_1)u(R(y_2 - C_2(C_1))) - R\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))C'_2(C_1)$$
$$+R^2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))C'_2(C_1)^2 - R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C''_2(C_1)$$
$$-R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Big](dy_1 + dT)\Big] = 0 \tag{B-10}$$

From equation (B-10) it is straightforward to show that,

$$\frac{\partial T^*}{\partial y_p} = \frac{\eta v''(C_p)}{A1} \tag{B-11}$$

$$\frac{\partial T^*}{\partial y_1} = -\frac{A2}{A1} \tag{B-12}$$

where,

$$A1 \equiv \eta v''(C_p) + (1-\eta)\Big[u''(C_1) + \beta''_{2,p}(C_1)u(C2(C_1)) + 2\beta_{2,p}(C_1)'u'(C2(C_1))C2'(C_1)$$
$$+\beta_{2,p}(C_1)u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}(C_1)u'(C2(C_1))C2''(C_1)$$
$$+\beta''_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1))) + 2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$-R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + \beta_{2,p}(C_1)\beta''_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + R^2\beta_{2,p}(C_1)\beta_{3,p}(C_1)u''(R(y_2 - C_2(C_1)))C'_2(C_1)^2$$
$$-R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C''_2(C_1) - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Big] \tag{B-13}$$

$$
\begin{aligned}
A2 \equiv (1-\eta)\Bigg[& u''(C_1) + \beta''_{2,p}(C_1)u(C2(C_1)) + 2\beta_{2,p}(C_1)'u'(C2(C_1))C2'(C_1) \\
& + \beta_{2,p}(C_1)u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}(C_1)u'(C2(C_1))C2''(C_1) \\
& + \beta''_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1))) + 2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1))) \\
& - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + \beta_{2,p}(C_1)\beta''_{3,p}(C_1)u(R(y_2 - C_2(C_1))) \\
& - R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + R^2\beta_{2,p}(C_1)\beta_{3,p}(C_1)u''(R(y_2 - C_2(C_1)))C'_2(C_1)^2 \\
& - R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C''_2(C_1) - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) \\
& - R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Bigg]
\end{aligned}
\tag{B-14}
$$

Hence,

$$
\begin{aligned}
\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} &= \frac{\eta v''(C_p) + A2}{A1} = \frac{A1}{A1} = 1 \\
&\Leftrightarrow Redistributive \quad Neutrality
\end{aligned}
\tag{B-15}
$$

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