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“The Bunny and the Batteries: Advertising to Consumers with Limited Attention”

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The Bunny and the Batteries: Advertising to Consumers with Limited Attention∗

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Abstract

An important role of advertising is to convince consumers that they want the product and to buy it from the brand advertising it. However, when consumers’ attention is limited, an advertisement that induces a consumer to enter the market might not lead him to purchase from the brand being advertised. An ad is salient when it makes consumers remember the brand. We analyze the welfare implications of salient advertising in a homogeneous goods market with consumers with limited attention. Firms engage in at least some degree of salient advertising if the marginal cost of advertising is not high. There are cases where welfare is highest with an intermediate advertising technology that leads to an equilibrium with positive but imperfect salience. When salient advertising substitutes for lower prices, and the total cost is sufficiently low, expected firm profits are greater than without advertising. If consumers are sensitive to advertising and increases in salience are smaller for higher prices, then the average price paid by consumers is less than without advertising, so consumer utility increases as well.

Keywords: Advertising, Salience, Price Dispersion, Oligopolistic Competition

JEL Classification Numbers: D21, D43, L13, M37.

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1 Introduction

According to Advertising Age’s annual assessment of advertising, total spending on advertising in the United States in 2009 was approximately 250 billion, almost two percent of GDP. Much of this expenditure corresponds to firms’ investment in advertising. The economics literature focuses on three functions of firm advertising: altering consumer preferences (Braithwaite, 1928; Kaldor, 1950), increasing consumer information about firms and their goods (Stigler, 1961; Telser, 1964; Butters, 1977; Grossman and Shapiro, 1984; Robert and Stahl, 1993), and augmenting the utility of consuming a particular good (Stigler and Becker, 1977; Becker and Murphy, 1993).

Experimental studies suggest that, in many purchasing situations, there is a lack of motivation to devote substantial cognitive processing efforts to brand selection. In this context, an important role of advertising is to get noticed and remembered by consumers. In particular, we say that an ad is salient when its main purpose is to make consumers remember the brand. The existing literature on advertising might suggest that salient advertising decreases overall welfare, since it does not contain any new information about the product. However, in a context where consumers devote limited attention to purchasing decisions, the welfare effects become less obvious.

To understand the effects of salient advertising, we first consider how consumers in the market decide which brand to buy. Building on research in psychology, experimental studies of consumers posit that once the consumer decides to buy the product, he does not necessarily consider all available alternatives. Instead,

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2 See Bagwell (2007) for an extensive overview of advertising.
he considers his list of viable brand alternatives, referred to as his consideration set (Miller and Berry, 1998; and Romaniuk and Sharp, 2004). He evaluates other attributes such as price of the brands in his consideration set, and then makes his decision of which brand to buy. Bullmore (1999) succinctly summarizes this decision process: “Most of us have clusters of brands which we find perfectly satisfactory. We will allocate share of choice within this repertoire according to chance, promotions, advertising, availability, price, impulse or recommendation.” For example, a consumer shopping for toothpaste at the supermarket has an idea of which brands he is willing to buy even before stepping into the toothpaste aisle. He then compares the prices of the first couple of brands that come into his mind and buys the cheapest. Alternatively, a consumer who is considering buying a car chooses to visit a subset of local car dealerships. After looking at the different makes, he purchases the one with the best gas mileage. As such, firms find it crucial to be on top of consumers’ minds. By engaging in salient advertising, firms attempt to increase the probability of entering consumer consideration sets. That is, firms must not only convince consumers that they want to buy the product, but, most importantly, that they want to buy it from the brand advertising it. When an ad is salient, it increases the probability a consumer will evoke that brand when forming his consideration set.

However, a firm’s attempt to make an ad salient might fail. Since most modern advertising occurs in the midst of multi firm competition, if we are to think of advertisements as a means to reinforce a brand in consumer memory, it is important to consider the effects of rival ads on memory as it pertains to purchasing decisions. Experimental evidence suggests that in the face of myriad ads from different firms (Kent, 1993; Kent and Allen, 1993), consumers are likely to get confused about
which brand a particular ad refers to. This occurs because there is typically a lag between consumers’ exposure to advertising and their decision to purchase the advertised good (Keller, 1987) and is accentuated when consumers review similar ads without an intention of purchasing what is advertised (Burke and Srull, 1988). In a particularly well known instance, the Energizer Bunny ad ended up promoting just batteries instead of Energizer: viewers could not remember which brand the ad belonged to because Duracell –Energizer’s main competitor– had also used bunnies in previous ads (Lipman, 1990). Therefore, advertising can function as a public good: the fact that a firm engages in salient advertising does not guarantee that all consumers will perceive the ad as salient. If an ad is not perceived as salient by consumers it will only serve to promote the product (as opposed to the brand), and, will not affect any of the existing brands’ probability of being remembered.

This paper sets up a theoretical model of salient advertising in order to analyze its effects on welfare. In our model, firms have the option of engaging in imperfect salience. That is, to choose the probability with which the ad will turn out to be salient in one realization. Firms can also engage in perfect salience (choosing ads that are always salient) or no salience at all (ads that do not attempt to promote the brand). Consumers form consideration sets by making a number of evocations. The probability that a particular firm is evoked is influenced by firms’ investment in the degree of salience. We find that in equilibrium, if salient advertising is either sufficiently expensive or sufficiently cheap, all firms either forgo or engage in perfect salient advertising, respectively. In both cases, the equilibrium price distributions are identical and both firms have an equal probability of being evoked. This means that profit is strictly lower in the low cost case, making advertising wasteful. The effect of salient advertising is lost because both firms advertise with
equal intensity. Thus, the high-cost advertising technology that leads to the no-
salience equilibrium Pareto dominates the low-cost technology that leads to the
perfect salience equilibrium. Intermediate levels of advertising cost lead to an equi-
librium with imperfect salient advertising. Unlike in the previous two cases where
advertising is ineffective, there are realizations in which firms can successfully use
advertising to get consumers to pay attention to their brand. The effects on prices
and firm profit are ambiguous whenever imperfect salience is involved, depending
on the advertising technology, consumer valuation and production cost. There are
cases where welfare is highest under an intermediate advertising technology that
leads to an equilibrium with positive but imperfect salience. When effective salent
advertising substitutes for lower prices and the total cost is low enough, expected
firm profits are greater than without advertising. However, if consumers are sen-
sitive to advertising and increases in salience are smaller for higher prices, then
the average price paid by consumers is less than without advertising, so consumer
utility increases as well.

The remainder of this paper is organized as follows. Section 2 sets up the model
and analyzes its equilibrium. Section 3 analyzes the welfare impact of technology
changes, Section 4 does a brief literature review, and Section 5 concludes.

2 The Model

There are two identical firms that compete to sell a homogeneous good. Each firm
pays a constant marginal cost of $c$ to produce the good. Firm $i$ makes a twofold
decision: it sets price, $p_i$, and determines the degree of salience in its advertising
$a_i^s \in [0, 1]$, incurring a cost of $A(a_i^s)$, where $A$ is strictly convex. The cost of
choosing no salience, \(A(0)\), is normalized to zero. If a firm chooses a certain degree of salience \(a_i^s \in (0, 1)\), it means that firm \(i\)’s ad will turn out to be salient with probability \(a_i^s\) (imperfect salience). Likewise, choosing \(a_i^s = 1\) means that firm \(i\)’s ad will always turn out to be salient (perfect salience), and choosing \(a_i^s = 0\) means that firm \(i\)’s ad will never turn out to be salient (no salience).

On the other side of the market, there is a unit mass of identical consumers with valuation \(v > c\) for the good. Before making a purchase, consumers form their consideration set by making two evocations. The degree of salience in the ads \((a_1^s, a_2^s)\) determines the probability that an individual consumer evokes a particular firm. With probability \((1-a_1^s)(1-a_2^s)\), none of the ads will be salient, so consumers will evoke each firm with equal probability. With probability \(a_1^s(1-a_2^s)\), firm 1’s ad will be salient and firm 2’s ad will not be salient. In this case, a consumer will evoke firm 1 with \(\mu > 1/2\) and firm 2 with probability \(1 - \mu\), where \(\mu\) is a fixed parameter of the model. Finally, when both ads turn out to be salient, which happens with probability \(a_1^s a_2^s\), consumers will evoke each firm with equal probability. That is, when both firms’ ads are salient, the effect of salience is lost because both ads cancel each other out, and the result will be the same as in the case where none of the ads is salient.

To form their consideration sets, consumers make two evocations with replacement. If only one firm is evoked, the consumer purchases from that firm. If two different firms are evoked, the consumer chooses the firm offering the lower price. If both firms have the same price, each of them has an equal probability of being chosen. In this framework, salient advertising raises the probability that a firm \(i\) will be evoked, but does not guarantee that it will be the only firm to be evoked.

Firms and consumers play the following game. First, the two firms in the
market simultaneously choose prices and choose the degree of salient advertising. Then, consumers in the market observe each firm’s ads and form their consideration sets. Then, consumers observe the prices of the firms in their consideration sets and make a purchasing decision. A pricing strategy for firm $i$ is a price distribution $F_i$. An advertising strategy is a degree of salient advertising $a_i^* \in [0, 1]$. 

2.1 Equilibrium

We restrict attention to the symmetric Nash equilibrium and dispense with subscripts on $F$, $a^*$ and $p$. First, we consider the baseline case with no advertising, where firms only compete in prices. Consumers make two evocations to form their consideration sets, and each firm is remembered with equal probability. In the baseline case, when salient advertising is not allowed, there is price dispersion, as shown in the following Proposition.

Proposition 1. In the baseline case, where consumers have limited attention but salient advertising is not allowed, firms play a mixed pricing strategy. The lower bound of the support is $p = c + \frac{1}{3}(v - c)$, the upper bound is the consumer valuation $v$, and the firm price distribution is $F(p) = 1 - \frac{1}{2} \left( \frac{v - p}{v - c} \right)$. Expected profit is equal to $\frac{v - c}{4}$.

The proof of Proposition 1 relies on the following Lemmas.

Lemma 1. There are no atoms in the equilibrium price distribution.

Lemma 2. The upper bound of the equilibrium firm price distribution is $v$.

The proof of Lemma 1 follows from the proof of Lemma A, shown in the appendix. The reasoning is that, if both firms have an atom at a certain price
p, they split all consumers with two firms in their consideration sets. Firms can benefit by undercutting the other firm, thus getting all consumers with two firms in their consideration sets. Lemma 2 follows because firms make no profit at prices above v, but they always have an incentive to raise prices to v for those consumers who only evoked them into their consideration sets. Price dispersion occurs for the following reason. Some consumers only evoked one firm into their consideration sets, so firms want to exploit their willingness to pay. However, other consumers evoked both firms into their consideration sets and will buy from the lowest priced firm, so firms also want to charge lower prices to attract these consumers. Equal profit on the support of the firm price distribution allows us to solve for the lower bound and the firm price distribution. All proofs are in the Appendix.

When there is advertising, there is still price dispersion in equilibrium (see Lemma A in the Appendix). By strict convexity of the cost of advertising function A, there exists a unique optimal level of advertising for each price in the support of the firm price distribution. Therefore, an optimal advertising strategy is a mapping from the support of the firm price distribution F to [0, 1]. The characterization of equilibrium depends on the marginal cost to marginal benefit of advertising ratio at the top of the firm price distribution, $\frac{A'(a^*(v))}{(v - c)}$. When the marginal cost to marginal benefit ratio of advertising is high or low, both firms either forgo advertising or engage in perfect salient advertising, respectively. On the other hand, intermediate values of this ratio lead to an equilibrium with imperfect salient advertising.

Let $\beta = \int_{\frac{1}{2}}^{v} a^*(x)dF(x) \in [0, 1]$. Expected firm profit is given by the following expression:
\[ \Pi(p, a^s(p)) = a^s(p)\beta\Pi_{ss}(p) + (1 - a^s(p))\beta\Pi_{ns}(p) \\
+ a^s(p)(1 - \beta)\Pi_{sn}(p) + (1 - a^s(p))(1 - \beta)\Pi_{nn}(p) - A(a^s(p)) \]

Where
\[ \Pi_{nn}(p) = \Pi_{ss}(p) = \left[ \frac{1}{4} + \frac{1}{2}(1 - F(p)) \right](p - c) \]
\[ \Pi_{ns}(p) = \left[ (1 - \mu)^2 + 2\mu(1 - \mu)(1 - F(p)) \right](p - c) \]
\[ \Pi_{sn}(p) = \left[ a^2 + 2\mu(1 - \mu)(1 - F(p)) \right](p - c) \]

The following Proposition summarizes the equilibrium result.

**Proposition 2.** There exists a unique symmetric Nash equilibrium where both firms have supports \([p, v]\). The lower bound \(p\), the firm price distribution and the degree of salient advertising depend on the advertising technology as follows.

1. **High-cost Advertising technology**, \(A'(0) > \frac{\mu^2}{3} - \frac{1}{4}\): no salience in equilibrium, \(a^s(p) = 0 \forall p \in [p, v]\). The lower bound of the support is \(p = c + \frac{1}{3}(v - c)\) and the firm price distribution is \(F(p) = 1 - \frac{1}{2} \left( \frac{v - p}{v - c} \right)\). Expected profit is equal to \(\frac{v - c}{4} - A(1)\).

2. **Low-cost Advertising Technology**, \(A'(1) > \frac{\mu^2}{3} - \frac{1}{4}\): perfect salience in equilibrium, \(a^s(p) = 1 \forall p \in [p, v]\). The lower bound of the support is \(p = c + \frac{1}{3}(v - c)\) and the firm price distribution is \(F(p) = 1 - \frac{1}{2} \left( \frac{v - p}{v - c} \right)\). Expected profit is equal to \(\frac{v - c}{4} - A(1)\).

3. **Medium-cost Advertising Technology**, \(A'(0) \leq \frac{A'(a^s(v))}{v - c} < \frac{1}{4} - (1 - \mu)^2\) and \(A'(1) \geq \frac{A'(a^s(v))}{v - c} > \mu^2 - \frac{1}{4}\): imperfect salience in equilibrium, \(a^s(p) \in (0, 1) \forall p \in [p, v]\). Expected profit is equal to \(\frac{v - c}{4} - A(1)\).

There are other two cases corresponding to medium-high and medium-low cost of advertising technologies, which are a combination of cases 1 and 3 and 2 and 3 respectively. We decided not to include them because we can’t get closed form solutions and the conditions are hard to interpret.
[p, v]. The lower bound of the support is

\[ p = c + \frac{A'(a^s(p))}{2(\mu - \frac{1}{2}) - \frac{A'(a^s(v))}{v-c}} \]

and the firm price distribution is

\[ F(p) = 1 - \frac{1}{2} \left[ \frac{A'(a^s(v))}{v-c} - \frac{A'(a^s(p))}{p-c} \right] \]

The expected probability that a firm’s ad is salient is given by

\[ \beta = \frac{(\mu^2 - \frac{1}{4}) - \frac{A'(a^s(v))}{v-c}}{2(\mu - \frac{1}{2})^2} \]

Expected profit is equal to

\[ (v-c) \left[ s(v)(1 - \beta)(\mu^2 - \frac{1}{4}) - (\mu - \frac{1}{2})(\frac{3}{2} - \mu)\beta(1 - a^s(v)) \right] + \frac{1}{4}(v-c) - A(a^s(v)) \]

As in the baseline case, the proof relies on Lemma A and Lemma 2 (see Appendix).

In the medium-cost advertising technology case, the equilibrium expected degree of salience depends on the ratio of the gain of having a salient ad when the opponent firm does not to the loss generated by the opponent’s ad being salient at the upper bound of the price distribution. When firm 1 is pricing at \( v \) and its ad realization is salient but firm 2’s ad realization is not salient, the proportion of the population who will only evoke firm 1 into their consideration set increases in \( \mu^2 - \frac{1}{4} \). Thus, there will be more consumers who pay full price for the good because they failed to remember firm 2, generating an expected gain of \((\mu - \frac{1}{4})(v-c)\). Because of the medium cost of advertising, this expected gain will exceed the marginal cost of investing in the degree of salience \( a^s(v) \). However, when firm 2’s
ad is salient, it decreases the efficiency of firm 1 having a salient ad realization. When firm 2’s ad realization is salient, the expected gain of firm 1 is given by \[
\left[ \frac{1}{4} - (1 - \mu^2) \right] (v - c). \]
That is, a salient realization of firm 2 generates a loss to firm 1, equal to \[
\left[ (\mu^2 - \frac{1}{4}) - \left( \frac{1}{4} - (1 - \mu^2) \right) \right] (v - c). \]
The higher the expected loss provoked by the opponent firm’s ad being salient, the lower the expected degree of salience, and vice versa.

The equilibrium firm price distribution is determined by a similar reasoning. When firm 2’s ad is salient, which happens with probability \( \beta \), firm 1 having a salient ad increases the proportion of the population that will evoke both firms into their consideration set. When firm 2’s ad is not salient, firm 1 having a salient ad increases the proportion of the population that will only have firm 1 in their consideration set, therefore decreasing the proportion of people with two firms in their consideration set. Depending on the value of \( \beta \), firm 1’s ad being salient will generate an expected increase or decrease in the proportion of people who remember both firms. When \( \beta < \frac{1}{2} \), firm 1’s salience generates an expected decrease in the proportion of the population whose consideration set consists of both firms equal to \( 2[\frac{A'(a^*(v))}{v-c} - (\mu - \frac{1}{2})] \). When this is the case, the increase in the proportion of people whose consideration consists of only firm 1, given by \( \frac{A'(a^*(v))}{v-c} \), who will definitely buy from firm 1 and generate a marginal benefit of \( p - c \), should exceed the marginal cost of engaging in the degree of advertising \( a^*(p) \). That is, salience functions as a substitute for lower prices. On the other hand, when \( \beta > \frac{1}{2} \), when firm 1’s ad is salient, the expected number of people who remember both firms increases in \( 2[(\mu - \frac{1}{2}) - \frac{A'(a^*(v))}{v-c}] \). This implies that salient advertising is complementary to charging lower prices. Also, the benefits earned by firm 1 when it is the only firm in consumers’ consideration sets are not enough to cover the
cost of salience.

The lower bound of the firm price distribution depends on the expected increase in the proportion of the population who will remember firm 1, given by \(2(\mu - \frac{1}{2}) - \frac{A'(s^*(v))}{v-c}\). Given that firm 1 has the lowest possible price, being remembered is sufficient for making a sale. If this number is high, then the lower bound of the price distribution will be closer to the marginal cost of production, since the firm will need to extract less from each individual consumer to compensate for the marginal cost of engaging in salient advertising. If, on the other hand, this number is low, \(p\) will be higher in the medium-cost case. The ratio \(\frac{A'(s(p))}{2(\mu - \frac{1}{2}) - \frac{A'(s^*(v))}{v-c}}\) determines how above the marginal cost of production a firm has to charge in order to cover the marginal cost of engaging in degree of salient advertising \(a^*(p)\). If this is smaller than \(\frac{1}{3}(v - c)\), then the lower bound of the price distribution is lower in the medium-cost advertising technology case than in the other two cases. This is more likely to happen in markets where at least one of the following is true: consumer valuation is high, production cost is low, or the probability of a salient firm being remembered, \(\mu\), is closer to 1.

**Definition 1.** We say that consumers are responsive to salience at the lower bound of the firm price distribution \(p\) whenever 
\[
2\left(\mu - \frac{1}{2}\right) - \frac{A'(s^*(v))}{v-c} > \frac{A'(s(p))}{\frac{1}{3}(v-c)}.
\]

It follows from the previous discussion that whenever consumer are responsive to salience at \(p\), the lower bound of the firm price distribution in the medium cost case is smaller that the lower bound in the high and low cost cases.
3 Welfare Analysis

In this section we analyze the welfare impacts of switching the advertising technology, given by the cost of advertising function $A$. We fix the consumer valuation $v$ and cost of production $c$, and focus solely on the advertising technology, represented by $A_i$ for $i = h, m, l$ referring to high-cost, medium-cost and low-cost advertising technologies respectively.

**Proposition 3.** Switching advertising technology from $A_l$ to $A_h$ always improves welfare.

The proof is straightforward, and follows from observing that the firm profit stated in Proposition 2 case 1 is always strictly greater than the firm profit stated in Proposition 2 case 2. Consumer welfare remains unchanged because the supports and the firm price distributions are identical. That is, the low-cost technology is always Pareto dominated by the high-cost technology.

Whenever the medium-cost technology is involved, the welfare analysis is ambiguous. First of all, firm profit might increase or decrease when switching from high-cost to medium-cost technology. Imperfect salience introduces a potential gain, given by the realizations in which only one firm’s ad turns out to be salient. In this case, the salient firm is remembered with a higher probability, and, thus, has a higher likelihood of being the only firm in the consumer consideration set. This means that the likelihood that a firm sells at higher prices increases. However, we must also take into account that there is an associated potential loss, generated by the opposite case. That is, when only one firm’s ad realization is not salient, it has a lower probability of being in the consumer consideration set, so its likelihood of selling at higher prices falls. A sufficient condition for imperfect
salience to generate an expected gain is that the advertising function \( a^*(p) \) be increasing in \( p \), which implies that \( a^*(v) > \beta \). This means that advertising works as a substitute for lower prices: firms engage in a higher degree of salient advertising for higher prices to increase the likelihood of being the only brand in the consumer consideration set. However, for lower prices, it becomes less important to engage in salient advertising because the likelihood of having the lowest price increases.

The effect of switching from high-cost technology to medium-cost technology on prices is also ambiguous. As mentioned in the previous section, when consumers are responsive to salience at the lower bound of the price distribution, both firms charge lower prices in the medium-cost of advertising case. Also, it must be the case that salient advertising \( a^*(p) \) is increasing in prices, to guarantee that the marginal cost of advertising increases in prices. In order to guarantee lower expected prices, it must be the case that the equilibrium firm price distribution in the high and low cost cases, denoted by \( F_c \), first order stochastically dominates the equilibrium firm price distribution from the medium cost case, denoted by \( F_m \). Whenever the marginal cost of salient advertising is strictly concave in prices, \( F_c \) always first order stochastically dominates \( F_m \). A necessary condition for strict concavity of marginal cost with respect to price is that \( a^*(p) \) is concave in \( p \), which means that the rate at which advertising increases should decrease as prices get higher. If the marginal cost of advertising is not strictly concave in prices, a sufficient condition to guarantee first order stochastic dominance is that the density at the upper bound of the firm price distribution obeys \( f(v) < \frac{1}{2(v-c)} \).

The following proposition summarizes the case where the medium-cost technology Pareto dominates the high-cost technology.

**Proposition 4.** Welfare is higher under \( A_m \) rather than under \( A_h \) whenever all
of the following conditions hold:

1. Imperfect salience generates an expected gain that exceeds its total cost at the upper bound of the price distribution, $A_m(a^*(v))$.

2. The degree of salient advertising is increasing in prices.

3. Consumers are responsive to salience at the lower bound of the price distribution.

4. The marginal cost of advertising is strictly concave in $p$ or the density at the upper bound of the firm price distribution $f(v) < \frac{1}{2(v-c)}$.

When the degree of salient advertising is increasing in prices, investing in imperfect salience generates an increase in expected revenue. Whenever this increase is sufficiently high to make up for the cost of investing in salience, firm profit is strictly higher under imperfect salience than under no-salience. Expected prices are lower due to conditions 3 and 4, as explained above. For a more detailed proof, see the Appendix.

In the following case, we establish conditions for the low-cost technology to be strictly worse than the medium-cost technology. Since the equilibrium price distribution and its lower bound are identical to the ones from the high-cost advertising technology, the intuition is very similar to that of the previous result. However, in this case, having to incur the cost of perfectly salient advertising makes firm profit even lower. The complete conditions for this case are stated in the following proposition.

**Proposition 5.** Welfare is higher under $A_m$ rather than under $A_h$ in the following cases.
1. When $A(t) > A_m(a^*(v))$ and all of the following conditions hold.

   (a) The degree of salient advertising is increasing in prices.

   (b) Consumers are responsive to salience at the lower bound of the price distribution.

   (c) The marginal cost of advertising is strictly concave in $p$ or the density at the upper bound of the firm price distribution $f(v) < \frac{1}{2(v-c)}$.

2. When $A(t) < A_m(a^*(v))$ and all of the following conditions hold.

   (a) The degree of salient advertising is increasing in prices.

   (b) Imperfect salience generates an expected gain that exceeds $A_m(a^*(v)) - A(t)$.

   (c) Consumers are responsive to salience at the lower bound of the price distribution.

   (d) The marginal cost of advertising is strictly concave in $p$ or the density at the upper bound of the firm price distribution $f(v) < \frac{1}{2(v-c)}$.

4 Literature Review

In the Economics literature, other models have also considered consumer purchasing decisions as a two-step process. Manzini and Mariotti (2007) and Lleras et al. (2010) suggest that having too many options to choose from is overwhelming for consumers, and hence, they end up restricting attention to subsets (possibly strict) of all feasible alternatives. In both models, a consumer forms his consideration set using one particular rationale or consideration filter (such as salience or
and then chooses one alternative from the consideration set using a different rationale (such as utility maximizing). Other models have studied how firm advertising can be used to shape or discriminate from consideration sets. Eliaz and Spiegler (2011) allow advertising to modify consideration sets in the following way. First, the consumer chooses a preliminary consideration set out of the set of all feasible alternatives. Then, salient advertising seeks to convince the consumer that other products are close substitutes for the product under consideration and should be included in his consideration set. In Tyson (2011), salient advertising cannot determine which elements enter the consideration set, but can be used as a filter to choose an element from the consideration set whenever it contains more than one brand.

As in Burdett and Judd’s (1983) noisy sequential search framework, we get price dispersion because every consumer has a positive probability of evoking one or more firms. However, in this paper, the distribution is endogenously determined by advertising. This model also differs from models of informative advertising with price dispersion (e.g., Butters, 1977; Robert and Stahl, 1993) where consumers cannot purchase from a particular firm without either obtaining a price ad from that firm or having paid a cost to search that firm. A consumer who enters the market for a good is aware of all potential competitors in that market, but he only compares the prices of brands he has evoked. This contrasts both Chioveanu’s (2008) model of persuasive advertising, where all consumers choose the lowest priced good unless they are “convinced” by advertising to become loyal to a certain brand and the framework of Haan and Moraga-Gonzlez (2009) where consumers are more likely to search firms with more salient advertising, but where search is required before a firm is considered.
5 Conclusion

In this paper, we have analyzed salient advertising to consumers with limited attention. Ads that are salient remind consumers of the brand that is being advertised in buying situations and may increase the probability of being the only brand in a consumer’s consideration set. Ads that are not salient focus on promoting the product (as opposed to the brand), and do not affect the probability of any firm being evoked into the consumer consideration set. When the advertising technology is expensive, firms do not engage in salient advertising at all. On the other hand, when the advertising technology is cheap, firms engage in perfectly salient advertising, which means that their corresponding ads are always salient. Finally, when the advertising technology involves medium costs, firms engage in imperfect salient advertising. This means that a firm’s ad has a certain expected probability of being salient. As a consequence, a firm will benefit when its ad realization is salient and the opponent firm’s ad realization is not salient. Similarly, a firm will be harmed when its own ad realization is not salient and the opponent firm’s ad realization is salient. That is, only in the imperfect salience case will salient ads have an effect on consumers and their consideration sets. When firms engage in perfect salience, the effect of salience is lost because both firms ads cancel each other out.

In terms of welfare, the high-cost advertising technology case with no salience always Pareto dominates the low-cost advertising technology case with perfect salience. While consumer welfare remains unchanged because the equilibrium price distributions and their supports are identical, firms are strictly worse off with perfect salience because they have to incur the additional cost of perfectly salient

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ads (that are ineffective in the end). This equilibrium result resembles a prisoners’ dilemma or a tug of war competition when costs of advertising are low. In the low-cost advertising technology case, banning advertising would favor both firms, leading to an increase in profit.

The welfare implications of salient advertising are ambiguous whenever the medium-cost technology is involved. Under certain circumstances, imperfect salience leads to lower expected prices and higher firm profits than the no-salience case, so welfare is higher with a medium-cost advertising technology rather than with a high-cost technology. This happens in markets where consumers are very sensitive to salient advertising or have a very high willingness to pay for the product, or the product is relatively cheap to produce. In this case, a ban in advertising strictly reduces both consumer and firm welfare.

An example of the regulation implications of this model is the ban on cigarette advertising in the United States, that became effective in 1971 for radio and television ads, and extended to all forms of outdoor (i.e. billboards) advertising in 1999. The main purposes of the ban are to reduce cigarette consumption among smokers. The ads that permeated the media before the ban suggest that cigarette advertising was perfectly salient (the Marlboro man, Joe Camel, Virginia Slims, etc.). In this context, our model has different predictions than the Becker and Murphy model of advertising. According to Becker and Murphy, Marlboro smokers get utility from seeing the Marlboro man. After the ban on advertising, consumers cannot feel like the Marlboro man anymore, so their utility of smoking decreases. Therefore, they should reduce their smoking, and hence, improve their health. In

\footnote{Public Health Cigarette Smoking Act, 1970.}

\footnote{Tobacco Master Settlement Agreement, 1998.}
our model, smokers are going to smoke anyways. They cannot see the Marlboro man anymore, but they cannot see Joe Camel either. In cigarette buying situations, they will remember Marlboro with the same probability they would have when there was cigarette advertising and keep on smoking the same quantity. Welfare does improve due to the ban, but the ones who benefit from it are the tobacco companies, who do not have to invest in salient advertising anymore.
Appendix

A Proof of Lemma 1

Before introducing the proof of Lemma 1 we need to introduce the following Lemma, which is a general version of Lemma 1.

**Lemma A.** Suppose that a firm that engages in degree of salient advertising \( a^s(p) \) at price \( p \). The equilibrium firm price distribution has no atoms.

**Proof.** Suppose the equilibrium price distributions have an atom at price \( \hat{p} \). Let \( \hat{a}^s(\hat{p}) \) be the degree of salient advertising at \( \hat{p} \). Firm \( i \)'s expected profit at \( \hat{p} \) is

\[
\mathbb{E}\Pi(\hat{p}, \hat{a}^s(\hat{p})) = \hat{a}^s(\hat{p})\beta\Pi_{ss}(\hat{p}) + (1 - \hat{a}^s(\hat{p}))\beta\Pi_{ns}(\hat{p}) + \hat{a}^s(\hat{p})(1 - \beta)\Pi_{sn}(\hat{p})
\]

\[+ (1 - \hat{a}^s(\hat{p}))(1 - \beta)\Pi_{nn}(\hat{p}) - A(\hat{a}^s(\hat{p})) \tag{2}\]

where

\[
\Pi_{nn}(\hat{p}) = \Pi_{ss}(\hat{p}) = \left\{ \frac{1}{2} + \frac{1}{2}[1 - F(\hat{p}) + \frac{1}{2}\mathbb{P}(p = \hat{p})] \right\} (\hat{p} - c)
\]

\[
\Pi_{ns}(\hat{p}) = \left\{ (1 - \mu)^2 + 2\mu(1 - \mu)[1 - F(\hat{p}) + \frac{1}{2}\mathbb{P}(p = \hat{p})] \right\} (\hat{p} - c)
\]

\[
\Pi_{sn}(\hat{p}) = \left\{ a^2 + 2\mu(1 - \mu)[1 - F(\hat{p}) + \frac{1}{2}\mathbb{P}(p = \hat{p})] \right\} (\hat{p} - c)
\]

Suppose that firm \( i \) shifts mass from \( \hat{p} \) to \( \hat{p} - \varepsilon \), without changing the degree of salient advertising. Expected profit becomes

\[
\mathbb{E}\Pi(\hat{p} - \varepsilon, \hat{a}^s(\hat{p})) = \hat{a}^s(\hat{p})\beta\Pi_{ss}(\hat{p} - \varepsilon) + (1 - \hat{a}^s(\hat{p}))\beta\Pi_{ns}(\hat{p} - \varepsilon)
\]

\[+ \hat{a}^s(\hat{p})(1 - \beta)\Pi_{sn}(\hat{p} - \varepsilon) + (1 - \hat{a}^s(\hat{p}))(1 - \beta)\Pi_{nn}(\hat{p} - \varepsilon) - A(\hat{a}^s(\hat{p})) \tag{3}\]

where
\( \Pi_{nn}(\hat{p} - \varepsilon) = \Pi_{ss}(\hat{p} - \varepsilon) = \left\{ \frac{1}{4} + \frac{1}{2}(1 - F(\hat{p} - \varepsilon)) \right\} (\hat{p} - \varepsilon - c) \)

\( \Pi_{ns}(\hat{p} - \varepsilon) = \left\{ (1 - \mu)^2 + 2\mu(1 - \mu)(1 - F(\hat{p} - \varepsilon)) \right\} (\hat{p} - \varepsilon - c) \)

\( \Pi_{sn}(\hat{p} - \varepsilon) = \left\{ a^2 + 2\mu(1 - \mu)(1 - F(\hat{p} - \varepsilon)) \right\} (\hat{p} - \varepsilon - c) \)

For \( \varepsilon \) small enough, profit at \( \hat{p} - \varepsilon \) is strictly higher than profit at \( \hat{p} \), a contradiction.

Since Lemma 1 refers to the baseline case with no salient advertising, its proof follows by setting \( a^*(p) = 0 \) for every \( p \) in the support.

**B Proof of Lemma 2**

*Proof.* A firm pricing at \( \bar{p} > v \) makes no profit. Suppose that the upper bound is \( \bar{p} < v \). At \( \bar{p} \), firm \( i \) will only sell if it is evoked both times, since it will be underpriced for sure. Thus, it can increase its profit by raising its price to \( v \), a contradiction.

**C Proof of Proposition 1**

*Proof.* Since there is no advertising, expected profit is given by

\[
\mathbb{E}\Pi(p, a^*(p)) = \left[ \frac{1}{4} + \frac{1}{2}(1 - F(p)) \right] (p - c)
\]  

(4)

Setting profit at \( v \) equal to profit at \( p \), solve for the price distribution

\[
F(p) = 1 - \frac{1}{2} \left[ \frac{v - p}{p - c} \right]
\]

(5)

Setting \( F(p) \) in Expression (5) equal to zero, solve for the lower bound of the
price distribution

\[ p = c + \frac{1}{3}(v - c) \quad (6) \]

Evaluating Expression (4) at \( v \), we get that expected profit is equal to \( 1/4(v-c) \). 

\[ \square \]

D Proof of Proposition 2

We will prove each of the three cases separately, by solving for equilibrium. There is price dispersion, as follows from Lemma A and Lemma 2

D.1 High Marginal Cost of Advertising

Proof. When the marginal cost of advertising is high, firms choose no advertising in equilibrium for all prices in the support \([p, v]\). Thus, expected profit is given by Expression (4) in the proof of Proposition 1. For a corner solution with no advertising, it must be the case that the marginal benefit of no advertising is less than the marginal cost of no advertising for every price in the support, given that the opponent chooses not to advertise.

\[ [\mu^2 - \frac{1}{4} - 2(\mu - \frac{1}{2})(1 - F(p))] (p - c) < A'(0) \quad (7) \]

Note that the left hand side of Expression (7) is increasing in \( p \). Therefore, a necessary and sufficient condition for Expression (7) to hold at every \( p \in [p, v] \) is

\[ A'(0) \geq (a^2 - \frac{1}{4})(v - c) \quad (8) \]
Setting profit at $v$ equal to profit at $p$, solve for the price distribution, given by Expression (5) in the proof of Proposition 1. Setting $F(p)$ in Expression (5) equal to zero, solve for the lower bound of the price distribution, shown in Expression (6). As in the baseline case, evaluating Expression (4) at $v$, we get that expected profit is equal to $1/4(v - c)$.

D.1.1 Low Marginal Cost of Advertising

Proof. When the marginal cost of engaging in salient advertising is low, in equilibrium, firms choose to engage in perfectly salient advertising. That is, firms choose $a_s(p) = 1$ for all $p \in [p, v]$. In this case, expected profit is given by

$$
\mathbb{E} \Pi(p, a_s(p)) = \{a_s(p) \left[ (\mu - \frac{1}{2})(\frac{3}{2} - \mu) + 2(\mu - \frac{1}{2})^2(1 - F(p)) \right] - \left[ (\mu - \frac{1}{2})(\frac{3}{2} - \mu) + 2(\mu - \frac{1}{2})^2(1 - F(p)) \right] + \frac{1}{4} + \frac{1}{2}(1 - F(p)) \} \{p - c\} - A(a_s(p))
$$

For this to be an equilibrium, it must be the case that

$$
[(\mu - \frac{1}{2})(\frac{3}{2} - \mu) + 2(\mu - \frac{1}{2})^2(1 - F(p))] > A'(1)
$$

The equilibrium firm price distribution is also given by Expression (5), and the lower bound by Expression (6). Taking the derivative of Expression (9) with respect to $a^s$, we find that the marginal benefit is increasing in $p$. Therefore, a sufficient condition for Expression (10) to hold at every $p \in [p, v]$ is
In this case, expected profit is strictly lower than in the equilibrium with no salience. This happens because firms incur an additional expenditure, $A(1)$, which yields an expected profit of $1/4(v - c) - A(1)$ whenever $1/4(v - c) \geq A(1)$.

\[ A'(1) \leq \frac{1}{3} (\mu^2 - \frac{1}{4}) \quad (11) \]

D.1.2 Medium Marginal Cost of Advertising

Proof. When the marginal costs of advertising are in a medium range, the equilibrium level of advertisement $s(p) \in (0, 1)$ for every $p \in \lbrack p, v \rbrack$. Expected profit is given by

\[
\mathbb{E} \Pi(p, a^*(p)) = \{a^*(p)\beta \left[ -2(\mu - \frac{1}{2})^2 + 4(\mu - \frac{1}{2})^2(1 - F(p)) \right] \\
+ a^*(p) \left[ \mu^2 - \frac{1}{4} - 2(\mu - \frac{1}{2})^2(1 - F(p)) \right] \\
- \beta \left[ (\mu - \frac{1}{2})(\frac{3}{2} - \mu) + 2(\mu - \frac{1}{2})^2(1 - F(p)) \right] \\
+ \frac{1}{4} + \frac{1}{2}(1 - F(p)) \} (p - c) - A(a^*(p))
\]

Where $\beta = \int_0^v a^*(x)f(x)dx \in (0, 1)$ is the other firm’s expected investment in salience.

An interior solution requires that marginal benefit be equal to marginal cost for every $p$.
\[
\{ \beta \left[ -2(\mu - \frac{1}{2})^2 + 4(\mu - \frac{1}{2})^2(1 - F(p)) \right] \\
+ \left[ \mu^2 - \frac{1}{4} - 2(\mu - \frac{1}{2})^2(1 - F(p)) \right] (p - c) = A'(a^*(p))
\]  

(Equation 13)

Evaluating Equation (13) at \( v \), we can solve for \( \beta \)

\[
\beta = \frac{\left( \mu^2 - \frac{1}{4} \right) - A'(a^*(v))}{2(\mu - \frac{1}{2})^2}
\]  

(14)

Necessary and sufficient conditions for \( \beta \in (0, 1) \)

\[
A'(0) \leq A'(a^*(v)) \leq (\mu^2 - \frac{1}{4})(v - c)
\]  

(15)

\[
A'(1) \geq A'(a^*(v)) \geq (\mu - \frac{1}{2})(\frac{3}{2} - \mu)(v - c)
\]  

(16)

Also from Equation (13), solve for \( F(p) \)

\[
F(p) = 1 - \frac{1}{2} \left[ \frac{A'(a^*(v))}{v-c} - \frac{A'(a^*(p))}{p-c} \right]
\]  

(17)

Set Expression (17) equal to zero to solve for the lower bound \( p \)

\[
p = c + \frac{A'(a^*(p))}{2(\mu - \frac{1}{2}) - \frac{A'(a^*(v))}{v-c}}
\]  

(18)

Condition (15) guarantees that the denominator in Expression (18) is positive, so \( p > c \).

In order to obtain the density function, take derivative with respect to \( p \) in Expression (17)

\[
f(p) = \frac{1}{2} \left[ \frac{A''(a^*(p))a^*(p)}{p-c} - \frac{A'(a^*(p))}{(p-c)^2} \right]
\]  

(19)
Note that \((\frac{3}{2} - \mu)(\mu - \frac{1}{2}) \leq (\mu - \frac{1}{2}) \leq (\mu + \frac{1}{2})(\mu - \frac{1}{2})\). The equilibrium firm price distribution function in Expression (17) must be a number between zero and one. Also, the density function in Expression (19) must be nonnegative. Thus, we can identify two cases.

1. \(\frac{A'(a^s(v))}{v-c} < (\mu - \frac{1}{2})\) and \(A''(a^s(p))a^s'(p) \leq \frac{A'(a^s(p))}{(p-c)}\): The denominator of Expression (17) is negative, so the numerator must also be negative. This does not add any information about the sign of \(s'(p)\).

2. \(\frac{A'(a^s(v))}{v-c} > (\mu - \frac{1}{2})\) and \(A''(a^s(p))a^s'(p) \geq \frac{A'(a^s(p))}{(p-c)}\): The denominator of Expression (17) is positive, so the numerator must also be positive. This implies that \(a^s(p)\) must be increasing in \(p\).

Using Expression (14) and Expression (19)

\[
\frac{1}{2} \left[ \frac{A'(a^s(v))}{v-c} - (\mu - \frac{1}{2}) \right] \int_v^\mu a^s(x) \left( \frac{A''(a^s(x))a^s'(x)}{x-c} - \frac{A'(a^s(x))}{(x-c)^2} \right) dx = \frac{\mu^2 - \frac{1}{4} - \frac{A'(a^s(v))}{v-c}}{2(\mu - \frac{1}{2})^2}
\]

(20)

Equation (20) solves for \(a^s(v)\). Then, set Equation (12) evaluated at \(p\) equal to Equation (12) evaluated at \(v\) to solve for \(a^s(p)\).

Also, we need to make sure that profit is positive

\[
\{-a^s(v) \left[ \mu^2 - \frac{1}{4} - \frac{A'(a^s(v))}{v-c} \right] \frac{\mu - \frac{1}{2}}{2(\mu + \frac{1}{2})} + (\mu^2 - \frac{1}{4})a^s(v) \} (v-c) + \frac{1}{4}(v-c) - A(a^s(v)) \geq 0
\]

(21)
E Proof of Proposition 4

Proof. Whether profit is higher in the high-cost technology case or in the medium-cost technology case depends on whether the following inequality holds.

\[
(v - c) \left[ -\beta(1 - a^s(v))(\frac{1}{4} - (1 - \mu)^2) + a^s(v)(1 - \beta)(\mu^2 - \frac{1}{4}) \right] > A_m(a^s(v)) \tag{22}
\]

The left hand side of the inequality represents the change in expected revenue generated by investing in imperfect salience. Total revenue is equal to the term on the left hand side of the inequality plus \(\frac{1}{4}(v - c)\), which is also the equilibrium expected revenue from the high-cost advertising technology. If the left hand side of the inequality is positive, it means that investing in imperfect salience generates an increase in expected revenue. A necessary and sufficient condition for the term inside the brackets to be positive is that \(a^s(p)\) be increasing in \(p\). This guarantees that \(a^s(v) > \beta\). If the inequality in Expression (22) holds, investing in imperfect salience generates an expected benefit that exceeds its total cost. Hence, profit is strictly higher in the medium-cost advertising technology case than in the high-cost technology case.

As explained at the end of the equilibrium analysis, whenever consumers are responsive to salience at the lower bound of the equilibrium price distribution corresponding to the medium-cost advertising technology case, from now on denoted \(p_m\) ans \(a^s(p)\) is increasing in \(p\), \(p_m\) is strictly lower than the lower bound of the high-cost firm price distribution, denoted \(p_c\). Additionally, \(a^s(p)\) increasing in \(p\)
guarantees that \( F_m(p_c) > 0 \). In order to ensure first order stochastic dominance of \( F_m \), it must be the case that for every \( p \in [p_c, v] \), \( F_c(p) < F_m(p) \). Rearranging terms, we get that it must be the case that

\[
\frac{A'(a^*(v)) - (\mu - \frac{1}{2})}{v-c} < \frac{A'(a^*(p)) - A'(a^*(p))}{p-c} \quad (23)
\]

Note that the left hand side of the inequality is constant. If the right hand side is increasing in \( p \), given that the inequality holds at \( p_c \), the Expression \((23)\) holds for each \( p \in [p_c, v] \). This is always the case whenever the composite function \( M(p) = A'(a^*(p)) \) is strictly concave. Otherwise, if the right hand side is non increasing in \( p \), it must be the case that when \( p \) approaches \( v \), the inequality still holds. Taking limits on both sides as \( p \to v \), we get that Expression \((23)\) holds whenever \( f(v) < \frac{1}{2(v-c)} \), where \( f(v) \) is the density function from Expression \((19)\).

\section{Proof of Proposition 5}

\textbf{Proof.} Comparing expected profit from the low-cost technology case, equal to \( 1/4(v-c) - A_l(1) \) and expected profit from the medium-cost case, given by Expression \((21)\), we get that expected profit is higher in the medium-cost case whenever

\[
\left[ \frac{\mu - \frac{1}{2}}{2(\mu + \frac{1}{2})} A'_m(a^*(v)) + \frac{1}{2}(\mu + \frac{3}{2})(\mu - \frac{1}{2})(v - c) \right] a^*(v) \geq A_m(a^*(v)) - A_l(1) \quad (24)
\]

If \( A_m(a^*(v)) < A_l(1) \), then it is not necessary for imperfect salience to generate an expected increase in revenue. As long as the expected decrease in revenue is smaller than \( A_l(1) - A_m(a^*(v)) \), profit is higher in the medium-cost case. On
the other hand, if $A_m(a^*(v)) > A_l(1)$, then engaging in imperfect salience should generate an increase in expected revenue sufficient to cover $A_m(a^*(v)) - A_l(1)$. Remember that a sufficient condition for imperfect salience to generate an increase in expected revenue is that $a^*(p)$ be increasing in $p$.

Given that the equilibrium firm price distribution and its lower bound are identical in the lo-cost technology case and in the high-cost technology case, the analysis for prices is identical to that in the proof of Proposition 4.
References


