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***“A Model of Stigma in the  
Fed Funds Market”***

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# A Model of Stigma in the Fed Funds Market\*

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## Abstract

It is often the case that banks in the US are willing to borrow in the fed funds market (the interbank market for funds) at higher rates than the ones they could obtain by borrowing at the Fed's discount window. This phenomenon is commonly explained as the consequence of the existence of a stigma effect attached to borrowing from the window. Most policymakers and empirical researchers consider the stigma hypothesis plausible. Yet, no formal treatment of the issue has ever been provided in the literature. In this paper, we fill that gap by studying a model of interbank credit where: (1) banks benefit from engaging in intertemporal trade with other banks and with outside investors; and (2) physical and informational frictions limit those trade opportunities. In our model, banks obtain loans in an over-the-counter market (involving search, bilateral matching, and negotiations over the terms of the loan) and hold assets of heterogeneous qualities which in turn determine their ability to repay those loans. When asset quality is not perfectly unobservable by outside investors, information about the actions taken by a bank in the credit market may influence the price at which it can sell its asset. In particular, under some conditions, discount window borrowing may be regarded as a negative signal about the quality of the borrower's assets. In such cases, some of the banks in our model, just as in the data, are willing to accept loans in the interbank market at higher rates than the ones they could obtain at the discount window.

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# 1 Introduction

It is often the case that some banks in the US are willing to occasionally borrow in the fed funds market (the interbank market for funds) at higher rates than the ones they could obtain by borrowing from the central bank, at the Fed's discount window (Peristiani, 1998, Furfine, 2001, Darrat et Al. 2004). This phenomenon is commonly explained as the consequence of the existence of a stigma effect attached to borrowing from the window. The general argument is that market participants may eventually identify with some accuracy which banks have borrowed at the discount window and take such activity as a sign of weakness in the financial conditions of the borrowing institution.<sup>1</sup>

While most policymakers and empirical researchers consider the stigma hypothesis plausible, no formal treatment of the issue has ever been provided in the literature. In this paper, we fill that gap by studying a model of interbank credit where: (1) banks benefit from engaging in intertemporal trade with other banks and with outside investors; (2) physical and informational frictions limit those trade opportunities; and (3) under some specific conditions, which we clearly identify, a stigma effect like the one observed in the data may arise.

Understanding the reluctance of banks to use the discount window is essential to address many important policy issues. For example, the prevalence of stigma may limit the ability of the central bank to effectively implement a "hard ceiling" on the range of interest rates observable in the interbank market. Partly in an effort to address such issues, in 2003, the Federal Reserve completely changed the terms of operation of its credit facilities. In spite of such effort, evidence of stigma remained in the market (Furfine, 2005).<sup>2</sup>

More recently, the reluctance of banks to borrow from the window bedeviled the central bank's attempts to inject liquidity in the market. As explained by Chairman Bernanke (2009), "In August 2007, ... banks were reluctant to rely on discount window credit to address their funding needs. The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness – the so-called stigma problem. The perceived stigma of borrowing at the discount window threatened to prevent the Federal Reserve from getting much-needed liquidity into the system." The creation of the Terms Auction Facility (TAF), and some of its particular organizational features, can actually be regarded as trying to limit the possibility of stigma associated with accessing this source of central bank liquidity.<sup>3</sup> In this paper, we will discuss

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<sup>1</sup>Journalist Mathew Cowley expresses this popular view succinctly in his column at Dow Jones Newswires: "*There's traditionally been a stigma associated with borrowing [from the Fed's discount window], which is initiated by the financial institution and is therefore regarded as a sign of weakness.*" (1 August 2008).

<sup>2</sup>The issue of "stigma" in the interbank market for funds has a long history in the US. For example, when Friedman and Schwartz (1963) discuss the creation in January 1932 of the Reconstruction Finance Corporation (RFC) which had the authority "to make loans to banks and other financial institutions, as well as to railroads" they say that: "... a provision of an act passed in July 1932 was interpreted as requiring publication of the names of banks to which the RFC had made loans in the previous month, and such publication began in August. The inclusion of a bank's name on the list was correctly interpreted as a sign of weakness, and hence frequently led to runs on the bank. Hence banks were fearful of borrowing from the RFC."

<sup>3</sup>"The TAF, apparently because of its competitive auction format and the certainty that a large amount of credit would be made available, appears to have overcome the stigma problem to a significant degree." (Bernanke, 2008)

specific conditions under which stigma may arise in the context of our formal model. We believe that the resulting insights are useful for evaluating alternative arrangements and policy options directed to reduce the incidence of stigma in the interbank market.

Banks in our model obtain loans in an over-the-counter market, involving search, bilateral matching, and negotiations over the terms of the loans. To repay these loans, banks sell assets of heterogeneous quality to outside investors. When asset quality is (at least partially) observable, information about the actions taken by banks in the credit market may influence the price at which they can sell their assets later. In particular, under some conditions, discount window borrowing may be regarded as a negative signal of the quality of the borrower's assets. In such cases, some of the banks in our model, just as in the data, are willing to accept loans in the interbank market at higher rates than the ones they could obtain at the discount window.

Aside from the possibility of stigma, our model generates some interesting outcomes in the interbank market, even when discount window borrowing is not possible.<sup>4</sup> For example, we find that under some parameter values, the asset market and the interbank market for loans may simultaneously shut down. This outcome occurs in our model due to the existence of an equilibrium interconnection between the two markets. If participants expect that the asset market will shut down (meaning that prices for assets of unobserved quality are expected to be equal to zero in that market), then they will not be willing to lend in the (prior) interbank market, and since there is no lending in the interbank market, no high (unobserved) quality assets are actually sold, which makes the expected zero price an equilibrium. In other words, in our model, when the asset market shuts down, the interbank market shuts down, because of adverse selection (in the asset market) and the consequent repayment risk (in the interbank market).

We make some stark assumptions in our model. It is fairly easy to see that many of them could be readily generalized. However, our main objective here is to formalize in as simple a framework as possible an argument that is often used to explain certain apparently abnormal trading patterns in the US interbank market for funds. Abstracting from some realistic features allow us to better capture the mechanisms at play. After stripping the problem from inessential details, some previously unappreciated components get revealed. Market frictions and bilateral negotiations, for example, play a prominent role in our formal explanation but not necessarily on the more heuristic ones used in policy circles. We believe that identifying these and other features is one of the main contributions of our paper.

The model in this paper combines several elements that are commonly regarded as important in explaining the nature of financial (and, in particular, interbank) market outcomes. First, as in Freeman's (1996) article, and the large literature that followed, spatial separation plays a key role in

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<sup>4</sup>There is now a large literature providing formal treatment of various issues related to the functioning of the interbank market. Some prominent examples are Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas and Holthausen (2004). More recent contributions include Freixas and Jorge (2007), Allen et Al. (2009), and the working papers by Acharya et Al. (2007), Heider et Al. (2008), Freixas et Al. (2009) and Bolton et Al. (2009). For a good discussion of this literature see the introduction of Allen et Al. (2009).

limiting the ability of some agents (banks) to trade with other agents (outside investors) at a certain point in time. Second, search and bilateral negotiations determine the terms of trade in the market, as in Duffie et Al. (2005) and Lagos and Rocheteau (2009).<sup>5</sup> Third, informational asymmetries and asset-quality heterogeneity play a crucial role in determining equilibrium interest rates and prices (as, for example, in Eisfeldt, 2004). Furthermore, the theory in this paper is in line with the long tradition, launched by Leland and Pyle (1977), of studying the role of signaling in financial markets.

The paper is organized as follows. In the rest of this section we discuss some evidence that has often been regarded as indicating the presence of stigma attached to lending from the Fed's discount window. Then, in the next section, we introduce our formal model of the interbank market, based on intertemporal trade with (physical and informational) frictions. In Section 3 we study equilibrium in the basic framework when the discount window is not available to banks. This section is intended to provide a description of the basic economics involved in the model. In Section 4 we introduce discount window lending and derive the equilibrium conditions for this, more complicated case. In Section 5, then, we study an equilibrium in which discount window lending becomes a negative signal and, hence, results in stigma. In that context, we discuss the particular conditions that can give rise to such phenomenon. In Section 6 we briefly discuss other possible equilibrium configurations and, in Section 7, we provide a summary discussion and conclusions.

## 1.1 Interest rates in the fed funds market and the discount window

During the 1980s and 1990s, the Fed provided discount window loans to banks at a rate below the fed funds target rate (i.e., the rate announced as the target for monetary policy). Supervisory scrutiny was used to control the amount borrowed by banks. On January 9, 2003, the US Federal Reserve dramatically changed its discount window lending policy and started to operate a standing facility, offering loans to eligible depository institutions at an interest rate higher than the fed funds target rate (at the time, the spread was set at 100 basis points). No other restriction or especial supervision was associated to lending from the discount window. In principle, under this new regime, the rate at the discount window (plus the implicit cost of collateral) should act as a ceiling for the fed funds market rate. However, there exists extensive evidence showing that it is common for banks to choose to borrow from another bank at a higher rate than the one they could get at the Fed's discount window. More generally, the evidence seems to indicate a persistent reluctance of US banks to borrow from the discount window.

For example, Furfine (2003) compares the amount of borrowing at the discount window after January 2003 with the volume that one would have predicted in advance, given the historical (pre-standing facility) empirical distribution of across-banks interest rates paid in the market. He finds that borrowing from the primary credit facility was significantly lower than what one might have expected based on that past data. Also, he finds that, at rates equal or higher than that offered

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<sup>5</sup>Ashcraft and Duffie (2007) argue that these are realistic features of the US interbank market for funds.

by the central bank, for each day during the period from January 9, 2003 to March 31, 2003, on average, there was more than 57 times more borrowing activity in the fed funds market than at the discount window.

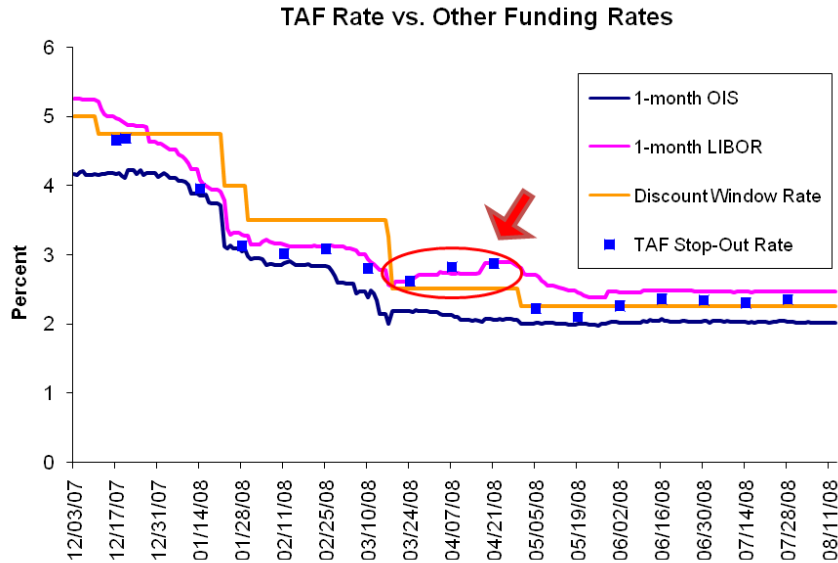


Figure 1

Interestingly, evidence of similar behavior can be found even before the system’s overhaul in 2003. For example, Furfine (2001) studies activity in the fed funds market during the operation of a temporary Lombard-type Fed lending facility (the SLF) put in place from October 1, 1999 to April 7, 2000 to respond to possible spikes in liquidity demand associated with the Y2K event. He finds that whenever market interest rates rose noticeably, borrowing in the overnight federal funds market at 150 or more basis points above the Fed’s target rate dwarfed lending at the SLF. Furfine, then, concludes that commercial banks were extremely reluctant to borrow from the Fed during that time.

In August 2007, as a response to the incipient liquidity crisis, the Fed lowered the spread in the discount window rate and started to allow eligible institutions to borrow funds at longer terms (instead of just overnight, as it was usually the case). The change generated little to no additional borrowing. In December 2007, the Fed created the Term Auction Facility (TAF), a biweekly auction of a fixed amount of 28-day funding for depository institutions eligible to obtain (unrestricted) credit at the discount window. Several features of the TAF made it less likely to generate stigma.<sup>6</sup> From its inception, borrowing at the TAF was in high demand. Interestingly, during March and April 2008 the stop-out rate at the TAF (i.e., the rate at which funds were allotted) was higher than the

<sup>6</sup>The auction of a fixed, large amount of funds guaranteed participation of multiple bidders, making more likely that they will remain anonymous. Furthermore, a period of three days was set between the auction day and the settlement day (when the funds were transferred to the winners of the auction). This delay might have helped to decrease the perception that participants were in desperate need for funding (Bernanke, 2009).

discount window rate (see Figure 1). That is, banks preferred to borrow at the TAF at a higher rate than the one they could obtain at the discount window.<sup>7</sup>

<b>Date</b>	<b>Daily effective fed funds rate</b>	<b>Range</b>		<b>Standard Deviation</b>
		<b>Low</b>	<b>High</b>	
<b>09/08/08</b>	<b>1.92</b>	<b>1.00</b>	<b>2.50</b>	<b>0.22</b>
<b>09/09/08</b>	<b>1.96</b>	<b>1.62</b>	<b>2.19</b>	<b>0.07</b>
<b>10/09/08</b>	<b>2.12</b>	<b>1.50</b>	<b>2.62</b>	<b>0.18</b>
<b>09/11/08</b>	<b>2.00</b>	<b>1.00</b>	<b>2.50</b>	<b>0.19</b>
<b>09/12/08</b>	<b>2.10</b>	<b>1.75</b>	<b>2.94</b>	<b>0.10</b>
<b>09/15/08</b>	<b>2.64</b>	<b>0.01</b>	<b>7.00</b>	<b>1.72</b>
<b>09/16/08</b>	<b>1.98</b>	<b>0.01</b>	<b>6.00</b>	<b>1.10</b>
<b>09/17/08</b>	<b>2.80</b>	<b>0.25</b>	<b>6.00</b>	<b>0.78</b>
<b>09/18/08</b>	<b>2.16</b>	<b>0.00</b>	<b>6.00</b>	<b>1.13</b>
<b>09/19/08</b>	<b>1.48</b>	<b>0.00</b>	<b>3.50</b>	<b>1.01</b>
<b>09/22/08</b>	<b>1.51</b>	<b>0.00</b>	<b>3.50</b>	<b>0.92</b>
<b>09/23/08</b>	<b>1.46</b>	<b>0.06</b>	<b>3.00</b>	<b>0.69</b>
<b>09/24/08</b>	<b>1.19</b>	<b>0.01</b>	<b>3.25</b>	<b>0.66</b>
<b>09/25/08</b>	<b>1.23</b>	<b>0.00</b>	<b>3.00</b>	<b>0.66</b>
<b>09/26/08</b>	<b>1.08</b>	<b>0.00</b>	<b>3.00</b>	<b>0.69</b>

Table 1

As recently as in September 2008, for many days, the (average) interest rate in the fed funds market was above the discount window rate. The rate for discount window primary credit was 2.25 during that period. As is clear from Table 2, the high average and standard deviation during the week of September 15 imply that a relatively high proportion of the trade volume was executed at rates significantly above the discount window rate.

## 2 The model

The economy last for three periods,  $t = 1, 2, 3$ . There are a large number of banks and investors which can interact with each other over time, subject to some specific limitations described below. Each bank owns an asset which pays a return  $v$  in period 3 if held to maturity (and zero in periods 1 and 2). The return  $v$  can take one of two values,  $R$  or 0. If the return of the asset is  $R$  we say that the asset is high quality. If it is 0, then we say that the asset is low quality. The probability that

<sup>7</sup>This tendency to borrow at the TAF and not at the discount window is even more evident when the effective cost of discount window funding is computed as the 30-day OIS rate plus the primary credit spread. See Armantier, Krieger, and McAndrews (2008).

the asset is high quality is  $p$ . Then, with probability  $1 - p$  the asset is low quality. Asset quality is realized in period 1 even though the return only becomes available in period 3.<sup>8</sup>

The asset has some degree of specificity that determines that, if sold to another bank, its return  $v$  becomes much lower. Investors, instead, have the ability to manage the asset appropriately after buying it from a bank, and hence to maintain the potential return virtually unaffected. While it could be interesting to consider the case where also some banks are good managers of purchased assets, we do not consider this case here and maintain the stark distinction between banks and investors throughout, just for simplicity. Also for simplicity, assume that investors have deep pockets and can always store their funds from one period to the next in a technology that gives a return  $i$  per unit stored.

At the beginning of period 1, banks get a liquidity shock: Some banks need to make a payment of size 1 and, hence, become illiquid; and some banks receive a payment of size 1 which makes them liquid.<sup>9</sup> Assume that half the banks become liquid and the other half, illiquid. Liquid banks have access to the same storage technology that investors and hence can always obtain the return  $i$  on their positive fund holdings. For many banks participating in the fed funds market this ‘reservation’ return is given by the interest on reserves paid by the central bank. The return  $i$  will play such role in our model.

Illiquid banks, on the other hand, when unable to make their required payment by the end of period 1, suffer a penalty  $\rho$ , with  $i < \rho < R$ . We broadly interpret the penalty  $\rho$  as the costs (explicit and implicit) for the bank of not being able to fund a preestablished commitment through the ‘normal’ funding channels. The premium from incurring an overnight overdraft in the bank’s account at the central bank is one (pecuniary) component of this cost, but there are many other (non-pecuniary) components that are just as important (Clouse and Dow, 2002, p. 1792). For simplicity, we assume the penalty  $\rho$  is all non-pecuniary in our model.<sup>10</sup>

In period 1, after the quality of the assets and the liquidity shocks are realized, banks can interact in an over-the-counter market for funds. Illiquid banks search for liquid banks to obtain immediate funding. Investors cannot participate in this market. An illiquid bank finds a liquid bank with probability  $\sigma$ . When two banks match, the liquid bank can costlessly verify the quality of the asset held by its illiquid counterparty.<sup>11</sup> The two banks in the match then decide whether or not to enter a lending agreement with each other and, finally, bargain over the terms of the loan. For simplicity,

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<sup>8</sup>Our simple three-period model is (in principle) suitable to be incorporated into an infinite-horizon general equilibrium framework like the one studied by Lagos and Rocheteau (2007) following the technical innovations in Lagos and Wright (2005). The research in this paper could be regarded as a first step in that direction.

<sup>9</sup>It is possible to provide further microfoundation for the liquidity shock experienced by banks by introducing a set of depositors with random withdrawal demands as, for example, in Bhattacharya and Gale (1987) and Allen et Al. (2009). For a sophisticated model of banks’ demand for fed funds, including many realistic features, see Clouse and Dow (2002).

<sup>10</sup>The assumption  $\rho < R$  simplifies dealing with payments feasibility, but it is not essential.

<sup>11</sup>In our model, then, lenders have perfect information about the financial conditions of their counterparties in the interbank market. Including information frictions at the lending stage would be an interesting extension to consider. For an early contribution in this direction see Flannery (1996).

we assume that the outcome of the negotiations is determined according to Nash bargaining with  $\theta$  being the bargaining power of the lender.<sup>12</sup> Loan maturity is one period and, at the time of repayment (i.e., in period 2), if a bank is not able to pay back the loan, then it has to surrender its asset to the lender.

In period 2, banks and investors participate in a centralized market in which participants can trade funds and assets with each other, and make payments to each other. Each bank has a probability  $\alpha$  that the quality of its asset becomes publicly observable at the beginning of period 2. With probability  $1 - \alpha$ , the quality of the asset remains unknown to investors and other banks. At the end of period 2 all banks and investors part ways and, consequently, there are no possible (business) interactions in the economy during period 3.

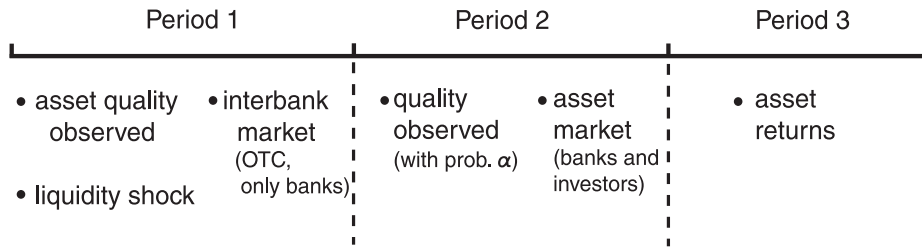


Figure 2: Timing

We assume that investors in period 2 cannot observe whether a bank has borrowed from another bank in period 1. If borrowing were observable, it could act as a signal about the quality of the asset. For example, if being able to take a loan in period 1 were regarded as a positive signal of the quality of the asset, then even a liquid bank with no urgent needs for funds may want to take a loan in period 1 when holding a bad asset. Of course, if the liquidity position of the bank were observable, this would undermine the previous strategy. Alternatively, if not only the loan but also the terms of the loan were observable, this information could become fully revealing in some cases. All this different specifications seem interesting and, in principle, worth studying formally. However, to isolate the possible signaling role of discount window lending and, in this way, maintain the focus of our analysis, we will assume that bank loans taken in period 1 are not observable to investors in period 2.

Basically, the set of frictions that characterize our environment are designed to capture a situation where some banks own illiquid assets but have an immediate need for funding. While, in principle, there is enough funds in the economy (on investors' hands) to cover all immediate needs, banks cannot access such liquidity directly. Instead, in the short run, illiquid banks can only trade with other banks in a market with frictions. Trade in this market is based on the premise that banks will have access to investors' funds in the medium term. In summary, illiquid banks have resources in period 3 that they need in period 1. They effectively transfer (at least part of) those resources

<sup>12</sup>See Bartolini et Al. (2005) for evidence that suggests that the relative bargaining power of borrowers and lenders plays a significant role in the determination of interest rates in the fed fund market.

to period 2 by trading with investors and, to period 1, by taking loans from liquid banks. We are interested in studying the implications of private information in this process of intertemporal reallocation of funds via borrowing and asset trading.<sup>13</sup>

### 3 Equilibrium

We will solve for a Perfect Bayesian Equilibrium of this economy. To do so, we proceed by backward induction. We start by computing asset prices in period 2 given investors' beliefs about the trading strategies of banks. Then, in period 1, illiquid banks look for liquid banks and when matched, negotiate over the terms of a loan taking into account their equilibrium prediction about asset prices in period 2. In equilibrium, the interactions in period 1 confirm the beliefs of investors in period 2.

Two important properties of the equilibrium loan contracts result directly from the assumed isolation of agents in period 3. First, all contract among banks involve one-period loans (from period 1 to period 2); and second, upon default in period 2, the lending bank will take possession of the borrower assets and sells them immediately to investors in the market.

#### 3.1 Observable asset quality

Suppose that  $\alpha = 1$ ; that is, investors in period 2 can perfectly observe the quality of the assets being sold in the market. Then, since low quality assets give zero return in period 3, investors are not willing to pay any positive amount for low quality assets in period 2. On the other hand, due to competition among investors, a high quality asset can be sold at price  $R$  in period 2.

In period 1, if an illiquid bank holding a low quality asset finds a liquid bank, the former will not be able to borrow from the latter. The lender in this case will anticipate that the borrower will have no funds to pay back the loan in period 2. Furthermore, by taking possession of the asset in period 2, the lender cannot sell the asset for any positive amount. In other words, an illiquid bank holding a low quality asset has no borrowing capacity in period 1 and, hence, will get no loan.

The situation is different if the illiquid bank is holding a high quality asset. In this case, if the illiquid bank finds a liquid bank then it will be able to take a loan from the liquid bank. After agreeing on a loan, the two banks will bargain over the interest rate, denoted by  $r_H$ . In particular, the interest rate will solve the following problem:

$$\max_{r_H \leq R} (r_H - i)^\theta [(R - r_H) - (R - \rho)]^{1-\theta}$$

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<sup>13</sup>See Acharya et Al. (2007) for another model where the interaction between the interbank market and the asset market plays a crucial role. In the terminology of Bolton et Al. (2009), we assume that there are two distinct sources of outside liquidity, bank loans and investors funds, and no inside liquidity. While bank loans are available in short notice, access to investors funds take time and involve the sale of assets of (possibly) uncertain quality. We study the interaction between these two markets of outside liquidity.

Since  $\rho < R$  the solution to this problem is given by  $r_H = i + \theta(\rho - i)$ . Note that the interest rate is increasing on the bargaining power of the liquid bank and is always below the penalty rate  $\rho$ .

In equilibrium, only illiquid banks that find a liquid bank and hold a high quality asset are able to take loans in the interbank market in period 1. Hence, the interbank market interest rate is given by  $r_H$ . Note that whenever  $\theta$  is positive  $r_H$  is greater than the risk-free opportunity cost of funds in period 1 which is given by  $i$ . This premium over the risk-free rate is the result of bargaining power by lenders and not default risk. In this equilibrium, banks that could be expected to default do not get loans in period 1 and, hence, do not influence the *observed* interest rates in the interbank market.

### 3.2 Unobservable asset quality

Suppose now that the quality of the asset held by a bank becomes observable in period 2 only with a probability less than one; that is, assume that  $0 \leq \alpha < 1$ . As in the previous section, when the quality of the asset becomes observable the price is equal to  $R$  if the asset is high quality and zero if the asset is low quality.

The more interesting case is when the quality of the asset is not observed. In this case, pricing in period 2 will depend on the beliefs of investors about the relative prevalence of high and low quality assets in the market (as in Eisfeldt, 2004). Let  $q$  be the (equilibrium) belief of investor that a given asset being sold in the market in period 2 is high quality. Then, the price of an asset of unobserved quality,  $P_U$ , will be equal to  $qR$ .

We need to determine now the possible equilibrium values of  $q$ . The first thing to note is that whenever  $q < 1$  all liquid banks holding assets of high unobserved quality will not want to sell the asset to investors in period 2. Similarly, whenever  $q \geq 0$  all banks (liquid and illiquid) holding assets of low unobserved quality will want to sell their assets in the market. What makes equilibrium determination nontrivial is the action of illiquid banks holding assets with high unobserved quality. These banks may or may not take a loan in period 1 depending on the value of  $q$ . In turn, whether these banks take a loan or not determines the relative prevalence of high quality assets in the market and, hence, the values of  $q$  consistent with equilibrium.

**Proposition 1.** When  $\rho - i - (1 - \alpha)R < 0$  there is an equilibrium with  $q = 0$ .

**Proof:** Recall that we are assuming that  $R > \rho > i$ . Then, we can rewrite the condition in the proposition as  $\alpha R - (R - \rho) < i$ . We will now show that when this condition holds and  $q = 0$  no loans are made in period 1. The reason for this is as follows. If an illiquid bank with a high quality asset does not take a loan in period 1, his payoff is  $R - \rho$ . Hence, this bank should get at least as much in expected terms from entering a loan contract. Since  $q = 0$  the maximum expected payoff obtainable from the asset is  $\alpha R$ . Then, a borrower can get a maximum expected repayment equal to  $\alpha R - (R - \rho)$ , but he can get  $i$  from not making the loan. So, under the condition of the proposition, if banks expects that investors will not be willing to pay for an asset of unobserved quality (i.e., if

$q = 0$ ), it is not possible to have the liquid and illiquid bank agreeing on a feasible loan contract. But then, since illiquid banks with assets of high unobserved quality do not have a loan to repay, they have no reason to sell their assets (they get zero from doing so, instead of  $R$ ). Therefore, only low quality assets will be put for sale in period 2, which is consistent with the belief expectation  $q = 0$ . #

The proposition gives us a condition under which the asset market in period 2 could shut down and, in anticipation of that fact, illiquid banks get screened out of the loan market in period 1 even when they are holding a high quality asset. It is interesting to note that the condition is more likely to hold when the probability  $\alpha$  that the quality of an asset will become observable in period 2 is low; that is, when the information frictions in the asset market are expected to be large.<sup>14</sup>

This “no credit” equilibrium does not exist if  $\rho - i - (1 - \alpha)R > 0$ . Furthermore, even if the condition of the proposition is satisfied, another equilibrium with credit in the interbank market may be possible. We study such equilibrium with credit next.<sup>15</sup>

Suppose now that  $q > 0$  in equilibrium. In such case, we know that in period 2 there will be  $(1 - \alpha)(1 - p)$  low unobserved quality assets in the market. Furthermore, for  $q > 0$  to be (part of) an equilibrium, it must be true that the high quality assets of illiquid banks that manage to obtain a loan in the interbank market are put for sale in period 2 (otherwise  $q$  would be equal to zero). We provide parameter conditions below for which this is the case.<sup>16</sup>

Since in this equilibrium the total amount of high quality assets in the market will then be equal to  $(1 - \alpha)\frac{1}{2}p\sigma$ , consistent beliefs are given by:

$$q^* \equiv \frac{p\sigma}{2(1 - p) + p\sigma}.$$

Let  $TS(q) = \rho - i - (1 - \alpha)(1 - q)R$  be the expected total surplus from a loan relationship between a liquid bank and an illiquid bank holding a high quality asset. As we will see, if  $TS(q^*) > 0$  then an illiquid bank holding high quality assets will be able to obtain a loan from a liquid bank whenever the two of them match in period 1. The negotiated value of  $r_H$  will, in turn, determine how this surplus gets divided between the two parties.

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<sup>14</sup>Changes in the quality of the asset, as reflected by changes in the return  $R$ , have two opposing effects. On the one hand, an increase in  $R$  increases the availability of funds for repayment; but, on the other hand, it increases the outside option for the potential borrower, reducing his incentives to take the loan. In our setup, the second effect dominates and, as a consequence, increases in  $R$  make the possibility of a shut-down of the interbank credit market compatible with a larger set of values for the other relevant parameters.

<sup>15</sup>Heider, Hoerova, and Holthausen (2009) study a model of the interbank market with private information about the riskiness of banks’ asset holdings and, hence, about banks’ repayment risk. They discuss the possibility of equilibria with no credit, similar to the one studied here. In their model, however, adverse selection in the interbank market is the result of (exogenously) assuming that safer assets have lower liquidation costs. For a model where the market for bank assets freezes due to reasons unrelated to asymmetric information and adverse selection, see Diamond and Rajan (2009).

<sup>16</sup>Note that, in this equilibrium, if the borrowing bank does not pay the loan, the high quality asset is still sold in the market by the liquid bank which acquires it upon default. Alternatively, we could assume that whenever indifferent the borrower will sell the asset in the market and try to pay as much as possible of its debt.

Given a value of  $r_H$  the surplus for the borrower is given by:

$$S_b(r_H, q^*) = \alpha(R - r_H) + (1 - \alpha) \max \{q^*R - r_H, 0\} - (R - \rho)$$

and the surplus for the lender is given by:

$$S_l(r_H, q^*) = \alpha r_H + (1 - \alpha) \min \{q^*R, r_H\} - i.$$

Note, of course, that  $S_b(r_H, q^*) + S_l(r_H, q^*) = TS(q^*)$ . The equilibrium interest rate, then, solves the following Nash bargaining problem:

$$\max_{r_H} S_l(r_H, q^*)^\theta S_b(r_H, q^*)^{1-\theta}$$

subject to  $S_l(r_H, q^*) \geq 0$  and  $S_b(r_H, q^*) \geq 0$ . Define the functions  $\tilde{r}_H(\theta, q^*) \equiv i + \theta TS(q^*)$  and  $\hat{r}_H(\theta, q^*) \equiv \frac{1}{\alpha} [i - (1 - \alpha)q^*R] + \frac{\theta}{\alpha} TS(q^*)$ . Then, we have that the solution to the Nash bargaining problem is given by:

$$r_H(\theta, q^*) = \begin{cases} \tilde{r}_H(\theta, q^*) & \text{if } \theta < \theta^T(q^*) \\ \hat{r}_H(\theta, q^*) & \text{if } \theta \geq \theta^T(q^*) \end{cases}$$

where  $\theta^T(q^*) = \max \{0, \min \{(q^*R - i)/TS(q^*), 1\}\}$ . Note that when  $\theta^T(q^*) \in (0, 1)$  we have that  $\tilde{r}_H(\theta^T(q^*), q^*) = \hat{r}_H(\theta^T(q^*), q^*) = q^*R$ .<sup>17</sup>

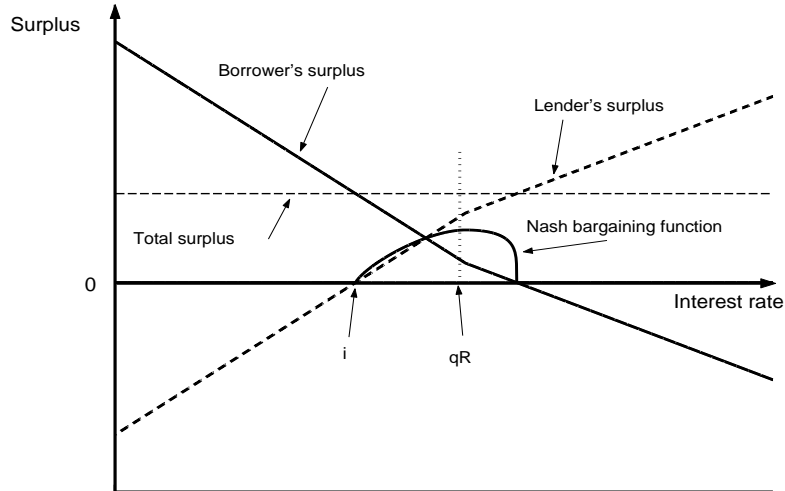


Figure 3: The Nash bargaining solution

After some algebra, it can be shown that  $S_b(r_H(\theta, q^*), q^*) = (1 - \theta)TS(q^*)$  and  $S_l(r_H(\theta, q^*), q^*) =$

<sup>17</sup>Since  $R > \rho$  we have that  $r_H(\theta, q^*) < R$  for all  $(\theta, q^*)$  and, in consequence,  $r_H(\theta, q^*)$  is always reasonable in the sense that there are at least some borrowers that are able to pay as much for a loan.

$\theta TS(q^*)$ . Hence, as long as  $TS(q^*)$  is positive both the liquid and illiquid bank in a match will agree to participate in a loan agreement. Define the threshold value  $q^T$  as:

$$q^T \equiv \max \left\{ 0, \frac{(1 - \alpha)R - (\rho - i)}{(1 - \alpha)R} \right\},$$

such that  $TS(q) \geq 0$  for all  $q \geq q^T$ . For concreteness, let us assume that whenever indifferent, banks enter a loan relationship. Then, we have the following proposition that provides conditions on parameter values such that an equilibrium with interbank credit exist.

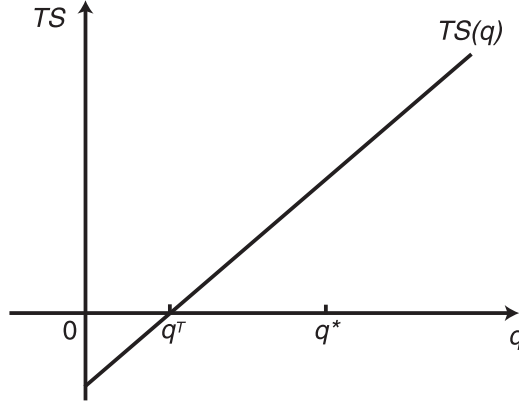


Figure 4: The surplus function

**Proposition 2.** When  $q^* \geq q^T$  there is an equilibrium with interbank credit.

**Proof:** Suppose that investors conjecture that, of the unobserved quality assets for sale in period 2, a proportion  $q^*$  are high quality. Then, the expected price of unobserved quality assets in period 2 is  $q^*R$ . Since  $q^* > 0$  all banks holding a low unobserved quality asset will sell it in period 2. Similarly, if a bank holding a high unobserved quality asset in period 2 has taken a loan in period 1, then its asset will be sold in the market in period 2, either by the bank itself or by its loan counterparty. Also, since  $q^* < 1$ , banks who do not manage to find a counterparty in the interbank market in period 1 and are holding a high unobserved quality asset, will not sell their asset in period 2 (since they have no loan to repay). Finally, since  $q^* \geq q^T$  we have that  $TS(q^*) \geq 0$  and, hence, illiquid banks holding a high quality asset take a loan from liquid bank whenever they find a counterparty in period 1. We have, then, that the assets of all banks with low unobserved quality, and of all banks with high unobserved quality and a loan to repay, will be sold in the market in period 2. This implies that the expected value of assets of unobserved quality being sold in period 2 is  $q^*R$ , which is consistent with the investors' initial conjecture. #

Note that, in this equilibrium, illiquid banks holding low quality assets may or may not receive credit depending on whether  $(1 - \alpha)q^*R$  is greater than or less than  $i$ , respectively. To see this, denote by  $r_L$  the interest rate arranged by these banks in a loan agreement. It only makes sense

to consider values of  $r_L$  less than or equal to  $q^*R$ , since this is the maximum amount that a lender could obtain in period 2 from a borrower holding a low quality asset. It is clear, then, that the lender, in expected terms, can get no more than  $(1 - \alpha)q^*R$  from the borrower, and if this quantity is less than  $i$ , the lender would not agree to participate in the loan. Note, also, that this implies that a loan may not take place even if the surplus from the loan agreement  $\rho - i$  is positive (as we assume it is).<sup>18</sup>

**Corollary 1.** When  $q^* \geq q^T > 0$  there are two equilibria, one with interbank credit and one where interbank credit shuts down.

**Proof:** Note that the parameters determining  $q^*$  are different from those determining  $q^T$ . Then, we can easily find parameters such that the conditions in the corollary hold. In such case, since  $q^T > 0$  implies that the condition in Proposition 1 holds, we have that a "no credit" equilibrium exists. Furthermore, since  $q^* \geq q^T$ , by Proposition 2, an equilibrium with interbank credit also exist. #

This corollary tells us that, for a set of the parameter values, the model is consistent with multiple equilibria. Furthermore, these equilibria have significantly different implications for the outcomes on the interbank credit market. If banks expect that pessimistic investors will price assets of uncertain quality, then they may not be willing to enter into loan relationships, and this shutdown of the credit market, in turn, will result in a selective reduction of bank participation in the asset market (i.e., high-quality-asset holders will be out of the market), which would justify investors' initial pessimism.

**Corollary 2.** When  $(1 - \alpha)R \leq \rho - i$  we have that  $q^T = 0$  and there is a unique equilibrium with interbank credit. The equilibrium price of the unobserved quality asset is  $P_U = q^*R$ .

**Proof:** Since  $(1 - \alpha)R \leq \rho - i$  we have that  $TS(q) \geq 0$  for all  $q \in [0, 1]$ . Then, high quality illiquid banks that find a match in the interbank credit market always establish a loan relationship. This implies that the assets of those banks will be put for sale in the market in period 2 and, hence, that the only consistent equilibrium value of  $q$  is  $q^*$ . #

**Lemma 1.** When the threshold value  $q^T$  is greater than zero, it is increasing in  $R$  and  $i$  and decreasing in  $\rho$  and  $\alpha$ . The equilibrium value  $q^*$  is increasing in  $p$  and  $\sigma$ .

**Proof:** The results can be obtained by simple differentiation of the expression for  $q^T$  and  $q^*$ . #

The results on this lemma can be given interesting interpretations. For example, according to the lemma, higher values of  $\alpha$  make the credit equilibrium consistent with a larger set of parameter values. More broadly, then, we can say that less informational frictions in the asset market make

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<sup>18</sup>Recall that  $\rho$  is a cost incurred by the bank if it cannot fund its liquidity shock in period 1. When the surplus from the loan agreement is positive and the loan does not happen, the illiquid bank would like to be able to use some of the resources dedicated to cover  $\rho$  to make a payment to the potential lender. We assume that  $\rho$  includes non-pecuniary costs (such as increased scrutiny by regulators) so these resources are not available for such type of payment. Alternatively, we could think that the emergency funding used to pay  $\rho$  is restricted and cannot be use in period 2 for the purpose of debt repayment.

interbank credit more likely in our model.<sup>19</sup> Similarly, since  $q^*$  is increasing in  $\sigma$ , we can say that the potential for a more liquid credit market makes the credit equilibrium more likely to arise. Perhaps somewhat less intuitive is the case of different values for  $R$ , the return of high quality asset. Higher values of  $R$  make the credit equilibrium less likely. The reason for this fact is that when a bank holding a high quality asset enters a credit relationship, it anticipates that with some probability it will have to sell the asset when its quality is not observed. This sale entails a loss proportional to  $R$  which discourages credit relationships ex ante.

The last proposition of this section provides a characterization of the equilibrium interest rate in the interbank market for credit when such a market is open. An important aspect of this characterization is the determination of the conditions under which a borrower is able to pay the arranged interest rate when the quality of its asset is not observable by investors. Recall that if the borrower cannot pay the interest rate, the asset changes hands and gets sold by the lender. Whether this happens or not depend on how  $r_H(\theta, q^*)$  compares with  $q^*R$ . If  $r_H(\theta, q^*) > q^*R$  then, when the quality of the asset is not observed by investors, the borrower cannot pay the arranged interest rate.

**Proposition 3.** Assume  $q^* \geq q^T$ . If  $\alpha i \geq \rho - (1 - \alpha)R$  then there is an equilibrium with interbank credit in which the interest rate is given by  $r_H(\theta, q^*) = \hat{r}_H(\theta, q^*) > q^*R$  for all  $\theta$ . If  $\alpha i < \rho - (1 - \alpha)R$  then there is an equilibrium with interbank credit in which the interest rate is given by the following expressions:

- 1) If  $q^*R \leq i$  then  $r_H(\theta, q^*) = \hat{r}_H(\theta, q^*) > q^*R$  for all  $\theta$ .
- 2) If  $i < q^*R < \frac{1}{\alpha}[\rho - (1 - \alpha)R]$  then  $\theta^T(q^*) \in (0, 1)$  and

$$r_H(\theta, q^*) \begin{cases} \tilde{r}_H(\theta, q^*) < q^*R & \text{if } \theta < \theta^T(q^*) \\ \hat{r}_H(\theta, q^*) > q^*R & \text{if } \theta \geq \theta^T(q^*) \end{cases}$$

- (3) If  $\frac{1}{\alpha}[\rho - (1 - \alpha)R] \leq q^*R$  then  $r_H(\theta, q^*) = \tilde{r}_H(\theta, q^*) < q^*R$  for all  $\theta$ .

**Proof:** See the Appendix. #

In this section, we have studied the functioning of an interbank market for funds in the presence of frictions that limit the ability of banks to trade with each other. As a result of these frictions, some banks are not able to borrow during period 1 even when they hold high quality assets that have a present value larger than the face value of the loans that the banks seek to obtain. A natural question to ask is how would equilibrium outcomes change if a central bank lending facility (i.e., a discount window) is available to all banks in this environment. In the next section, we extend the model to allow for discount window lending and describe the equilibrium conditions corresponding to such case.

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<sup>19</sup>Note, however, that higher values of  $\alpha$  imply higher repayment risk for holders of low-quality asset. Hence, higher values of  $\alpha$  may be associated with less volume of trade in the interbank market, as only banks with high quality assets get credit.

## 4 Discount window lending

Assume now that all banks have access in period 1 to the central bank’s discount window, where they can obtain loans at the interest rate  $r_W > 0$ . We will assume that discount window lending is uncollateralized and perfectly observable by all agents in the economy. These are extreme, simplifying assumption, but not essential.<sup>20</sup>

Discount window loans in the US are fully collateralized. We could easily amend our notation to include the opportunity cost of holding collateral as part of the cost of borrowing from the discount window. A more delicate issue would be to deal with a more realistic treatment of asset holdings by banks in our model. The Fed carefully assesses the value of the assets that are pledged as collateral for discount window borrowing purposes. Presumably, banks holding what we are calling low quality assets would not be able to access the Fed’s primary credit program.<sup>21</sup> However, it should be kept in mind that asset characteristics are very stark in our simple framework. Introducing a more complete taxonomy of assets, with various degrees of riskiness and liquidity, would permit us to discriminate between banks in sound financial condition and those that, while able to pledge collateral with the central bank, are regarded as representing a significant repayment risk in the interbank market for overnight loans. Our extreme assumption here makes this discrimination simple and allows us to, then, more clearly identify the economic mechanism that results from it.

Similarly, in terms of information assumptions, we also trade off realism for simplicity. In the U.S., discount window borrowing is not perfectly observable. The Federal Reserve periodically announces the total amount of discount window lending granted the previous two weeks. This is, potentially, a very noisy signal of the participation of particular banks in the reported discount window activity. However, under certain circumstances, market participants may be able to put together various pieces of information (like a prior funding request by a particular institution, for example) which, in combination with the Fed’s reported number, may partially reveal the identity of the borrowing banks (Furfine, 2001). While we consider these issues interesting, they are secondary to the main objective of this paper. Again, it would not be difficult to make our model more general in this direction. All we really need for our results is that agents obtain informative signals about the other banks’ activity at the discount window.

In the U.S., in principle, banks could borrow funds at the discount window to lend them later to other banks in the fed funds market. We do not consider this possibility in our model. Liquid banks

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<sup>20</sup>In Acharya et Al. (2007) discount window loans play a related funding role by reducing illiquid banks’ exposure to the risk of having to sell their assets in the market at a very significant loss. Here, the market for assets is actually closed at the time the illiquid bank needs funding in period 1. In a sense, our assumption is an extreme version of that considered in Acharya et Al. (2007).

<sup>21</sup>Depository institutions in the US have access to three types of discount window credit: primary credit, secondary credit, and seasonal credit. Primary credit is available to depository institutions that are in sound financial condition. Its provision is associated with minimal administrative requirements and its usage is essentially unrestricted. Secondary credit is available to depository institutions that are not eligible for primary credit. It is provided only in particular situations and the institutions borrowing from the secondary credit program are closely monitored by the Fed. Seasonal credit is provided to assist small depository institutions to manage seasonal swings in loans and deposits.

meet at most one illiquid bank in period 1. Since they have access to funds at the opportunity cost  $i$ , they have no incentives to take loans from the discount window at rates higher than  $i$ . In our analysis below, we assume that  $r_W$  is greater than  $i$  in all cases, so the lend-to-borrow strategy is not profitable for liquid banks. Some illiquid banks could also try to borrow extra at the window to later lend to other illiquid banks. For simplicity, we assume that each bank can take only one side in the market. This assumption is a feature of the matching technology and rules out the lend-to-borrow strategy for illiquid banks. More generally, all one needs to assume is that search frictions in the market for interbank loans limit the ability of banks to arbitrage interest-rate differentials by following the lend-to-borrow strategy at the discount window. This general premise, and not the particular details used to capture it in the model, is what is crucial for the theory in our paper.

We solve, again, for Perfect Bayesian equilibrium. To do so, it is convenient to start by identifying possible outcomes in the market for assets during period 2. As before, when the quality of an asset becomes observable, the price of the asset is either  $R$  or zero depending on whether the asset is of high or low quality, respectively.

When the quality of the asset is not observable, things become more complicated. Whether a bank borrowed at the discount window or not could be an informative signal about the quality of the asset that the bank is trying to sell. This possibility is the result of two important assumptions in our model. On the one side, banks in the interbank market are able to obtain some accurate information about the quality of the asset held by counterparties, which in turn influences their lending behavior. On the other side, sometimes investors are not able to observe directly the quality of the asset being traded, nor the asset-holder's private dealings in the interbank market, but do get to observe the asset-holder's past transactions with the central bank.

In period 2, then, investors form beliefs about the quality of a given asset of unobserved quality which depend on whether the asset holder has borrowed or not at the discount window. Let  $q_W$  be the belief probability that the asset is high quality if the holder has borrowed at the window; and let  $q_N$  be the corresponding probability (of high quality) if the holder has not borrowed at the window. These are equilibrium beliefs that will depend also on the decisions taken by all banks, given those beliefs. We study bank decision next.

Illiquid banks that do not find a liquid counterparty in the interbank market have to decide whether or not to borrow from the discount window. For this decision, the bank compares the payoff of taking each possible action. To calculate this payoff, we assume that all banks which have borrowed at the window sell their asset in period 2 to pay back the loan (in full or partially).<sup>22</sup> Define  $P(q_W, r_W) \equiv \max \{q_W R - r_W, 0\}$ . Then, an illiquid bank that has not found a counterparty

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<sup>22</sup>It could be interesting to consider alternative treatments of those borrowers that cannot repay discount window loans in full. In the simple case we study here, loans from the discount window differ from loans granted by private counterparties only in the way the interest rate is determined. At the window, the rate is exogenously set and is not contingent on asset quality.

and is holding a high quality asset will borrow at the window if:

$$\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W) \geq -\rho + R;$$

and a bank holding a low quality asset will borrow at the window if:

$$(1 - \alpha)P(q_W, r_W) \geq -\rho + (1 - \alpha)q_N R.$$

Illiquid banks that do find a counterparty in period 1 must decide among three possible alternatives: they could either borrow from the liquid bank, from the window, or not borrow at all. If an illiquid bank borrows in period 1, it will have to sell its asset in period 2 to repay (all or some of) the loan. Define  $P(q_N, r_j) \equiv \max\{q_N R - r_j, 0\}$  with  $j = H, L$ , where  $r_j$  is the interest rate on a loan from a liquid bank to an illiquid bank holding a  $j$  asset. Then, an illiquid bank that finds a liquid counterparty and is holding a high quality asset will agree to take the loan if:

$$S_b(r_H, q_N, q_W) = \alpha(R - r_H) + (1 - \alpha)P(q_N, r_H) - \max\{\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W), -\rho + R\} \geq 0.$$

An illiquid bank that finds a liquid bank in period 1 and is holding a low quality asset will enter a lending relationship with the liquid bank if:

$$S_b(r_L, q_N, q_W) = (1 - \alpha)P(q_N, r_L) - \max\{(1 - \alpha)P(q_W, r_W), -\rho + (1 - \alpha)q_N R\} \geq 0.$$

Finally, we need to consider the decision of liquid banks upon entering a match with an illiquid bank. If the illiquid bank is holding a high quality ( $H$ ) asset, then the liquid bank will agree to make a loan if and only if:

$$S_l(r_H, q_N) = \alpha r_H + (1 - \alpha) \min\{q_N R, r_H\} - i \geq 0.$$

Similarly, when the illiquid bank is holding a low quality asset, the liquid bank will agree to make a loan if and only if:

$$S_l(r_L, q_N) = (1 - \alpha) \min\{q_N R, r_L\} - i \geq 0.$$

Define the total surplus in a match as  $TS_j(q_N, q_W) = S_b(r_j, q_N, q_W) + S_l(r_j, q_N)$  for  $j = H, L$ . Whenever the total surplus in a match is positive, banks will agree to enter a lending relationship and will negotiate over the interest rate. The outcome of such negotiation is the solution to the following Nash problem for  $j = L, H$ :

$$\max_{r_j} S_l(r_j, q_N)^\theta S_b(r_j, q_N, q_W)^{1-\theta}$$

subject to  $S_l(r_j, q_N) \geq 0$  and  $S_b(r_j, q_N, q_W) \geq 0$ . Call the solution to this problem  $r_j(\theta, q_N, q_W)$  for

$j = L, H$ .

In period 2, those banks that have taken a (private or discount window) loan in period 1 will sell their asset in the market. If  $q_N > 0$  then all banks holding a low quality asset will sell their asset even if they do not have a loan to repay. If  $q_N < 1$  then banks holding a high quality asset that do not have a loan to repay will not sell their asset. These cases exhaust all the possibilities.

A Perfect Bayesian equilibrium, then, can be characterized by a set of beliefs  $(q_N, q_W)$ , loan agreements with the corresponding interest rates, and asset sales and prices such that: (1) all agents make optimal lending and asset sale decisions given those beliefs (as described above); (2) asset prices reflect those beliefs; and (3) the agents' decisions validate those equilibrium beliefs in the sense that they are the result of applying Bayes Rule on equilibrium outcomes (i.e., a fixed point in beliefs).

We close this section by providing some general lemmas that can be use to simplify certain equilibrium expressions and to facilitate the construction of an equilibrium in our model.

**Lemma 2.** In any equilibrium with private lending to illiquid banks holding low quality asset the following condition holds:

$$\frac{i}{1 - \alpha} \leq r_L(\theta, q_N, q_W) \leq q_N R.$$

**Proof:** The maximum amount that an illiquid bank can repay is  $q_N R$ . Hence, only values of  $r_L$  lower that  $q_N R$  are relevant. Furthermore, if  $(1 - \alpha)r_L < i$  the liquid bank will not accept to lend. #

Note that this lemma can be used to simplify the equilibrium expressions for  $S_b(r_L, q_N, q_W)$  and  $S_l(r_L, q_N)$ . The next lemma shows that, in equilibrium, if an illiquid bank holding a high quality asset borrows at the discount window when it cannot find a counterparty, then so does an illiquid bank that is holding a low quality asset and is in the same situation (i.e., cannot find a counterparty).

**Lemma 3.** If the condition  $\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W) \geq -\rho + R$  holds, then condition  $(1 - \alpha)P(q_W, r_W) \geq -\rho + (1 - \alpha)q_N R$  also holds.

**Proof:** The proof follows from the fact that  $q_N \leq 1$  and  $r_W > 0$ . #

In principle, there are many possible configurations of equilibrium outcomes in this model, depending on parameter values. Furthermore, as in the previous section, for some parameter values, multiple equilibria may exist. Studying the different cases can provide interesting insights about the influence of discount window policy on interbank lending activity. In the next section, we take a partial step in this direction. To keep the analysis focus on the issue of stigma, first we study equilibrium for a particular range of parameter values for which the possibility of stigma is present. Later, we discuss other possible equilibrium configurations.

## 5 Stigma

The objective in this section is to study, in the context of the model, the empirical and theoretical arguments discussed in the introduction of the paper. In particular, we want to construct an equilibrium in which stigma is attached to lending from the discount window and, for this reason, some banks take loans in the interbank market at rates higher than the discount window rate  $r_W$ . In the process, we identify conditions under which such a situation is theoretically possible and draw some conclusions about its empirical plausibility.

We consider the case when  $r_W = i + \tau$  for  $\tau > 0$ ; that is, lending at the discount window involves paying a penalty rate (i.e., the rate  $r_W$  is greater than the opportunity cost of funds,  $i$ ).

**Proposition 4.** Define  $\xi(p) = \frac{p-\sigma p}{1-\sigma p}$  and assume that  $\tau \in (0, A)$  where  $A = \min\left\{\rho - i, \frac{\alpha}{1-\alpha}i\right\}$ . Then, there exist a threshold value  $\bar{p}_\tau < 1$  such that if  $p \in (\bar{p}_\tau, 1)$  then there is an equilibrium with both interbank credit and discount window lending in which  $q_W = \xi$ ,  $q_N = 1$ , and  $r_H = i + \theta TS_H(q_N, q_W)$ .

**Proof:** See the Appendix. #

In this equilibrium, only those illiquid banks that find a counterparty in the interbank market and are holding a high quality asset, borrow from another bank. Illiquid banks that do not find a counterparty, plus those banks that do but are holding a low quality asset, borrow from the discount window. Note that this configuration does not necessarily imply unrealistic levels of borrowing at the discount window. In fact, if both  $p$  and  $\sigma$  are close to unity, as they are likely to be in the empirically relevant case, then most banks actually borrow from the interbank market in this equilibrium.

Note that if  $\tau$  is greater than  $A$ , that is, if the penalty rate from borrowing at the discount window is too large, then all illiquid banks that find a counterparty in the market will take a private loan. If this is the case and all banks borrowing at the window do so for exogenous reasons (i.e., because they did not find a counterparty) and not as the product of their choice, then there would be no adverse selection associated with banks' participation in central bank lending. As a consequence, having borrowed at the window could not be, in equilibrium, an informative signal about the quality of the assets held by the bank, and stigma would no longer be possible.

The following corollary provides our main result dealing with the comparison between the equilibrium interest rate in the interbank market and the discount window rate.

**Corollary 3.** In the equilibrium described in Proposition 4, for  $\theta$  close enough to unity,  $r_H > r_W$ .

**Proof:** Note that  $\theta$  appears only in the condition that determines the surplus splitting rule between the liquid and the illiquid bank holding a high quality asset. Then, the existence of the equilibrium described in Proposition 4 is independent of the value of  $\theta$  and the equilibrium exists for any value of  $\theta$ , including those arbitrarily close to one. Since in such equilibrium we have that  $r_H = i + \theta TS_H(q_N, q_W) = i + \theta [(1 - \alpha)(1 - q_W)R + r_W - i]$  which approaches  $(1 - \alpha)(1 - q_W)R + r_W > r_W$  when  $\theta$  approaches unity, the corollary holds. #

This corollary demonstrates that when the bargaining power of lenders is high, the equilibrium in our model may involve some banks that are willing to take a loan in the interbank market at a rate higher than the rate that they could obtain at the discount window. That is, under certain conditions, our model predicts the empirical pattern of interest rate that we discussed in the introduction.

The equilibrium premium of the observed private rate  $r_H$  over the discount window rate  $r_W$  is given by:

$$r_H - r_W = \theta(1 - \alpha)(q_N - q_W)R - (1 - \theta)\tau$$

which is actually decreasing in  $\tau$ . Hence, lowering the spread  $\tau$ , a policy parameter, would tend to reduce the extent to which some transactions in the market are executed at a rate higher than the discount window rate. This is an interesting finding. A common reaction by policymakers to the reluctance by banks to borrow from the window has been to lower the penalty spread.<sup>23</sup> Our model predicts that such a change should also reduce the extent by which observed market rates are higher than the discount window rate as a result of stigma.

In the equilibrium of Proposition 4, the proportion of banks lending from the discount window is equal to  $1 - \sigma p$ . During normal times, most banks in the US do not borrow from the discount window. This empirical regularity suggests that the cases where the proportion of banks borrowing in the market,  $\sigma p$ , is high are the relevant ones. Since  $q_W$  depend on  $\sigma p$ , accommodating this fact has implications for the level of stigma that can be obtained in the model. However, it should be noted that, in principle, the model is consistent with relatively low values of  $q_W$  even when  $\sigma p$  is high. Then, significant levels of stigma and equilibrium interest rate premia, as measured by  $r_H - r_W \in (0, (1 - \alpha)(1 - q_W)R)$ , can be obtained in “realistic” versions of the model.

The threshold value of  $A$  in Proposition 4 is positively related with the value of  $\alpha$ , the probability that the quality of the asset held by a bank becomes observable by investors. In other words, when information about asset quality is more certain, the range of values of  $r_W$  for which the equilibrium in Proposition 4 can exist is larger. However, Corollary 3 also shows that the range of premia over the discount window rate that an illiquid bank would be willing to accept from a private counterparty is decreasing in the value of  $\alpha$ . Then, while more information in asset markets can make stigma more likely for a given value of the discount window rate, the intensity with which equilibrium stigma is reflected in the observable variables actually decreases with such information.

In our model, the level of the interest rate that is observed in the market is given by  $r_H$ , which does not involve any repayment risk. Banks with low quality assets (which could be regarded as the risky ones in our setup) do not receive loans from private banks. They are just cut out of the interbank market. Furthermore, all banks borrowing at the market pay the same interest rate. In the data, however, any given day there is a distribution of rates observed in the market. We could

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<sup>23</sup>For a long time, since 2003, the Fed provided discount window credit at a rate 100 basis points over the target fed funds rate. Recently, with the advent of the 2007 financial crisis and, in part, to respond to the persistent reluctance of banks to borrow from the window, the Fed lowered the spread significantly, first to 50 basis points in August 2007, and later to 25 basis points, in March 2008.

generalize the model to capture these different rates by, for example, introducing some heterogeneity in the bargaining power of different banks. This modification may, in fact, be realistic (Ashcraft and Duffie, 2007). Banks in need for funds in a given day may find that their usual counterparty has no funds available that day. In that case, they need to search in the market for alternative counterparties, and depending on their network connections, they may find their bargaining power much reduced. In our model, illiquid banks with low bargaining power will pay higher interest rates. In fact, this kind of heterogeneity will be consistent with the fact that, most of the times, only some banks pay interest rates that are higher than the one they could obtain at the discount window.

An interesting feature of the equilibrium in Proposition 4 is that it requires  $p$  to be above a certain positive threshold. If we think that lower values of  $p$  are associated with a general deterioration of asset quality, our model tells us that the kind of equilibrium we are considering will not be possible when asset quality deteriorates beyond some point. A reason for why our equilibrium may break down as the value of  $p$  becomes lower is that some banks may restrain from borrowing at the window if the equilibrium value of  $q_W$  is too low (as it needs to be if the value of  $p$  is low). To be concrete, if the value of  $q_W$  is too low, it may be the case, for example, that it is no longer beneficial for banks holding low quality assets to borrow at the window (see the analysis in the next section). But then, illiquid banks that find a match will borrow in the market and illiquid banks that do not find a match will borrow at the window. In consequence, the composition of banks borrowing at the window would be the same as that of banks borrowing in the market (a proportion  $p$  of banks holding high quality assets and a proportion  $1 - p$  holding low quality asset). Hence, borrowing at the discount window will not be regarded as a negative signal and the equilibrium of the model will no longer be consistent with the possibility of stigma.

Note that higher values of the probability of finding a match in the interbank market,  $\sigma$ , are associated (*ceteris paribus*) with lower equilibrium values of  $q_W$ . In fact, very high values of  $\sigma$  will undermine the possibility of an equilibrium with stigma of the type we consider in Proposition 4. The reason for this fact is that, in our equilibrium, when a larger proportion of banks find a match in the interbank market the composition of banks borrowing at the window shifts towards a relative abundance of banks holding low quality assets and, hence,  $q_W$  becomes smaller. For small enough values of  $q_W$  a bank that finds a match and is holding a low quality asset will actually prefer to take a loan from its private counterparty, an action that is inconsistent with the equilibrium proposed in Proposition 4.<sup>24</sup> If we think that the likelihood of finding a counterparty in the interbank market is an approximate measure of the liquidity in that market, then we can conclude that the possibility of equilibrium stigma is associated with low levels of liquidity in the interbank market. In other words, when the market for interbank loans is very liquid, the type of equilibrium with stigma studied in this section is likely to break down.

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<sup>24</sup>If  $q_W$  is very low, taking a loan at the discount window entails a large (expected) discount at the time of selling the asset in period 2, on the eventuality that the quality of the asset is not observed by investors.

Finally, notice that the possibility of an equilibrium with stigma is directly associated with the ability of investors to discriminate between those banks that have borrowed from the window and those that have not. In situations where the activity of borrowing from the central bank becomes less than perfectly observable, the incidence of stigma is likely to be low. For example, if central bank lending is done using an auction format which guaranties the participation of multiple institutions (like in the case of the TAF in the US), and such arrangement makes virtually impossible to draw good inferences about the particular identity of participating borrowers, then stigma is likely to vanish from equilibrium outcomes.

## 6 Other possible equilibria

There are other possible equilibrium configurations, depending on parameter values.<sup>25</sup> One interesting alternative is the equilibrium in which illiquid banks that find a counterparty in the interbank market borrow from other banks in that market, and only those illiquid banks that do not find a counterparty borrow at the discount window. In such an equilibrium, there is no stigma attached to borrowing from the window. The following proposition provides an explicit characterization.

**Proposition 5.** Assume that  $\frac{\alpha}{1-\alpha}i < \tau < \rho - i$ . There is a threshold value  $\bar{p}_\tau < 1$  such that if  $p \in (\bar{p}_\tau, 1)$  then there is an equilibrium with both interbank credit and discount window lending in which  $q_W = q_N = p$ , and  $r_H = \theta r_W + (1 - \theta)i < r_L = \theta r_W + (1 - \theta)\frac{i}{1-\alpha} < r_W$ .

**Proof:** See the Appendix. #

Note that this proposition requires  $\tau$  to be greater than  $A$ , as defined in Proposition 4. Basically, the proposition in this section deals with equilibrium outcomes in the case in which the interest rate at the discount window,  $r_W$ , is relatively *high*. As it turns out, when  $r_W$  is relatively high, there is no equilibrium stigma at the discount window. Furthermore, note that  $r_j < r_W$  for  $j = H, L$ , and hence observed market interest rates (and any weighted average of them) will be below the rate at the discount window. This result is in sharp contrast with the finding stated in Corollary 3.

Another interesting feature of the equilibrium in Proposition 5 is that, since  $r_H < r_L$ , some dispersion in interest rates is observed in the interbank market. In particular, banks with high quality assets obtain credit at lower interest rates than banks with low quality assets.<sup>26</sup> Finally, note that since  $\frac{p-\sigma p}{1-\sigma p} < p < 1$ , a bank borrowing at the window receives a higher price for an assets of unobserved quality than in the equilibrium in which borrowing at the window is regarded as a

<sup>25</sup>Under some conditions, the equilibrium of Proposition 4 can coexist with a no-credit equilibrium of the type considered in Section 3 where  $q_N = 0$ . In such no-credit equilibrium, all banks would borrow from the window and no possibility of stigma would arise. The issue of stigma in economics is often discussed in relation to the possibility of multiple equilibria of this kind (where "everybody does it" because there is no stigma, and there is no stigma because "everybody does it" as, for example, in the tax evasion example of Kim, 2003). Since the focus of this paper is on the nature of transactions in the interbank market for funds, the fact that the equilibrium in which stigma vanishes involves no trade in the interbank market makes this case less interesting.

<sup>26</sup>King (2008) presents evidence suggesting that banks with higher repayment risk tend to pay higher interest rates for loans in the US interbank market.

negative signal (i.e., the equilibrium of Proposition 4). However, for banks that do not borrow at the window, the price of the unobserved quality asset is actually lower in this equilibrium without stigma.

An implication of Propositions 4 and 5 is that stigma happens at moderate levels of the discount window (rate) spread  $\tau$ . A natural question to ask is whether equilibria with no stigma could be constructed for low levels of  $r_W$ . It is easy to see that, for low values of  $r_W$ , one can construct equilibria where all banks borrow from the discount window. In such construction, off equilibrium beliefs become important, since investors need to form beliefs about the probability that a (deviating) bank not borrowing at the window holds a high quality asset. These beliefs are necessarily arbitrary in our model and can affect equilibrium outcomes and the interpretation of stigma. This kind of technical complications is common in signaling environments and a thorough discussion of the issues involved is beyond the scope of this paper.

## 7 Conclusion

In this paper, we have provided a formal model of the interaction between the interbank market for funds and the asset market. Our model is capable of reproducing certain trading patterns by banks that are consistent with a situation in which outside investors attach some degree of stigma to the activity of borrowing from the central bank's discount window. The intention was to introduce the minimum number of elements in the model to allow us to capture such stigma effect. In this kind of signaling setting there is a delicate balance between information and frictions, which needs to be maintain in order to have that, while some information flows to the market, the equilibrium does not become fully revealing. Achieving this balance in a parsimonious way is the main justification for most of our basic assumptions.

The main components of our model are the following. On the one hand, participants in the interbank market have (some) information about the quality of their counterparty's assets. On the other hand, private dealings in the interbank market are not observable by third parties. Actions in the interbank market, if observable, would reveal asset quality and, hence, pin down asset prices independent of any signal. Repayment risk is endogenous in the model and depends on the equilibrium in the asset market. In turn, banks' activities in the asset market depend on their ability to borrow in the interbank market. In particular, some banks in the asset market may be selling their assets because they need to repay their interbank loans. Others banks, however, may be selling their assets just because they know that their asset are low quality and that, at the prevailing equilibrium prices, they are effectively overpriced.

When we introduce the possibility that banks can borrow liquidity from the central bank, participation at the discount window may be subject to adverse selection and, as a consequence, stigma may arise. We make information about borrowing activity at the window observable by outside investors. The idea of stigma clearly relies on some degree of observability; the underlying logic

is that information held by participants in the interbank market, which would otherwise remain private, can flow to asset market participants through reported activity at the discount window. Another important aspect associated with the possibility of stigma is that loan repayment risk, while partly endogenous, also depends on the true quality of the assets held by the banks. This fact is what implies that, in equilibrium, banks holding bad assets are more likely to not obtain loans in the interbank market and, hence, be borrowing at the discount window. Consequently, there is adverse selection in participation decisions at the discount window, justifying that borrowing from the central bank is regarded as a negative signal and, in this way, making stigma an equilibrium phenomenon.

This was a paper on positive economics. We made no attempt to address most of the relevant policy issues associated with our general subject of inquiry. The incidence of stigma in the activity of borrowing from the discount window has potentially important policy implications. For example, if some amount of discount window lending is optimal and stigma makes banks reluctant to access such liquidity (as it has often been argued), then the effectiveness of policy may be seriously impaired as a result. While addressing the policy questions is, of course, very important, we think that in the process of reaching reliable conclusions, an essential first step is to develop a better understanding of the fundamental nature of stigma in the interbank market. Taking such first step was the objective of this paper.

## 8 Appendix

**Proof of Proposition 4:** Consider an arbitrary value of  $\tau \in (0, A)$ . We want to show that for a set of values of  $p$  there is an equilibrium with both interbank credit and discount window lending in which  $q_W = \xi$ , and  $q_N = 1$ . We organize the proof in five steps.

*Step 1.* Given  $\tau$ , pick the value  $p_1$  such that  $q_1 \equiv \frac{p_1 - \sigma p_1}{1 - \sigma p_1}$  satisfies  $q_1 = \frac{i + \tau}{R}$ .<sup>27</sup> Since  $i + \tau < i + \rho - i = \rho < R$  we have that  $q_1 < 1$  and hence  $p_1 < 1$ . Then, if  $p > p_1$  and  $q_W = \xi$  then we have that  $q_W R > q_1 R = r_W$ . Also note that:

$$\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W) = \alpha R + (1 - \alpha)q_W R - r_W = R - r_W - [(1 - \alpha)(1 - q_W)R].$$

Now, pick the value  $p_2$  such that  $q_2 \equiv \frac{p_2 - \sigma p_2}{1 - \sigma p_2}$  satisfies

$$q_2 = 1 - \frac{\rho - i - \tau}{(1 - \alpha)R}.$$

Since  $\rho - i - \tau > \rho - i - \rho + i = 0$  then  $q_2 < 1$  and  $p_2 < 1$  and if  $p > \max\{p_1, p_2\}$  then

$$R - r_W - [(1 - \alpha)(1 - q_W)R] > R - r_W - (\rho - i - \tau) = R - \rho > 0.$$

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<sup>27</sup>Note that the function  $q(p) = \frac{p - \sigma p}{1 - \sigma p}$  is increasing in  $p$  and that  $\lim_{p \rightarrow 1} q(p) = 1$  and  $\lim_{p \rightarrow 0} q(p) = 0$ .

Hence, whenever  $p > \max\{p_1, p_2\}$  illiquid banks holding a high quality asset will borrow at the discount window when they do not find a match in the interbank market. By Lemma 3, illiquid banks holding a low quality asset will do the same. Furthermore, notice that the inequalities above demonstrate that the relevant alternative for those illiquid banks that do find a match is for them to borrow (rather than not to borrow at all).

*Step 2.* For any  $\max\{p_1, p_2\} < p < 1$  we have that  $q_W = \xi < 1$  and  $TS_H = (1 - \alpha)(1 - q_W)R + r_W - i = (1 - \alpha)(1 - q_W)R + \tau > 0$ . Then,

$$R - r_H = R - i - \theta TS_H > R - i - TS_H = R - (1 - \alpha)(1 - q_W)R - r_W$$

which implies that  $R - r_H > \alpha R + (1 - \alpha)q_W R - r_W$  and hence illiquid banks that find a match in the interbank market and are holding a high quality asset borrow from the market.

*Step 3.* By Lemma 2, a liquid bank would agree to give a loan to an illiquid bank holding a low quality asset only if the agreed upon interest rate  $r_L$  is such that  $r_L \geq i/(1 - \alpha)$ . Pick the value  $p_3$  such that  $q_3 \equiv \frac{p_3 - \sigma p_3}{1 - \sigma p_3}$  satisfies

$$q_3 = 1 - \frac{\alpha i - (1 - \alpha)\tau}{(1 - \alpha)R}.$$

Since  $\alpha i - (1 - \alpha)\tau > \alpha i - (1 - \alpha)[\alpha i / (1 - \alpha)] = 0$  we have that  $q_3 < 1$  and  $p_3 < 1$ . If  $p > \max\{p_1, p_2, p_3\}$  then

$$(1 - \alpha)(q_W R - r_W) > (1 - \alpha) \left[ \left( 1 - \frac{\alpha i - (1 - \alpha)\tau}{(1 - \alpha)R} \right) R - r_W \right] = (1 - \alpha)R - i \geq (1 - \alpha)(R - r_L).$$

Hence, illiquid banks holding a low quality asset will borrow at the window even when they find a match in the interbank market. Note that the condition  $(1 - \alpha)(q_W R - r_W) > (1 - \alpha)R - i$  is equivalent to the condition  $TS_L < 0$ .

*Step 4.* Let  $\bar{p}_\tau = \max\{p_1, p_2, p_3\}$ . For any  $p \in (\bar{p}_\tau, 1)$ , illiquid banks holding high quality assets borrow in the interbank market if they find a match and all other banks borrow at the window. That is,  $1 - \sigma p$  banks borrow at the window, of which only a proportion  $p(1 - \sigma)$  are holding high quality assets. Hence,  $q_N = 1$  and  $q_W = p(1 - \sigma)/(1 - \sigma p) \equiv \xi$ .

*Step 5.* In this equilibrium, we have that

$$S_b(r_H, q_N, q_W) = (1 - \alpha)(1 - q_W)R - r_H + r_W$$

and

$$S_l(r_H, q_N) = r_H - i.$$

Solving the Nash bargaining problem we get that  $r_H = i + \theta[(1 - \alpha)(1 - q_W)R + r_W - i] = i + \theta TS_H$ . #

**Proof of Proposition 5:** Consider a value of  $\tau$  such that  $r_W = i + \tau \in \left(\frac{i}{1 - \alpha}, \rho\right)$ . We want to show

that for a set of values of  $p$  there is an equilibrium where illiquid banks that find a counterparty borrow from the interbank market (regardless of the quality of the asset they hold) and illiquid banks that do not find a counterparty borrow from the discount window.

Given  $\tau$ , pick the value  $\bar{p}_\tau = \max\left\{\frac{i+\tau}{R}, 1 - \frac{\rho-i-\tau}{(1-\alpha)R}\right\}$ . Since  $i + \tau = r_W < \rho < R$  we have that  $\bar{p}_\tau < 1$  and  $(\bar{p}_\tau, 1)$  is non-empty. Consider a value of  $p \in (\bar{p}_\tau, 1)$ . We want to show that:

(1) illiquid banks with a match borrow from the interbank market; that is:

$$\alpha(R - r_H) + (1 - \alpha)P(q_N, r_H) > \alpha(R - r_W) + (1 - \alpha)P(q_W, r_W),$$

and

$$(1 - \alpha)P(q_N, r_L) > (1 - \alpha)P(q_W, r_W);$$

(2) illiquid banks without a match borrow from the discount window; that is:

$$\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W) > -\rho + R,$$

and

$$(1 - \alpha)P(q_W, r_W) > -\rho + (1 - \alpha)q_N R;$$

(3) liquid banks are willing to lend to illiquid banks (regardless of asset quality); that is:

$$\alpha r_H + (1 - \alpha) \min\{q_N R, r_H\} > i,$$

and

$$(1 - \alpha) \min\{q_N R, r_L\} > i.$$

If these three sets of conditions hold, then it is easy to see that, of the banks borrowing in the market, a proportion  $p$  holds high quality assets. Furthermore, this proportion is also  $p$  for the banks borrowing at the discount window. Hence,  $q_n = q_W = p$  in equilibrium. Since  $p > \bar{p}_\tau \geq \frac{i+\tau}{R}$  we have that  $pR > r_W$ .

The total surplus from an interbank loan to a high-quality asset holder is  $TS_H = r_W - i > 0$  and the total surplus from a loan to a low-quality asset holder is  $TS_L = r_W - \frac{i}{(1-\alpha)} > 0$ . Bargaining over the terms of the loan will then result on  $r_H = i + \theta TS_H = \theta r_W + (1 - \theta)i < r_W$  and  $r_L = \frac{i}{1-\alpha} + \theta TS_L = \theta r_W + (1 - \theta)\frac{i}{1-\alpha} < r_W$ . In consequence, we have that  $pR > r_L > r_H$ .

Then, the two conditions in (1) hold since  $r_H > r_W$  and  $r_L > r_W$ ; and the two condition in (3) hold since  $r_H > i$  and  $r_L > \frac{i}{1-\alpha}$ . Since  $p > \bar{p}_\tau > 1 - \frac{\rho-i-\tau}{(1-\alpha)R}$  we have that the first condition in (2) holds as:

$$\alpha(R - r_W) + (1 - \alpha)P(q_W, r_W) = \alpha R + (1 - \alpha)pR - r_W > \alpha R + (1 - \alpha)R - (\rho - i - \tau) - r_W = -\rho + R.$$

Finally, the second condition in (2) holds because  $r_W < \rho < \frac{\rho}{1-\alpha}$ . This completes the proof. #

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