

Does a Seller Really Want Another Bidder?*

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Abstract

Jeremy I. Bulow and Paul D. Klemperer (*AER*, 1996) argue that the usual concerns of auction design miss the big picture, and show that a simple English auction without a reserve price and $N + 1$ bidders attains expected revenue in excess of any auction with N bidders. The issue of how this additional bidder might be attracted is not treated in their model. In fact, that an auction can convince another bidder it is worth his while to compete carries a critical message about expected revenue. In those many markets where potential bidders decide whether to compete in an auction based on the expected profitability of bidding, Bulow and Klemperer's conclusion is shown here to be overturned. I explore the symmetric equilibrium of a model where potential bidders first decide whether to participate in an auction, and then participants select bidding strategies. Expected revenue is increased by some degree of bidder *discouragement*, in that it is never optimal to have all N potential bidders participate with probability one, even for very small N .

D44; D82; C72; Keywords: affiliated-values auctions, auction revenue, number of bidders, increased competition, endogenous bidder participation

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1 Introduction

Most auction theory simply posits an exogenous number of bidders without comment,¹ and proceeds to characterize such concerns as the preferences of an expected-revenue-maximizing seller across selling procedures. Bulow and Klemperer (1996), in this journal, indicate that the issue left out of such models may be more important than those treated:

A seller with no bargaining power who can only run an English auction with no reserve price among $N + 1$ symmetric bidders will earn more in expectation than a seller with all the bargaining power, including the ability to make binding commitments, who can hold an optimal auction with N buyers. . . No amount of bargaining power is as valuable to the seller as attracting one extra bona fide bidder. (p. 180)

Bulow and Klemperer are correct to broaden a seller's purview to the amount of competition an auction generates. In many auction markets, however, their prescription would be misapplied. What they compare is a basic English auction with $N + 1$ bidders to an optimal auction with N bidders. In each case, the number of bidders is *exogenous*, with the bidders modeled as rational competitors, who reach the unique symmetric Bayes-Nash equilibrium.² For broad applicability, though, *both* the bidding once participating *and also* the (prior) decision to be a bidder ought to be presumed rationally determined.³

The key insight of the model presented here: an additional bidder's *deliberate* presence carries an inference about auction revenue not developed in Bulow and Klemperer's model. The additional bidder made a rational decision to enter the competition despite only 1 chance in $N + 1$ of winning the auction. Hence, the profitability of being the winning bidder must be high enough to cover in expectation the resource costs of competing in $N + 1$ such auctions. This higher expected profitability of competing, of course, comes largely at the expense of lower expected revenue.

The model below treats N potential bidders deciding whether to participate in an auction, with a symmetric equilibrium characterizing both this decision and the resultant bidding of those who do compete. An unambiguous prediction is that a seller always benefits by some measure

¹Cf. the Klemperer (1999) survey, the Vijay Krishna (2002) text, and references cited in these works.

²Their assumption (A.3), p. 187, and the footnote thereto.

³Bidders in many markets presumably compete in or bypass an auction due to the expected profitability of bidding: oil leases, airwaves licenses, privatization of governmental enterprises, timber sales, acquisitions of distressed corporations, initial public offerings, used cars, art, and scores of others.

of *discouragement* of additional bidders. Specifically, the revenue-maximal auction never has all N potential bidders competing with probability one. This result applies even when $N = 2$ in a common-value auction, and $N = 3$ in a general affiliated-values auction. Thus, a seller may not gain from an additional bidder if his presence carries the message that he sees sufficient expected profitability to bother competing.

2 A Simple Participatory Model

Consider a model in which $N \geq 2$ is the exogenous number of *potential bidders* who will make rational decisions whether to compete in a given auction, for a single indivisible asset. The number who participate will be denoted n , and the number who actually bid denoted a .

Initially suppose the special case of a common-value auction: asset value to any potential bidder is a random variable V with density $g(v)$ bounded above 0 on domain $D_V \subset [0, \infty)$. The generalization to affiliated-values auctions (for which results differ but insubstantially) is introduced after some notation.

The seller is assumed to pre-commit to an *auction mechanism* $M := (m, \varphi) \in \mathbb{M} \times \mathfrak{R}$, where φ is an *entry fee* (the model contemplates the possibility of a negative entry fee, which if used would be paid by the seller to a participant willing to submit a bid). Simply let \mathbb{M} restrict the seller to announcing m_E , an English auction, m_2 , a second-price auction, or m_1 , a first-price auction. Moreover, restrict the seller to a zero reserve price.⁴

As in Bulow and Klemperer (1996), all N potential bidders are assumed risk-neutral, and assumed to adopt symmetric strategies. A strategy for a potential bidder is $(\tilde{\pi}_M, \kappa_n, \beta_a)_{M \in \mathbb{M}, n=1, \dots, N, a=1, \dots, N}$, where $\tilde{\pi}_M$ is the probability that he participates in the auction, given that the seller has committed to auction mechanism M ; κ_n is the set of his types for which he becomes an actual bidder (continuing to compete), given that n is the number of potential bidders who become *participants*; and β_a is the function of his type specifying his bidding strategy, given that a is the number of participants who became *actual bidders*.

Potential bidders simultaneously select probabilities of participating. Not participating yields a

⁴The simplifications in this and the previous paragraph are without loss of generality, as shown in results provided below or cited from Harstad (2007). Moreover, these assumptions are conservative, in that the Bulow-Klemperer preference for an added bidder is overturned even when the seller is given a smaller set of options in the event of fewer bidders.

payoff of 0. Participation has two consequences: each participant j obtains some private information X_j about the asset's value to him (call this j 's *type* or *signal*), and each incurs a *participation cost*, $c > 0$ (not revenue for the seller).⁵ For simplicity, assume that the number n of participants becomes public information once simultaneous participation decisions have been made.⁶

Next, n privately informed participants make simultaneous decisions as to whether to pay the entry fee φ (or to receive it, if $\varphi < 0$). Not doing so ceases to compete, with a payoff of $-c$. Doing so makes the participant an “actual bidder.”

The a actual bidders follow symmetric equilibrium bidding strategies for auction form M . The high bidder earns a payoff $V - p - \varphi - c$, where p is the price paid under rules m ; other actual bidders attain payoffs $-\varphi - c$. The seller's payoff is $p + a\varphi$, price plus receipts of entry fees (“revenue” is this sum).

Participants draw types from the infinite sequence $\{X_1, X_2, \dots\}$, assumed exchangeable, positively affiliated, real-valued random variables with nonatomic measure ξ , and marginal ξ_1 onto support $\mathcal{X} \subset \mathfrak{R}_+$.⁷ Let $V_z = (\frac{1}{z}) \sum_{i=1}^z X_i$. Asset value $V = \lim_{z \rightarrow \infty} V_z$; to avoid trivialities, $2c < E_\xi[V] < \infty$.

To conclude this section, allow for the possibility that additional participants lead to the highest-valuing participant forecasting a higher asset value, which extends the model beyond the common-value setting. (This extension can safely be ignored by readers unconcerned about these details.) To do so, let V as used above be considered the *underlying asset value*, with asset value to a particular participant observing signal X_i a continuous function $t(V, X_i)$, increasing in both variables (common across participants, in that t does not have a subscript). Without loss of generality,

⁵Hence, in this model, potential bidders who have no private information when they decide whether to become informed and compete. This makes an $N + 1^{\text{st}}$ potential bidder's decision situation exactly analogous to those of the other N potential bidders. Thus, it is the natural analogue to Bulow and Klemperer's model, where the exogenously produced $N + 1^{\text{st}}$ bidder is added, and his impact on expected revenue calculated, at a point where his private information is to be drawn from the same distribution as the first N bidders. In a model like William F. Samuelson (1985), where an additional bidder incurs a resource cost to compete after freely acquiring his private information, the impact on expected revenue would depend entirely on the relation between that additional bidder's type and the two highest of the original bidders' types. Similarly, Dirk Bergemann and Juuso Välimäki (2002) consider the issue of additional information acquisition by bidders who already possess imperfect but private information about their valuations. In their model, the number of bidder is exogenous; the impact of an additional bidder would depend on the likelihood that he possessed one of the two highest valuations.

⁶Harstad (2007) also treats the case where the number of participants is not learned, with identical results. The issue of simultaneity of the participation decision is considered in concluding remarks.

⁷Affiliation is defined and characterized in Paul R. Milgrom and Robert J. Weber (1982), pp. 1098-1100 and 1118-1121; it is referred to as the MLRP (monotone likelihood ratio property) in several auction models. Roughly, affiliation means that higher realizations for any subset of the variables $\{X_1, X_2, \dots\}$ make higher realizations for any disjoint subset more likely. Exchangeability means that the joint distribution is unaffected by any finite permutation of the indices.

presume $t(V, X_i) \geq V \forall X_i \in \mathcal{X}$; the common-value setting will be the special case $t(V, X_i) = V$. The winning bidder's payoff becomes $t(V, X_i) - p - \varphi - c$; other payoffs are unchanged.

3 Equilibrium Revenue

In symmetric equilibrium for auction mechanism M , all potential bidders select the same probability of participating, $\pi(M) \geq 0$. Hence, $\pi(M) \in (0, 1)$ must leave any given potential bidder indifferent over participating when all $N - 1$ other potential bidders participate with probability $\pi(M)$. This requirement may informally be expressed as $w(\pi) E[T(M, \pi, n, \varphi, v) - p(M, \pi, a, n)] - \varphi e(M, \pi, a, n) - c \geq 0$, with strict inequality permitted only if $\pi = 1$. In this equation, $w(\pi)$ is the ex-ante probability that a potential bidder wins conditional on his participating; $T(M, \pi, n, \varphi, v)$ is the expected asset value to a potential bidder who will participate, conditional on his winning, and on π participation probability, n participants, φ entry fee, and a draw of v from $g(\cdot)$; $p(M, \pi, a, n)$ is an equilibrium expected price paid by the winning bidder; and $e(\cdot)$ is the ex ante probability that a potential bidder pays the entry fee conditional on his participating. This latter term, $e(M, \pi, a, n)$, sums (with Bernoulli weights for the probabilities of $n - 1$ rival participants) over the probability of drawing a type for which φ is not greater than the product of expected profitability given winning with type x times the probability of winning with type x against $n - 1$ rival participants.⁸

With a zero reserve price, the auction will see a transaction every time at least one participant is willing to pay the entry fee. With the seller's value of the asset normalized to zero, which is the minimum of the support of asset value to a participant, every transaction yields a gain from trade. The expected value of the gains from trade for mechanism M is thus

$$\bar{V}(M) = \sum_{1 \leq n \leq N} \sum_{1 \leq a \leq n} E[T[M, \pi(M), n, \varphi, V] | n, \{a > 0\}] \binom{n}{a} \mu_a(M, n) \beta_n,$$

where expected asset value is conditioned on at least one of n participants electing to pay the entry fee; $\mu_a(M, n)$ is the ex ante probability, conditional on n participants, that participants $1, \dots, a$ choose to pay the entry fee, and $a+1, \dots, n$ choose not to; and β_n is the probability of n participants given the equilibrium value of π .

⁸Details are laid out for a more general model in Harstad (2007), section 3. Note that $T(\cdot)$, $p(\cdot)$ and $e(\cdot)$ will typically be degenerate in at least one variable for a given m .

The expected number of actual bidders is

$$\bar{a}(M) = \sum_n \left\{ \sum_a a \binom{n}{a} \mu_a(M, n) \right\} \beta_n$$

and the expected number of participants is $\bar{n}(M) = \sum_n n \beta_n = N \pi(M)$.

Expected revenue sums the price and expected entry fee receipts: $\bar{R}(M) = p(M, \pi, a, n) + \bar{a}(M) \varphi$.

If each participant is indifferent between participating or not, then equilibrium requires the expected gains from trade to accrue to the seller, net of effectively compensating participants for their resource costs. This conclusion can be expressed as

$$\bar{R}(M) = \bar{V}(M) - c \bar{n}(M), \tag{1}$$

and has several antecedents in the auction literature.⁹

The seller can attain the entire interval of equilibrium participation probabilities, 0 through 1. Setting $\varphi = E[V]$ yields $\pi(M) = 0$, and setting $\varphi = -c$ yields $\pi(M) = 1$. Intermediate values are obtained from having enough continuity of expected profitability in the entry fee to apply the intermediate value theorem.¹⁰

A key characteristic of the symmetric equilibrium yielding (1) is that two different auction mechanisms attaining the same equilibrium participation probability π attain the same level of equilibrium revenue. Indeed, there exists a function $R(\pi)$ on $[0, 1]$ with the property that $\bar{R}(M) = R(\pi(M))$.¹¹

4 Discouraging Competition

The desired characterization is simply stated and shown. That $R(\pi)$ is continuous is straightforward. As its range is obviously bounded, it attains a maximum. Let π^H be the largest π attaining this maximum.

⁹Theorem 3 in Harstad (2007) encompasses as special cases corresponding prior results in William F. Samuelson (1985), Donald B. Hausch and Lode Li (1990), Harstad (1990) and Dan Levin and James L. Smith (1994), with presursors in Milgrom (1981), Steven Matthews (1984), and Kenneth R. French and Robert E. McCormick (1984). See also footnotes 17 and 18.

¹⁰A proof for the case of the second-price auction is Thm. 5 in Harstad (2007).

¹¹This characteristic is a corollary of Proposition 7, and the projection onto $[0, 1]$ to yield $R(\pi)$ is Corollary 8, in Harstad (2007).

Proposition 1 $\pi^H < 1$ in a common-value auction, or more generally if $N \geq 3$.

Proof. Consider $R(\pi)$ as π decreases incrementally from 1 to $1 - \Delta\pi$. This has three analytically separable effects on expected revenue. First, seller in equilibrium compensates bidders for their participation costs, and so this effect yields a gain in expected revenue as aggregate expected participation costs decrease. Second, the highest (across participants) expected value of the auctioned asset falls, as this no longer attaches probability one to the highest of N values (with small probability it becomes the highest of $N - 1$ values). To measure this effect, define $J(n) = E_{\xi} [\max_{i=1, \dots, n} t(V, X_i)]$; then this expected highest value is

$$K(\pi) = \sum_{n=1, \dots, N} \binom{N}{n} \pi^n (1 - \pi)^{N-n} J(n).$$

Third, the probability of 0 actual bidders increases, reducing expected revenue as there are then no gains from trade for seller to appropriate. As π decreases very near 1, the first effect shows a revenue gain that is linear in $\Delta\pi$, by (1); the second effect is absent under pure common values, and otherwise a revenue loss that is of the order $(\Delta\pi)^{N-1}$, by derivation of $\partial K / \partial \pi|_{\pi=1-\Delta\pi}$; and the third effect a revenue loss that is of the order $(\Delta\pi)^N$. For small $\Delta\pi$, the first effect dominates. ■

Hence, a seller prefers to discourage potential bidders from competing at least to some extent: relative to any revenue-maximal mechanism, there always exists an alternative auction mechanism exhibiting a larger expected number of participants. That is, a seller wishes to make the auction mechanism sufficiently extractive of surplus that it is not in a bidder's interest to participate if all rival potential bidders are participating with probability 1. Notice that in a common-value auction with $N = 2$ potential bidders, rather than an auction yielding $\pi = 1$, the seller will have some revenue-superior alternative which will lead potential bidder 1 to be indifferent over participating even when he infers that there will be at least a $(1 - \pi^H) > 0$ probability of facing no competition (and a zero reserve price). With a private-values element (a general affiliated-values setting), a seller never prefers to attract a third participant with probability one.¹²

¹² An illustrative example with $N = 2$: let $t(V, X_i) = X_i$ with $X_i \sim U[0, 1]$ (uniformly distributed independent private values). Then $sign(\partial R(\pi) / \partial \pi|_{\pi=1-\Delta\pi}) = sign((1/6) - c)$, from (1). As at $\pi = 1$ and $\varphi = 0$, a participant's equilibrium expected profitability is $1/6$, a seller will only attain a higher expected payoff at $\pi = 1$ than at $\pi = 1 - \Delta\pi$ if this payoff includes positive entry fee receipts, and thus will in this sense still be discouraging bidder participation.

This discouragement of potential bidders might possibly be handled by switching from a first-price to a second-price auction, or from either to an English auction. Each of these switches in auction form, and any other change that would extract more surplus from a number of bidders fixed exogenously, serves in this model to reduce π .¹³ While each of the switches in auction form has a discrete impact on π , and might reduce it too far, π responds continuously to changes in the entry fee.¹⁴

5 Concluding Discussion

Several auction papers have found expected revenue for a given auction form to be increasing in the number of competitors.¹⁵ What is striking about the Bulow and Klemperer result is that this effect of an exogenously specified arrival of further competition dominates all aspects of auction design, making the simple English auction with an additional bidder superior to any auction with a given level of competition.

However, this paper has offered a model for situations where the arrival of additional competition is itself a result of rational decision, and the seller rationally takes into account the inference that the additional participant is there *deliberately*.¹⁶ This inference, that the asset is expected to sell for far enough below its value as to make a smaller chance of winning ($1/[n+1]$ instead of $1/n$) still worthwhile, overturns the prior preference for more competition.¹⁷ Some degree of bidder

¹³This result is an interesting complement to Bergemann and Pesendorfer (2007). They consider an exogenously given number of bidders, and allow a seller to control both the allocation mechanism and the precision with which bidders learn their (independent) private values. Here, the precision of competitors' private information is exogenous, but seller through the allocation mechanism affects their incentives to acquire information.

¹⁴The proof in Theorem 5 in Harstad (2007) for the second-price auction is readily imitated for an English auction. Eric Maskin and John G. Riley (2000) establish a corresponding degree of continuity for a first-price auction.

¹⁵This is a direct implication in the theoretical models of Lawrence Friedman (1956), Michael H. Rothkopf (1969), Roger B. Myerson (1981), Milton Harris and Artur Raviv (1981) and Riley and Samuelson (1981), and an asymptotic result in Robert Wilson (1977), Wolfgang Pesendorfer and Jeroen Swinkels (1997) and Harstad, Ilia Tsetlin and Aleksandar Pekeć (2006). Simulations by Douglas K. Reece (1978) and empirical studies by Walter Mead (1969), Smith (1977), Kenneth M. Gaver and Jerold L. Zimmerman (1977), Kenneth Hendricks, Robert H. Porter and Bryan Boudreau (1987), and S. Michael Giliberto and Nikhil P. Varaiya (1989), among others (see surveys by Jean-Jacques Laffont (1997) and Porter (1995)), show this comparative static.

¹⁶Under the title, "What Really Matters in Auction Design," Klemperer (2002), after discussing collusion, states "The second major area of concern of practical auction design is to attract bidders, since an auction with too few bidders risks being unprofitable for the auctioneer." (p. 172) However, in discussing a variety of examples, especially airwaves auctions, Klemperer frequently recognizes the need for an additional bidder to see sufficient chance of winning in order to be attracted to compete, and mentions in his conclusions "even modest bidding costs may be a serious deterrent to potential bidders." (p. 186) The model presented here helps to formalize assertions implicit in his discussion of some of these examples.

¹⁷A similar inference can be drawn in the much more involved model of Jacques Crémer, Yossi Spiegel and Charles Z. Zheng (2006). They allow a seller to limit competition by having the capability to control when particular competitors

discouragement is always beneficial: Proposition 1 shows that the seller always had an option with a higher expected number of competitors than the expected-revenue-maximizing option.¹⁸

It is clear that this inference to be drawn from added competition that deliberately competes is more general than the simple model used here to illustrate it. One particular case is where added bidders arise via a sequential decision process as to whether to participate. The presence of the second participant implies that the expected profitability of being the winning bidder is at least twice the participation cost, when the price is determined by competition among two. The presence of a third participant would imply that the expected profitability of being the winning bidder is at least three times the participation cost, when the price is determined by competition among three. That higher level of expected profitability, of course, comes primarily at the seller's expense.

The assumption of symmetry in potential bidders' information, beliefs and behavior is critical to the above analysis, as it generates a unique equilibrium continuation for any auction mechanism. This uniqueness makes expected profitability calculations well-defined; without it, expected profitability calculations governing whether to participate and whether to pay the entry fee would depend on which of many (typically infinitely many) equilibrium continuations would follow.

Nonetheless, in many situations characterized by asymmetric sequential decisions about whether to compete in an auction or other contest, it may be reasonable that participants arrive in decreasing order of expected profitability of participating. That would suggest the negative inference

learn their private values. It is then often optimal to permit a single buyer to incur the cost of learning his private value, approach him with a take-it-or-leave-it price, and continue on to allowing one more buyer to learn his private value if that price is rejected. In this sense, added competition is essentially a last resort in their optimum. Their result is clearly dependent on a stringent assumption that the seller can in effect impose a bidder-specific lump-sum tax on all (potential) bidders at the beginning of the game. The current model shares with most auction theory papers the tradition of assuming that any surplus-extracting device a seller can employ is potentially distortive of bidders' behavior.

¹⁸The contrast with Bulow and Klemperer is relatively straightforward. More complex is the relationship of this result with the interesting characterizations in R. Preston McAfee (1993). McAfee builds a partly game-theoretic, partly competitive-equilibrium model of an outcome roughly akin to a steady state in an infinitely recurring market. Each seller auctions a single asset with a reserve price in each period; each independent-private-values buyer selects a seller's auction to compete in based on the profile of reserve prices. (In the steady state, all sellers select the same reserve price, and all buyers use the uniform mixed strategy to select an auction to attend.) A transaction leads to the seller and the successful buyer exiting the market. Each period then sees a replenishment of new sellers and new buyers in an exogenous ratio. McAfee's characterizations of the steady-state in-period reserve-price choice as efficient, and of the resulting steady-state expected revenue, are fully consistent with equation (1) above. An exogenous increase in the ratio of newly arriving buyers per arriving seller is beneficial to sellers in the steady state. This is partly because these added arriving buyers are exogenously specified; an arriving buyer's attendance at a particular seller's auction is endogenous, but his arrival at the market of competing sellers is not the result of any decision by him. It is also partly due to each seller responding to the higher buyer/seller ratio by increasing his in-period reserve-price, but this happens without any impact on the number of competitors, as all other sellers have identically increased their reserve price in the altered steady-state associated with the exogenous change in the buyer/seller ratio. The comparative static in this model that would correspond to all rival auctions becoming more extractive of surplus would be a reduction in c . The opportunity cost of competing in this auction, which will be lower if rival auctions have, for example, higher entry fees, is reflected in c . Clearly, a lower c benefits the seller.

associated with additional competitors developed here may be all the more dramatic.¹⁹

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¹⁹ Asymmetry raises a number of thorny issues, though, likely rendering broad conclusions misleading at best. In particular, Klemperer (2002) has examples where it is intuitively appealing to believe that changes in auction form may have a dramatic impact on the incentives of potential bidders with lower expected profitability to compete. James C. Cox, Sam Dinkins and James T. Swarthout (2001) outline a model with differential participation costs and an assumed symmetric equilibrium continuation.

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