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“Forward Guidance Under Uncertainty”

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Forward Guidance Under Uncertainty*

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Abstract

Increased uncertainty about the future can reduce a central bank’s ability to stabilize the economy. The inability to offset contractionary shocks at the zero lower bound endogenously generates downside risk for the economy. This increase in risk induces precautionary saving by households, which causes larger contractions in output and inflation and prolongs the zero lower bound episode. When the economy faces significant uncertainty, optimal monetary policy implies further lowering real rates by committing to a higher price-level target. Under optimal policy, the monetary authority accepts higher inflation risk in the future to minimize downside risk when the economy hits the zero lower bound. In the face of large shocks, raising the central bank’s inflation target can attenuate much of the downside risk posed by the zero lower bound.

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1 Introduction

With the federal funds rate currently near zero, the Federal Reserve cannot further stabilize the economy by lowering its short-term nominal policy rate. When constrained by the zero lower bound, research by Eggertsson and Woodford (2003), Wolman (2005), and others advocates using expectations about the future conduct of monetary policy to help support the economy. By committing to expansionary policy in the future, these papers argue that the central bank can mitigate the contractionary effects of the zero lower bound. In practice, central banks often refer to this policy tool as providing forward guidance about the future path of policy. However, much of this previous research relies on models where household decisions can be summarized by the lifetime path of real interest rates. These models fail to analyze how households respond to expectations of future monetary policy when they face increased uncertainty about the future evolution of the economy. Since the beginning of the Great Recession, many policymakers and economists have expressed significant uncertainty about future economic activity. For example, almost all Federal Open Market Committee (FOMC) participants recently indicated that their uncertainty about future output growth is higher than the norm during the previous two decades.\footnote{See page 53 of the Monetary Policy Report to the Congress on July 17, 2012.} Motivated by the current environment of increased uncertainty, this paper examines the ability of forward guidance to stabilize the economy when the future is more uncertain.

I show that increased uncertainty can reduce the central bank’s ability to stabilize the economy at the zero lower bound. When the economy hits the zero lower bound, the monetary authority can lower the expected path of real interest rates through expectations of future expansionary monetary policy. In making their consumption decisions, however, households care about both the expected path of real interest rates and the conditional distribution of future consumption. When the economy faces significant uncertainty about the future, the inability of the monetary authority to offset shocks endogenously generates higher expected volatility and downside risk for the economy. This increase in risk induces precautionary saving by households, which implies lower consumption for a given path of real interest rates. The decreased demand for consumption goods causes larger contractions in output and inflation when the economy encounters the zero lower bound. In addition, higher uncertainty can result in a dramatically prolonged zero lower bound episode.

To analyze the quantitative impact of uncertainty, I solve a general-equilibrium model with a zero lower bound constraint on the central bank’s nominal policy rate. I model increased uncertainty about the future as a higher volatility of the exogenous shocks hitting the economy. I examine the effects of increased uncertainty about future discount factors of the representative household, which have the interpretation as uncertainty about future aggregate demand. Using the model, I simulate various zero lower bound scenarios under either a low or high uncertainty calibration. My calibration strategy is motivated by the sub-sample maximum likelihood estimates of Ireland (2011) and Ireland (2003) or implied stock market volatility. I model the occasionally-binding constraint using the global solution method of Coleman (1990).
Increased uncertainty about the future both amplifies and propagates adverse fluctuations at the zero lower bound. Using the constant price-level targeting rule of Eggertsson and Woodford (2003), I simulate a decline in aggregate demand similar to the contraction during the Great Recession. The model predicts that increased uncertainty generates an additional 1.0% decline in the output gap and an additional 0.5% decline in inflation. If the increased uncertainty becomes realized as higher actual shock volatility, the economy experiences significant fluctuations and likely fails to escape the zero lower bound after several years. Without the higher realized shock volatility, price-level targeting can always fully stabilize the economy within a short period after the economy hits the zero lower bound.

Optimal monetary policy under uncertainty responds to the distribution of shocks that agents expect to hit the economy. Optimal monetary policy implies further lowering real rates by committing to a higher price-level target when the economy faces significant uncertainty about the future. To implement the optimal policy, the monetary authority commits to modestly extending its period of zero policy rates after the initial contraction in economic activity. To minimize the downside risk to the output gap and inflation when the economy hits the zero lower bound, the monetary authority must accept higher inflation risk in the future. Thus, the monetary policymaker faces a trade-off between the medium-run distribution of inflation and the short-run distributions of output and inflation. However, optimal monetary policy does not fully eliminate the downside risk in the economy posed by the zero lower bound. Even under optimal policy, the economy may still experience large fluctuations and fail to escape the zero lower bound for an extended period if the volatility of shocks hitting the economy is high. In the face of large shocks, raising the central bank’s inflation target can attenuate much of the downside risk posed by the zero lower bound.

The key parameter in my analysis is the volatility of the demand shocks hitting the economy. To ensure the reasonableness of my calibration, I simulate the model and compare the distribution of possible outcomes with recent macroeconomic data. I use the history-dependent interest-rate rule estimated by Gust, López-Salido and Smith (2013) as a description of recent FOMC behavior. After matching the initial conditions at the end of 2008, the macroeconomic data since the Great Recession falls within the simulated prediction intervals of the high uncertainty model. Thus, actual data from the U.S. economy is inline with the distribution of possible outcomes that the representative household uses in evaluating their decisions. This exercise provides some evidence that the level of uncertainty in the calibrated model is reasonable. The results suggest the combination of higher volatility and the zero lower bound may play a significant role in explaining the slow recovery of the United States economy. Without higher volatility and the propagation provided by the zero lower bound, the simple model is unable to generate recessions like the most recent macroeconomic data.
2 Intuition

This section formalizes the intuition from the introduction using several key equations of the model. For Section 2 only, I use Taylor series approximations of these equations to show how increased uncertainty about the future can affect the central bank’s ability to stabilize the economy. These approximations provide analytical tractability which is unavailable when examining the model equations in their original nonlinear form. In Section 4, I show that the intuition from these approximations is consistent with the computational results using the full nonlinear model.

2.1 Household Consumption Under Uncertainty

The household consumption Euler equation highlights why increased uncertainty about the future may reduce the central bank’s ability to stabilize the economy at the zero lower bound. Under constant relative risk aversion utility from consumption, the following equation links consumption $C_t$ by the representative household to the gross real interest rate $R_t$:

$$1 = E_t \{ \beta R_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \},$$

where $\beta$ is the household discount factor and $\sigma$ is the risk aversion parameter in the household’s utility function. Using a second-order Taylor series approximation around the steady state, Appendix A.1 shows Equation (1) can be written as follows:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r^r - r^r \right) - \frac{1}{2} \sigma \text{Var}_t c_{t+1}$$

where lowercase variables denote the log of the respective variable, $r^r$ is the steady state net real interest rate, and $\text{Var}_t c_{t+1}$ denotes the conditional variance of future consumption. Iterating Equation (2) forward and taking expectations at time $t$ implies the following solution for current consumption:

$$c_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} \left( E_t r^r_{t+i} - r^r \right) - \frac{1}{2} \sigma \sum_{j=0}^{\infty} \text{Var}_t c_{t+1+j}$$

When the economy encounters the zero lower bound, Equation (3) shows that the monetary authority can raise household consumption by lowering the expected path of real interest rates. However, Equation (3) also shows that households base their consumption decisions on both the expected path of real interest rates and the expected conditional distribution of future consumption. For any given path of real interest rates, households consume less if they expect a more volatile distribution of future consumption. To achieve a given level of consumption, the monetary authority must choose an even lower path for real rates when households face significant uncertainty about future consumption.

2.2 Consumption Uncertainty in General Equilibrium

To illustrate the general-equilibrium effects of the higher-order consumption moments, I embed the approximated household Euler equation into a simple general-equilibrium model. Using a simplified version of the
model outlined in Section 3, Appendix A.2 shows how to derive the following approximate higher-order version of a standard New-Keynesian model:

\[ x_t \approx E_t x_{t+1} - \frac{1}{\sigma} \left( r_t^e - r_t^n \right) - \frac{1}{2} \sigma \text{Var}_t x_{t+1} \]  

(4)

\[ r_t^i \approx r_t - E_t \pi_{t+1} + \frac{1}{2} \text{Var}_t \pi_{t+1} + \sigma \text{Cov}_t \left( x_{t+1}, \pi_{t+1} \right) \]  

(5)

\[ \pi_t \approx \beta E_t \pi_{t+1} + \kappa x_t \]  

(6)

These equations link the output gap \( x_t \) and inflation rate \( \pi_t \) to the nominal interest rate \( r_t \) and real interest rate \( r_t^i \). The output gap \( x_t \) is the percent deviation of equilibrium output from output in an equivalent economy without nominal price rigidities. Shocks in the economy cause changes in the natural real interest rate \( r_t^n \), which is the real interest rate that would prevail in the equivalent flexible-price economy. Changes in the natural rate can cause fluctuations in the output gap and inflation.\(^2\) The monetary authority can minimize these fluctuations by adjusting the nominal interest rate to offset shocks to the natural real rate. However, the zero lower bound \( r_t \geq 0 \) imposes a limit on the central bank’s ability to offset fluctuations in the natural real rate. When the natural real rate becomes negative, the monetary authority becomes constrained by the zero lower bound and must rely on expectations about future monetary policy to help stabilize the economy.

The expected volatility of the natural real rate governs the amount of uncertainty faced by the economy. Higher expected volatility in \( r_t^n \) makes the future harder to forecast, which increases the uncertainty about the future. Equation (5) augments the standard Fisher relation \( r_t^i = r_t - E_t \pi_{t+1} \) to include the impact of uncertainty about future inflation and its expected covariance with future output gaps. Since prices adjust slowly to changing economic conditions, changes in the nominal interest rate affect the economy by altering real interest rates. Solving Equations (4) and (6) forward:

\[ x_t = -\sum_{i=0}^{\infty} \left( E_t r_{t+i}^n - E_t r_{t+i}^r \right) - \frac{1}{2} \sigma \sum_{j=0}^{\infty} \text{Var}_t x_{t+1+j} \]  

(7)

\[ \pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \kappa x_{t+i} \right\} \]  

(8)

Equations (7) and (8) show that the evolution of the economy is summarized by the expected paths of real interest rates and the expected conditional variance of the output gap. The additional consumption risk in the household Euler equations adds the second-order moments of the output gap to the standard New-Keynesian model.

The transmission of the household consumption risk to the macroeconomy depends crucially on monetary policy’s ability to stabilize the economy. In absence of the zero lower bound, the monetary authority

\(^2\) For clarity of exposition, Equations (4) and (5) omit two additional covariance terms which are related to the exogenous process for the natural rate shocks. The coefficients on these terms are very small and they do not provide any additional intuition. See Appendix A.2 for additional details.
can always fully stabilize the economy by setting its nominal policy rate equal to the natural real rate. In this scenario, the conditional variances of the output gap and inflation are zero since the monetary authority can stabilize the economy in all future periods. However, suppose the natural real rates becomes negative and the zero lower bound prevents the central bank from fully stabilizing the economy. Households and firms internalize this reduced ability to offset future fluctuations at the zero lower bound. The higher expected volatility affects the economy through two channels. First, Equations (4) and (6) show that expectations of future output gap fluctuations depress current output and inflation for any given path of real interest rates. In addition, Equation (5) shows that a given level of the nominal interest rate and expected inflation are less effective at lowering the real interest rate if agents expect inflation volatility and correlated fluctuations in the output gap and inflation. When the future is more uncertain, the monetary authority must further lower the path of nominal and real interest rates to achieve a given level of the output gap.

2.3 Zero Lower Bound and Downside Risk

The intuition discussed thus far suggests that the zero lower bound endogenously generates a more volatile distribution of future consumption for the representative household. In addition to higher expected volatility, however, the asymmetric ability of the central bank to offset shocks generates negative-skewness in the expected distribution of consumption. While the central bank can fully offset expansionary shocks with higher nominal policy rates, the zero lower bound implies a constraint on its ability to offset contractionary shocks. Households internalize this constraint when forming expectations about future consumption. Increased uncertainty amplifies this asymmetry and produces significantly left-skewed distributions for consumption throughout the zero lower bound episode. Thus, the zero lower bound endogenously generates downside tail-risk in household consumption. Returning to Equation (1), a third-order approximation of the consumption Euler equation can be written as follows:

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r^r_t - r^r \right) - \frac{1}{2} \sigma \text{Var}_t c_{t+1} + \frac{1}{6} \sigma^2 \text{Skew}_t c_{t+1}, \]  

where \( \text{Skew}_t c_{t+1} \) denotes the conditional skewness of future consumption. Thus, the negative skewness introduced by the zero lower bound provides an additional mechanism that further reduces the responsiveness of consumption to real interest rates.

2.4 From Intuition to Model Simulations

The intuition of this section argues that increased uncertainty about the future can amplify adverse fluctuations at the zero lower bound. In the following section, I calibrate and solve a nonlinear model and show that the simulated zero lower bound scenarios are consistent with the intuition developed in this section. In addition, I show that the effects of increased uncertainty in the calibrated model are quantitatively significant. At the zero lower bound, the precautionary behavior by households amplifies and propagates shocks and dramatically prolongs the zero lower bound episode.
3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that I use my analysis. The baseline model shares many features with the models of Ireland (2003) and Ireland (2011). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. I allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). The baseline model considers fluctuations in the discount factor of households, which have the interpretation as demand shocks.

3.1 Households

In the model, the representative household maximizes lifetime expected utility over streams of consumption $C_t$ and leisure $1 - N_t$. The household receives labor income $W_t$ for each unit of labor $N_t$ supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and receives lump-sum dividends $D_t$. The household also has access to zero net supply nominal bonds $B_t$ and real bonds $B^R_t$. A nominal bond pays the gross one-period nominal interest rate $R_t$ while a real bond pays the gross one-period real interest rate $R^R_t$. The household divides its income from labor and its financial assets between consumption $C_{t+s}$ and the amount of the bonds $B_{t+s+1}$ and $B^R_{t+s+1}$ to carry into next period. The discount factor of the household $\beta$ is subject to shocks via the stochastic process $a_t$. An increase in $a_t$ induces households to consume more and work less for no technological reason. Thus, I interpret changes in the household discount factor as demand shocks for the economy.

The representative household maximizes lifetime utility by choosing $C_{t+s}, N_{t+s}, B_{t+s+1},$ and $B^R_{t+s+1}$, for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$\max E_t \sum_{s=0}^{\infty} a_{t+s} \beta^s \left( C_{t+s} (1 - N_{t+s})^{1-\eta} (1-\sigma)^{1-\sigma} \right)$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{1}{R_t} \frac{B_{t+1}}{P_t} + \frac{1}{R^R_t} B^R_{t+1} \leq \frac{W_t}{P_t} N_t + \frac{B_t}{P_t} + \frac{D_t}{P_t} + B^R_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\eta a_t C_t^{\eta(1-\sigma)-1} (1 - N_t)^{(1-\eta)(1-\sigma)} = \lambda_t$$  \hspace{1cm} (10)

$$\lambda_t W_t = \lambda_t$$  \hspace{1cm} (11)

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{R_t P_t}{P_{t+1}} \right) \right\}$$  \hspace{1cm} (12)

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) R^R_t \right\}$$  \hspace{1cm} (13)

where $\lambda_t$ denotes the Lagrange multiplier on the household budget constraint. Equations (13) - (14) represent the household intratemporal optimality conditions with respect to consumption and leisure, and Equations (15) - (16) represent the Euler equations for the one-period nominal and real bonds.
3.2 Intermediate Goods Producers

Each intermediate goods-producing firm \( i \) rents labor \( N_t(i) \) from the representative household in order to produce intermediate good \( Y_t(i) \). Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price \( P_t(i) \) each period. Firm \( i \) chooses \( N_t(i) \), and \( P_t(i) \) to maximize the discounted present-value of cash flows \( D_t(i)/P_t(i) \) given aggregate demand \( Y_t \) and price \( P_t \) of the finished goods sector. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production \( \Phi \).

Each intermediate goods-producing firm maximizes discount cash flows using the household stochastic discount factor:

\[
\max \mathbb{E}_t \sum_{s=0}^\infty \left( \beta^s \frac{\lambda_{t+s}}{\lambda_t} \right) \left[ \frac{D_{t+s}(i)}{P_{t+s}} \right]
\]

subject to the production function:

\[
\left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \leq N_t(i) - \Phi,
\]

where

\[
\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i) - 1} \right]^2 Y_t
\]

The first-order conditions for the firm \( i \) are as follows:

\[
\frac{W_t}{P_t} N_t(i) = \Xi_t N_t(i) \tag{14}
\]

\[
\phi_P \left[ \frac{P_t(i)}{\Pi P_{t-1}(i) - 1} \right] \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} \right] = (1 - \theta) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} + \theta \Xi_t \left[ \frac{P_t(i)}{P_t} \right]^{-\theta - 1} + \phi_P \mathbb{E}_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \frac{Y_{t+1}}{Y_t} \left[ \frac{P_{t+1}(i)}{\Pi P_t(i)} \right] - 1 \left[ \frac{P_{t+1}(i)}{\Pi P_t(i)} \right] \frac{P_{t+1}(i)}{P_{t+1}(i)} \right\}, \tag{15}
\]

where \( \Xi_t \) is the multiplier on the production function, which denotes the real marginal cost of producing an additional unit of intermediate good \( i \).

3.3 Final Goods Producers

The representative final goods producer uses \( Y_t(i) \) units of each intermediate good produced by the intermediate goods-producing firm \( i \in [0, 1] \). The intermediate output is transformed into final output \( Y_t \) using the following constant returns to scale technology:

\[
\left[ \int_0^1 Y_t(i) \frac{\lambda_{t+1}}{\lambda_t} di \right]^{\frac{\theta}{\theta - 1}} \geq Y_t
\]

Each intermediate good \( Y_t(i) \) sells at nominal price \( P_t(i) \) and each final good sells at nominal price \( P_t \). The finished goods producer chooses \( Y_t \) and \( Y_t(i) \) for all \( i \in [0, 1] \) to maximize the following expression of firm profits:

\[
P_t Y_t - \int_0^1 P_t(i) Y_t(i) di
\]
subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

\[ Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \]

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index \( P_t \) can be written as follows:

\[ P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \]

### 3.4 Monetary Policy

I assume a cashless economy where the monetary authority sets the one-period net nominal interest rate \( r_t = \log(R_t) \). Due to the zero lower bound on nominal interest rates, the central bank cannot lower its nominal policy rate below by zero. In my baseline model, I assume that the monetary authority sets its policy rate according to the following constant price-level targeting rule of Eggertsson and Woodford (2003):

\[ r_t \left( \theta (p_t - p^*) + x_t \right) = 0, \tag{16} \]

where \( p_t \) is the log of the price level, \( p^* \) is the constant price-level target of the central bank, and \( x_t \) is the gap between current output and output in the equivalent flexible-price economy. When the zero lower bound does not bind, the monetary authority uses the nominal interest rate \( r_t \) to close the output gap-adjusted price level in parenthesis. When the central bank encounters the zero lower bound, however, the monetary authority cannot perfectly stabilize the economy using its nominal policy rate. By committing to a stable price-level in the long-run, this rule promises to undo any deflation caused by the zero lower bound. By committing to higher inflation and more expansionary policy in the future, Eggertsson and Woodford (2003) shows that this policy rule can help mitigate some of the contractionary effects of the zero lower bound. By committing to a constant price-level target, this rule implies a zero percent inflation target for the central bank. In Section 6.1, I relax this assumption and consider a central bank which chooses a two or four percent inflation target.

### 3.5 Shock Processes

Shocks to the discount rate of households are the only exogenous stochastic process in the baseline model. The stochastic process for these fluctuations is as follows:

\[ a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma^a \varepsilon^a_t \tag{17} \]

Large negative innovations to this process imply a large decline in aggregate demand, which forces the economy to encounter the zero lower bound. The volatility of the preference shock \( \sigma^a \) controls the amount of uncertainty about the future faced by the economy. A higher expected volatility makes forecasting the future time path of the stochastic process more difficult. I specify the stochastic process in levels, rather
than in logs, to prevent the volatility $\sigma^a$ from impacting average value of $a_t$ through a Jensen’s inequality effect. In the model simulations, $a_t$ always remains significantly greater than zero.

### 3.6 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$ and employ the same amount of labor $N_t(i) = N_t$. Thus, all firms have the same cash flows and I define gross inflation as $\Pi_t = P_t/P_{t-1}$ and the markup over marginal cost as $\mu_t = 1/\Xi_t$. Therefore, I can model our intermediate-goods firms with a single representative intermediate goods-producing firm. Since fluctuations in household discount factors do not affect the equivalent flexible-price version of my baseline model, I define the output gap as output in deviation from its deterministic steady state $x_t = \ln(Y_t/Y)$. In addition, the gross natural real interest rate that would prevail in the equivalent flexible-price economy can be defined as $R_n^t = \beta^{-1}a_t(E_t a_{t+1})^{-1}$. Thus, shocks to the household discount factor act as fluctuations in the natural real rate for the economy.

### 3.7 Solution Method

To formally analyze the impact of the zero lower bound, I solve the model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method, as opposed to local perturbation methods such as linearization, allows me to model the occasionally binding zero lower bound constraint. This method discretizes the state variables on a grid and solves for the policy functions which satisfy all the model equations at each point in the state space. Appendix B contains the details of the policy function iteration algorithm.

### 3.8 Calibration

Table 1 lists the calibrated parameters of the model. I calibrate the model at quarterly frequency using standard parameters for one-sector models of fluctuations. Since the model shares features with the estimated models of Ireland (2003) and Ireland (2011), I calibrate many of the parameters to match the estimates reported by those papers. To assist in numerically solving the model, I introduce a multiplicative constant into the production function to normalize output $Y$ to equal one at the deterministic steady state. I choose steady-state hours worked $N$ and the model-implied value for $\eta$ such that the model has a Frisch labor supply elasticity of two. Household risk aversion over the consumption-leisure basket $\sigma$ is 2. The value for $\sigma$ implies an intertemporal elasticity of substitution of 0.5, which is consistent with the empirical estimates of Basu and Kimball (2002). The fixed cost of production for the intermediate-goods firm $\Phi$ is calibrated to $(\mu - 1)Y$, which eliminates pure profits in the deterministic steady state of the model.

The crucial parameter in my calibration is the volatility of the preference shock $\sigma^a$, which controls the amount of uncertainty about the future faced by the economy. I simulate various zero lower bound scenarios using both a low uncertainty calibration with $\sigma^a = 0.015$, and a high uncertainty calibration with $\sigma^a = 0.045$. The low uncertainty calibration is consistent with the maximum likelihood estimates of
Ireland (2003) over the post-1979 Great Moderation sample period. When the sample period includes the 2008-2009 Great Recession, however, Ireland (2011) estimates a much larger value for the volatility of the preference shock. My calibrated value for the high uncertainty calibration lies slightly below the estimate of Ireland (2011), but remains inside the standard errors of his estimates. This calibration strategy aims to model the views of the FOMC participants that uncertainty about future economic activity is higher than the norm during the previous two decades. Alternatively, my calibration strategy can also be motivated using implied stock market volatility. Using a similar model, Basu and Bundick (2012) calibrate changes in uncertainty using fluctuations in the VIX volatility index. The increase in uncertainty from the low to high uncertainty calibration roughly corresponds to a two standard deviation uncertainty shock, which is a conservative estimate for the increase in uncertainty during the Great Recession.

3.9 Transmission of Precautionary Saving to Macroeconomy

Before examining the computational results, this section shows how precautionary saving by households lowers output and inflation in the macroeconomy. As I discuss in the previous sections, a more volatile and negatively-skewed expected distribution of consumption induces precautionary saving by the representative household. Since consumption and leisure are both normal goods, lower consumption also induces “precautionary labor supply,” or a desire for the household to supply more labor for a given level of the real wage. Figure 1 illustrates this effect graphically in real wage and hours worked space. Through the forward-looking marginal utility of wealth denoted by $\lambda_t$, an increase in uncertainty shifts the household labor supply curve outward through a wealth effect. If prices adjust slowly to changing marginal costs, however, firm markups over marginal cost rise when the household increases its desired labor supply. For a given level of the real wage, an increase in markups decreases the demand for labor from firms. As Basu and Bundick (2012) discuss, the increase in markups depends crucially on the behavior of the monetary authority. At the zero lower bound, the central bank is unable to offset the increase in markups using its nominal interest rate. When the monetary authority aims to stabilize the economy using expectations about future policy, the contractionary higher markups reduce output and inflation throughout the initial recession and recovery. Thus, the higher markups act as contractionary headwinds in the economy during the zero lower bound episode. In a reasonably calibrated New-Keynesian model, the next section shows that these higher markups can significantly amplify and propagate adverse fluctuations at the zero lower bound.

4 Quantitative Effects of Uncertainty on Forward Guidance

4.1 Single Shock Model Responses

To analyze effects of uncertainty on the central bank’s ability to stabilize the economy, I simulate a large increase in the discount factor of the representative household. This shock acts like a large decline in

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3Basu and Bundick (2012) argue that the increase in uncertainty in late 2008 is consistent with over a three standard deviation uncertainty shock.
aggregate demand and causes the zero lower bound to bind for several periods. After the initial shock, I assume the economy experiences no further shocks and the stochastic process for $a_t$ returns to its steady state value using its autoregressive law of motion in Equation (17). Figure 2 plots the model responses under both low and high levels of uncertainty using the same time path for the discount factor process. The discount factor shock implies a negative natural real rate of about 6%, which Levin et al. (2010) argues is consistent with the initial economic contraction during the Great Recession. The calibrated autoregressive coefficient implies that the natural real interest rate remains negative for six quarters following the initial shock.

The constant price-level targeting rule is able to quickly stabilize the economy when uncertainty is low. After the contractionary shock fades and natural real interest rate becomes positive, the central bank maintains a zero nominal policy rate during the economic recovery. This expansionary policy lowers real interest rates, which stimulates household consumption and output by firms. Since the central bank commits to the price-level target, agents in the model fully internalize this future behavior of the monetary authority. This lower path of real interest rates mitigates much of the fall in output and inflation throughout the zero lower bound episode. Despite the severe contraction, the monetary authority is able to quickly stabilize the economy by maintaining a zero policy rate for an additional two quarters during the recovery. Approximately one year after the deflationary forces subside, the central bank is able to fully stabilize the output gap and inflation when uncertainty about the future is low.

Increased uncertainty, however, dramatically affects the ability of the constant price-level targeting rule to stabilize the economy. As the labor supply and labor demand figures illustrate, increased uncertainty raises firm markups and lowers equilibrium hours worked. The model responses show that markups are significantly higher while the natural real interest rate remains negative. These higher markups depress output and inflation through the first several periods of the zero lower bound episode. As the contractionary shock fades and the monetary authority maintains a zero policy rate, increased uncertainty continues to induce precautionary behavior by households and higher firm markups. These forces generate contractionary headwinds in the economy, which worse macroeconomics outcomes for a given path of real interest rates. As a result, the monetary authority must maintain a zero nominal policy rate for a substantially longer period to implement the same monetary policy rule. Even with the additional periods of zero lower rates, the central bank must implement positive output gaps for an additional three years to generate the necessary inflation to stabilize the price level. Although the negative natural real rates only lasted six quarters, the price-level targeting rule takes several years to fully stabilize the economy.

4.2 Model Simulations and Downside Risk in the Economy

In the previous zero lower bound scenarios, a single large shock takes the economy to the zero lower bound and the economy experiences no shocks in the following periods. This analysis allows for easy illustration of the effects of uncertainty at the zero lower bound. However, this method does not show the nonlinear
effects of higher uncertainty that becomes realized as higher actual shock volatility. To show the effects of both higher uncertainty and higher realized volatility, Figure 3 plots the simulated model predictions after the economy encounters the zero lower bound. To generate these responses, I simulate the same initial shock under both uncertainty calibrations. In the subsequent periods, however, I draw random shocks from either the low or high uncertainty calibration, respectively. I repeat this procedure 50,000 times for both the low uncertainty and high uncertainty calibrations. Figure 3 plots the median response across the simulations and 95% prediction intervals for each uncertainty calibration.4

The model simulations highlight how increased uncertainty about the future can affect the ability of the constant price-level targeting rule to stabilize of the economy at the zero lower bound. Under low uncertainty, the monetary authority can fully stabilize the economy in about four years and likely exits the zero lower bound in about two years. When the economy experiences higher realized volatility, however, the model responses show that the price-level target is much less able to stabilize the economy. Even five years after the initial decline in aggregate demand, the economy may still experience significant fluctuations in output and inflation. In roughly half of the simulated scenarios, the economy fails to escape the zero lower bound after four years. Even if the monetary authority perfectly commits to stabilizing a theoretically-motivated nominal variable, the economy may experience significant fluctuations and fail to escape the zero lower bound if the volatility of shocks in the economy are high.

The simulations at the zero lower bound also highlights a key mechanism in the model. In particular, Figure 3 shows that the simulations for the output gap under high uncertainty show a distinct negative skewness. This skewness is a result of the asymmetric ability of the monetary authority to offset shocks at the zero lower bound. While the central bank can fully offset expansionary shocks with higher nominal nominal policy rates, the zero lower bound implies a constraint on their ability to offset contractionary shocks. Increased uncertainty amplifies this asymmetry and produces significantly left-skewed distributions for expected real activity throughout the zero lower bound episode. Throughout the zero lower bound episode, agents internalize this constraint on the monetary authority when forming expectations about future economic activity. Figure 4 illustrates this effect by plotting the expected distributions of outcomes after the economy encounters the zero lower bound.5 Due to the higher uncertainty and the zero lower bound constraint, households and firms understand that a highly negative macroeconomic outcome is possible even one year in the future. As outlined in the Intuition section, this source of downside tail risk induces the precautionary behavior of households and reduces the ability of the price-level targeting rule to stabilize the economy.

---

4This exercise is similar to the generalized impulse response advocated by Koop, Pesaran and Potter (1996). However, since I am interested in the levels of the output gap, inflation, and the nominal interest rate, I do not difference out the simulated paths using a baseline simulation.

5The expected distributions are computed using a kernel density smoother. A histogram using the raw simulated data produces similar results.
4.3 Optimal Monetary Policy Under Commitment

The monetary authority in my baseline model follows the constant price-level targeting rule of Eggertsson and Woodford (2003). I choose this specification for the central bank to examine how uncertainty affects the ability of the same monetary policy rule to stabilize the economy at the zero lower bound. In this section, I show how an optimal policy maker under commitment responds when the economy faces significant uncertainty about the future. Appendix C outlines the optimal policy problem and its associated solution. Figures 5 - 7 replicate the previous numerical simulations under the assumption that monetary policy is conducted optimally under commitment.

When uncertainty is low, the zero lower bound hardly constrains the monetary authority’s ability to stabilize the economy. In response to a single adverse shock, Figure 5 shows that the central bank is able to quickly stabilize the economy by maintaining a brief period of zero policy rates after the natural rate becomes positive. As Eggertsson and Woodford (2003) discuss, optimal policy commits to a higher price-level target when the economy encounters the zero lower bound. The increase in expected inflation lowers the path of real interest rates and induces higher consumption and output during the initial economic contraction. Even when the economy is continually hit by shocks, Figure 6 shows that the economy facing low uncertainty is fully stabilized and almost always exits the zero lower bound in about four years. Despite implying different behavior for the price-level target, Figures 3 and 6 show that optimal policy and the constant price-level targeting rule generate similar macroeconomic outcomes when uncertainty is low. Thus, optimal monetary policy is closely approximated by the constant price-level target in a low uncertainty environment.

However, the zero lower bound becomes a serious constraint on policy when uncertainty about the future is high. Optimal monetary policy under commitment implies additional increases in the price-level target when the economy faces significant uncertainty about the future. In response to a large contractionary shock, Figure 5 shows that the monetary authority raises its price-level target by an additional 0.2% under increased uncertainty. To implement the optimal policy, the monetary authority extends its period of zero policy rates for an additional two quarters after the natural real rate becomes positive. The model shows that optimal monetary policy under uncertainty depends on the nature of shocks that agents expect to hit the economy. Even before the economy is hit by larger realized shocks, optimal monetary policy preemptively responds to the higher expected volatility by raising its price-level target. Even under optimal policy however, increased uncertainty generates an additional 0.7% decline in the output gap and a 0.2% drop in inflation after a single contractionary shock.

Under optimal policy, the monetary authority accepts higher inflation risk in the future to minimize the downside risk to output and inflation when the economy hits the zero lower bound. Despite conducting policy optimally, Figure 6 shows that the monetary authority cannot fully stabilize the economy when the volatility of the exogenous shocks is high. In comparison to the constant price-level target, however, the
distributions for one-year ahead expected inflation in Figure 7 are now positively-skewed. By committing to continually raise its price-level target in response to adverse fluctuations, the monetary authority is able to minimize some of the downside risk to short-run output and inflation. Thus, the monetary policymaker faces a trade-off between the distribution of medium-run inflation and the distribution of short-run output and inflation when the economy hits the zero lower bound. However, optimal monetary policy does fully eliminate the downside risk posed by the zero lower bound. If the volatility of shocks hitting the economy is high, model simulations show that the economy may still experience large fluctuations and fail to escape the zero lower bound for an extended period.

4.4 A Calibration Check Using Recent Macroeconomic Data

In the previous sections, I show that increased uncertainty about the future can reduce the central bank’s ability to stabilize the economy at the zero lower bound. A key parameter in my analysis is the volatility of the demand shocks hitting the economy. Thus, I want to ensure a reasonable calibration for the demand shock volatility. In this section, I simulate a version of my baseline model and compare the distribution of possible outcomes with some recent macroeconomic data since the Great Recession. This exercise allows me to examine whether the distribution of outcomes the representative household uses in evaluating their decisions is in line with the recent experience of the U.S. economy. Figure 8 plots the estimated output gap, inflation, and federal funds rate from the end of 2008 to the end of 2012. Since the output gap is difficult to measure precisely, I use the average output gap as computed by Weidner and Williams (2009) using a variety of methods. To match the inflation measure in the model, I use the annualized quarterly percent change in the core personal consumption expenditures price-index less a two percent inflation target. Since the end of 2008, the United States economy has experienced a significant period of output far below potential, relatively stable but below target inflation, and an extended period of zero nominal policy rates.

To compare the simulated model outcomes with actual macroeconomic data, I need to specify a more reasonable description of monetary policy. Despite the theoretical motivations, no central bank explicitly targets the nominal price level. However, recent empirical evidence suggests that the Federal Reserve’s recent behavior can be described by an interest-rate rule with significant history-dependence. Using Bayesian likelihood methods, Gust, López-Salido and Smith (2013) argues that the following history-dependent interest-rate rule can describe recent Federal Reserve behavior:

\[
\begin{align*}
    r^d_t &= (1 - \phi_r) r + \phi_r r^d_{t-1} + \phi_\pi (\pi_t - \pi) + \phi_x x_t, \\
    r_t &= \max(0, r^d_t),
\end{align*}
\]

where \( r^d_t \) is the desired policy rate of the monetary authority, and \( r_t \) is the actual policy rate subject to the zero lower bound. This history-dependent policy rule is motivated by the work of Reifschneider and Williams (2000). When the monetary authority is unconstrained by the zero lower bound, this policy rule responds exactly as a Taylor (1993)-type policy rule with interest-rate smoothing. However, when the
monetary authority encounters the zero lower bound, the history-dependent rule lowers future desired policy rates to offset the previous higher-than-desired nominal rates caused by the zero lower bound. Similar to a price-level target, the history-dependent rule commits to a lower path of nominal interest rates after the economy encounters the zero lower bound. Agents fully internalize this future conduct of policy, which helps attenuate the contractionary effects of the zero lower bound. I calibrate $\phi_r = 0.9$, $\phi_\pi = 0.8$, and $\phi_x = 0.25$, which are in line with the estimates of Gust, López-Salido and Smith (2013).

When volatility in the economy is high, the time-paths for the actual macroeconomic data fall within the simulated prediction intervals in Figure 8. In a similar exercise to Section 4.2, I simulate the model under the low or high uncertainty calibration using the history-dependent interest-rate rule. However, I calibrate the size of the initial shock such that both calibrations generates the same estimated output gap as the fourth quarter of 2008. After matching the initial conditions at the end of 2008, the macroeconomic data since the Great Recession falls within the simulated prediction intervals of the high uncertainty model. Thus, actual data from the U.S. economy is inline with the distribution of possible outcomes that the representative household uses in evaluating their decisions. This exercise provides some evidence that the level of uncertainty in the calibrated model is reasonable. Through the lens of the model simulations, however, the recent macroeconomic data is not be the most likely outcome for the economy. Clearly, the model lacks many other mechanisms which are likely important for fully explaining the dynamics of the economy. However, the results suggest that volatility and the zero lower bound may be important contributors to the slow recovery of the United States economy.

One potential criticism of the previous exercise is that increasing the volatility of the shocks hitting the economy simply increases the size of the prediction intervals. Thus, the model could generate any arbitrary outcome from the data by simply increasing the width of the prediction intervals. Figure 9 attempts to address this concern by simulating a version of the model under high uncertainty but without imposing the zero lower bound. Without the amplification and propagation mechanism provided by the zero lower bound, the high volatility economy cannot generate outcomes for inflation and the output gap similar to the recent data. Without the zero lower bound, I would need to roughly double the volatility of the high uncertainty calibration to make the recent macroeconomic outcomes fall within the simulated prediction intervals. Through the lens of a simple model, the results suggest that both higher volatility and the zero lower bound are jointly necessary to generate simulations that are consistent with the slow recovery after the Great Recession.

5 Additional Discussion and Connections with Existing Literature

5.1 Monetary Policy at the Zero Lower Bound

This paper shows that the zero lower bound changes the conditional distribution of consumption when the households face significant uncertainty about the future. This idea is related to work by Adam and Billi
(2006), Nakov (2008), and Billi (2008), which also examine the conduct of monetary policy at the zero lower bound. All three papers use a linearized New-Keynesian model, but solve the model using a global solution method where agents take account of the future shocks expected to hit the economy. Nakov (2008) shows that higher expected shock volatility causes larger declines in output and inflation even if monetary policy is optimal under commitment. Recent work by Billi (2008) argues that price-level targeting can effectively minimize downside risks in the economy. Returning the intuition from Equation (2), household consumption in the first-order linearized model is based on the expected value of consumption and the current real interest rate. Thus, the models in these papers are able to capture changes in the conditional mean caused by the presence of uncertainty at the zero lower bound. However, the models are unable to other changes in the consumption distribution caused by the zero lower bound. Figures 4 and 6 show that higher uncertainty about the future affects the conditional mean, variance, and skewness of the expected distributions at the zero lower bound. Thus, the linearized model likely underestimates the true effects of uncertainty and downside risks by restricting the analysis to changes in the conditional mean.

This paper is related to work by Wolman (2005), Nakata (2013), Fernández-Villaverde et al. (2012), Braun, Körber and Waki (2012), Christiano and Eichenbaum (2012), Gust, López-Salido and Smith (2013), and Judd, Maliar and Maliar (2012), which also solve a nonlinear New-Keynesian model with a zero lower bound constraint. These papers are complementary to this study as they examine a different set of economic questions. This study is closest to Nakata (2013), which studies optimal government spending and monetary policy at the zero lower bound. Nakata compares a deterministic economy ($\sigma_a = 0$) to an economy with uncertainty ($\sigma_a > 0$) and shows that optimal government spending under discretion increases when the economy faces uncertainty about the future. While Nakata (2013) and I use a similar model of households and firms, his work primarily focuses on the role of fiscal policy at the zero lower bound. In this paper, I focus on how uncertainty about the future can affect the monetary authority’s ability to stabilize the economy using expectations about future policy.

5.2 Contractionary Bias in the Nominal Interest-Rate Distribution

In addition to the precautionary working mechanism, increases in uncertainty at the zero lower bound can produce an additional source of fluctuations. This additional amplification mechanism, which Basu and Bundick (2012) defines as the contractionary bias in the nominal interest rate distribution, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges when the zero lower bound prevents the monetary authority from attaining its inflation goal on average. For this discussion, assume monetary policy sets its desired policy rate using the following simple rule:

$$r_t^d = r + \phi_\pi \left( \pi_t - \pi \right)$$

(20)

$$r_t = \max\left( 0, r_t^d \right)$$

(21)

For a given monetary policy rule, the volatility of the exogenous shocks determines the volatility of inflation. Through the monetary policy rule in Equation (20), the volatility of inflation dictates the volatility
of the desired nominal policy rate. However, since the zero lower bound left-truncates the actual policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates. Figure 10 illustrates this effect by plotting hypothetical distributions of the nominal interest rate under low and high levels of exogenous shock volatility. The plot shows that the average actual policy rate is an increasing function of the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule. Reifschneider and Williams (2000) first discuss this phenomenon and Mendes (2011) analytically proves this result using a simple New-Keynesian model.

Changes in the contractionary bias caused by higher uncertainty have dramatic general-equilibrium effects on the economy. Figure 11 plots the average Fisher relation $r = \pi + r^r$ and the average policy rule under both high and low levels of volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. An increase in volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility thus raises average real interest rates, since it implies a higher level of the nominal interest rate for a given level of inflation. All else equal, higher real interest rates discourages output and puts downward pressure on the average level of inflation in the economy. Appendix B of Basu and Bundick (2012) provides numerical evidence that small changes in the contractionary bias caused by higher expected volatility can have dramatic effects on the model economy. For example, Basu and Bundick (2012) shows that a small 0.25 percentage point increase in uncertainty about future demand produces a 0.35 percent decrease in aggregate output when monetary policy follows a simple Taylor (1993)-type rule at the zero lower bound.

Throughout this paper, I follow Reifschneider and Williams (2000) and focus on specifications for monetary policy that remove this alternative mechanism. The constant price-level target automatically removes the contractionary bias by offsetting any deflation with equivalent inflation in the future. When monetary policy is conducted optimally under commitment or the history-dependent interest rate rule, I add or subtract a constant bias correction to the steady state inflation rate $\Pi$ under high uncertainty to ensure the simulated model delivers zero inflation on average. This modeling strategy allows me to isolate the effects of the precautionary behavior by households and show how it makes the economy less responsive to the expected path of interest rates. Without these corrections, an increase in the contractionary bias caused by higher uncertainty implies that the monetary authority misses its unconditional inflation target simply because the zero lower bound binds in a few more states of the world.

This discussion of the contractionary bias helps clarify the economic mechanisms at work in some recent papers in the literature. Recent work by Nakata (2012) and Johannsen (2013) show that higher demand and fiscal uncertainty at the zero lower bound greatly depresses the economy. Both papers use nonlinear New-Keynesian models and assume that monetary policy follows a Taylor (1993)-type rule subject to the zero lower bound. However, neither of these papers make any adjustments for the contractionary bias. Therefore, their results contain the effects of both the contractionary bias and precautionary working
mechanisms. While quantitatively large, the effects of the contractionary bias channel emerge as a technical consequence of examining changes in uncertainty under a particular simple monetary policy rule. In addition, the uncorrected Taylor (1993)-type rule probably does not represent the actual conduct of Federal Reserve policy at the zero lower bound. Therefore, while these papers also examine the effects of uncertainty at the zero lower bound, they primarily rely on a very different economic mechanism to generate their results.

5.3 Uncertainty and the Effectiveness of Monetary Policy

Recent papers by Vavra (2012) and Aastveit, Natvik and Sola (2013) argue that monetary policy is less effective at altering economic activity when uncertainty is high. As the micro level, Vavra (2012) shows that the output response to a nominal shock in an $S_s$ pricing-model is substantially reduced when the volatility of firm-level productivity increases. Aastveit, Natvik and Sola (2013) shows that the responses of output and investment to an identified monetary policy shock are reduced under higher macro-level uncertainty. Both of these papers share a common message with this paper: Higher expected volatility can affect the transmission of monetary policy to the macroeconomy. However, these papers focus on the effects of higher uncertainty on different agents in the economy. Vavra (2012) examines how uncertainty affects the forward-looking decisions of price-setting firms, while Aastveit, Natvik and Sola (2013) follows the intuition of Bloom (2009) and emphasizes non-convex adjustment costs for investment. In this paper, I focus on precautionary saving and working behavior by households. I view these works as highly complementary to this paper as all three papers illustrate the various effects of uncertainty on the effectiveness of monetary policy. However, both of these papers are silent on the effects of the zero lower bound, which currently remains a real constraint on many central banks.

6 Extensions

6.1 Raising the Central Bank’s Inflation Target

In response to the recent macroeconomic outcomes around the world, some economists argue that central banks should raise their inflation targets above the conventionally accepted two percent. Ball (2013) states, “A four percent target would ease the constraints on monetary policy arising from the zero lower bound on interest rates, with the result that economic downturns would be less severe.” In the previous sections, I assume that the central bank targets a zero average inflation rate with a four percent average real interest rate. To examine the effects of a higher inflation target, I re-calibrate the steady state inflation rate $\Pi$ such that the average inflation rate is either two or four percent with a two percent real interest rate. Figure 12 shows the simulation results under two or four percent inflation targets under optimal monetary policy where the economy faces high uncertainty about the future.

In choosing an inflation target, the central bank faces a trade-off between the average level of inflation and the amount of fluctuations caused by the zero lower bound. Under the two percent inflation target, the
results are similar to the previous sections since the average nominal interest rate in the economy remains unchanged. In the face of large shocks, Figure 12 shows that a higher inflation target helps attenuate much of the fluctuations and downside risk associated with the zero lower bound. In addition, the economy exits the zero lower bound over two years earlier in the median simulation. While the model clearly does not properly model the costs of higher average inflation, the results are consistent with the ideas of Ball (2013) and Blanchard, Dell’Ariccia and Mauro (2010) that higher average inflation can reduce fluctuations associated with the zero lower bound. The results also supplement the work of Williams (2009) by showing that a two-percent inflation target may provide insufficient buffer even under optimal monetary policy.

7 Conclusions

The aim of this paper is to show how uncertainty about the future can affect the ability of the monetary authority to stabilize the economy. In the absence of the zero lower bound, monetary policy could simply alleviate the contractionary effects of uncertainty by lowering its nominal policy rate. When the monetary authority encounters the zero lower bound, the central bank must rely on expectations about the future path of policy. This paper shows that the monetary authority must commit to a more expansive policy when the future is more uncertain. This study emphasizes that policymakers must consider the entire distribution of possible outcomes when evaluating trade-offs at the zero lower bound.
Technical Appendix

A Derivation of Analytical Model Equations From Intuition Section

A.1 Approximation of Consumption Euler Equation

This section provides a detailed derivation of the equations from Section 2. Using the consumption Euler equation in Equation (1), complete the following steps to derive Equations (2) or (9):

1. Multiply and divide the right side of the Euler equation by the steady state values of the real interest rate $R^R$ and consumption $C$ raised to the power $-\sigma$. Apply the natural logarithm and exponential functions inside the conditional expectations. Denote $\hat{X}_t = \log(X_t/X)$ to write the variables in log-deviations from steady state.

$$1 = E_t \left\{ \beta R_t^R \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\} = E_t \left\{ \frac{R_t^R}{R^R} \left( \frac{C_t}{C} \right)^{\sigma} \left( \frac{C_{t+1}}{C} \right)^{-\sigma} \right\}$$

$$1 = E_t \left\{ \exp \left( \log \left( \frac{R_t^R}{R^R} \right) - \sigma \log \left( \frac{C_{t+1}}{C} \right) + \sigma \log \left( \frac{C_t}{C} \right) \right) \right\}$$

$$1 = E_t \left\{ \exp \left( \hat{R}_t^R + \sigma \hat{C}_t - \sigma \hat{C}_{t+1} \right) \right\}$$

2. Factor out the time $t$ variables outside of the conditional expectations, reorganize, and take the logarithm of both sides.

$$1 = E_t \left\{ \exp \left( \hat{R}_t^R + \sigma \hat{C}_t \right) \exp \left( -\sigma \hat{C}_{t+1} \right) \right\}$$

$$\left( \exp \left( \hat{R}_t^R + \sigma \hat{C}_t \right) \right)^{-1} = E_t \left\{ \exp \left( -\sigma \hat{C}_{t+1} \right) \right\}$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log \left( E_t \left\{ \exp \left( -\sigma \hat{C}_{t+1} \right) \right\} \right)$$

3. Replace $\exp \left( -\sigma \hat{C}_{t+1} \right)$ with its Taylor series expansion around $\hat{C}_{t+1} = 0$ and take conditional expectations at time $t$.

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log \left( E_t \left\{ 1 - \sigma \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \hat{C}_{t+1}^2 - \sigma^3 \hat{C}_{t+1}^3 + \ldots \right\} \right)$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log \left( 1 - \sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 E_t \hat{C}_{t+1}^2 - \sigma^3 E_t \hat{C}_{t+1}^3 + \ldots \right)$$

4. Define $Z = \sigma E_t \hat{C}_{t+1} - \frac{1}{2} \sigma^2 E_t \hat{C}_{t+1}^2 + \sigma^3 E_t \hat{C}_{t+1}^3 - O \left( C_{t+1}^4 \right)$ and use the Taylor series expansion of $\log(1 - Z) = -Z - (1/2)Z^2 - (1/3)Z^3 - O \left( Z^4 \right)$ to expand the previous equation. To compute a second-order approximation, drop all terms that are third-order or above. Reorganize the remaining terms to form the conditional variance $\text{Var}_t \hat{C}_{t+1} = E_t \hat{C}_{t+1}^2 - \left( E_t \hat{C}_{t+1} \right)^2$.

$$-\hat{R}_t^R - \sigma \hat{C}_t = \sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \text{Var}_t \hat{C}_{t+1}$$

$$\hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma} \hat{R}_t^R - \frac{1}{2} \sigma \text{Var}_t \hat{C}_{t+1}$$
5. Denote variables in logs using lowercase letters and normalize steady state consumption \( C \) to equal one to derive Equation (2):

\[
c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r_t' - r' \right) - \frac{1}{2} \sigma \text{Var}_t c_{t+1}
\]

To derive the third-order approximation in Equation (9), retain the third-order terms in Step 4 and reorganize the remaining terms to form the conditional skewness \( \text{Skew}_t \hat{C}_{t+1} = E_t \hat{C}_{t+1}^3 - 3E_t \hat{C}_{t+1}^2 \hat{C}_t + (E_t \hat{C}_{t+1})^3 \).

**A.2 Derivation of Higher-Order New-Keynesian Model**

In this section, I outline the derivation of the approximate higher-order New-Keynesian model from Section 2.2. To simplify the derivations, I assume that the exogenous process for \( a_t \) follows an autoregressive process in logs and the household utility function is additively separable in consumption and leisure. To derive Equations (4) - (6), I combine a second-order approximation of the household Euler equations with a first-order approximation of the remaining model equations. Clearly, this approach neglects some higher-order terms that are present in a complete second-order approximation of the underlying model. However, the approximations in this section provide analytical tractability which is unavailable when examining the model equations in their original nonlinear form. In Section 4, I show that the intuition from these approximations is consistent with the computational results using the full nonlinear model. To derive Equations (4) - (6), complete the following steps:

1. Apply Steps 1 - 5 from Appendix A.1 to the consumption Euler equations for real and nominal bonds and the Euler equation for the natural real interest rate. Use the law of motion \( \log(a_{t+1}) = (1 - \rho_a) \log(a_t) + \sigma^a \varepsilon_{t+1} \) to write \( a_{t+1} \) as function of \( a_t \) and \( \varepsilon_{t+1} \). Impose \( E_t \varepsilon_{t+1} = 0 \) and \( E_t \varepsilon_{t+1}^2 = 1 \).

\[
1 = E_t \left\{ \left( \beta \frac{a_{t+1}}{a_t} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-1} R_t^R \right\}
\]

\[
1 = E_t \left\{ \left( \beta \frac{a_{t+1}}{a_t} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-1} \frac{R_t}{\Pi_{t+1}} \right\}
\]

\[
1 = E_t \left\{ \left( \beta \frac{a_{t+1}}{a_t} \right) R_t^N \right\}
\]

\[
-\hat{R}_t^R - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = \log \left( E_t \left\{ \exp \left( \sigma^a \varepsilon_{t+1} - \sigma \hat{C}_{t+1} \right) \right\} \right)
\]

\[
-\hat{R}_t - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = \log \left( E_t \left\{ \exp \left( \sigma^a \varepsilon_{t+1} - \sigma \hat{C}_{t+1} - \hat{\Pi}_{t+1} \right) \right\} \right)
\]

\[
-\hat{R}_t^N + (1 - \rho_a) \hat{a}_t = \log \left( E_t \left\{ \exp \left( \sigma^a \varepsilon_{t+1} \right) \right\} \right)
\]

\[
- \hat{R}_t^R - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = -\sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \text{Var}_t \hat{C}_{t+1} - \sigma (\sigma^a) \text{Cov}_t \left( \hat{C}_{t+1}, \varepsilon_{t+1} \right) + \frac{1}{2} (\sigma^2)^2 \quad (22)
\]
\[ -\dot{R}_t - \sigma \dot{C}_t + (1 - \rho_a) \dot{a}_t = -\sigma E_t \dot{C}_{t+1} - E_t \dot{\Pi}_{t+1} \frac{1}{2} \sigma^2 \text{Var}_t \dot{C}_{t+1} + \frac{1}{2} \text{Var}_t \dot{\Pi}_{t+1} - \sigma (\sigma^a) \text{Cov}_t \left( \dot{C}_{t+1}, \varepsilon_{t+1} \right) + \sigma \text{Cov}_t \left( \dot{C}_{t+1}, \dot{\Pi}_{t+1} \right) - \sigma^a \text{Cov}_t \left( \dot{\Pi}_{t+1}, \varepsilon_{t+1} \right) + \frac{1}{2} (\sigma^2)^2 \] (23)

\[ -\dot{R}_t^N + (1 - \rho_a) \dot{a}_t = \frac{1}{2} (\sigma^2)^2 \] (24)

2. Subtract Equation (22) from Equation (23). Also, subtract Equation (24) from Equation (22).

\[ -\dot{R}_t^R + \dot{R}_t^N - \sigma \dot{C}_t = -\sigma E_t \dot{C}_{t+1} + \frac{1}{2} \sigma^2 \text{Var}_t \dot{C}_{t+1} - \sigma (\sigma^a) \text{Cov}_t \left( \dot{C}_{t+1}, \varepsilon_{t+1} \right) \]

\[ -\dot{R}_t + \dot{R}_t^R = -E_t \dot{\Pi}_{t+1} + \frac{1}{2} \text{Var}_t \dot{\Pi}_{t+1} + \sigma \text{Cov}_t \left( \dot{C}_{t+1}, \dot{\Pi}_{t+1} \right) - \sigma^a \text{Cov}_t \left( \dot{\Pi}_{t+1}, \varepsilon_{t+1} \right) \]

3. After reorganizing and dividing by \( \sigma \), the resulting equations can be written as follows:

\[ \dot{C}_t = E_t \dot{C}_{t+1} - \frac{1}{\sigma} \left( \dot{R}_t^R - \dot{R}_t^N \right) - \frac{1}{2} \sigma \text{Var}_t \dot{C}_{t+1} + (\sigma^a) \text{Cov}_t \left( \dot{C}_{t+1}, \varepsilon_{t+1} \right) \] (25)

\[ \dot{R}_t^R = \dot{R}_t + -E_t \dot{\Pi}_{t+1} + \frac{1}{2} \text{Var}_t \dot{\Pi}_{t+1} + \sigma \text{Cov}_t \left( \dot{C}_{t+1}, \dot{\Pi}_{t+1} \right) - \sigma^a \text{Cov}_t \left( \dot{\Pi}_{t+1}, \varepsilon_{t+1} \right) \] (26)

4. Under the assumption of separable utility from consumption and leisure, Equations (10) and (11) are now modified as follows:

\[ \eta a_t C_t^{-\eta} = \lambda_t \] (27)

\[ \chi a_t N_t^\eta = \lambda_t \frac{W_t}{P_t} \] (28)

\( \eta \) is now the inverse Frisch labor supply elasticity and \( \chi \) is a constant which pins down steady state hours worked.

5. Using a first-order linear approximation of Equations (27), (28), (14), (15), the aggregate production function, and the national income accounting identity, the following equations can be derived:

\[ x_t = \dot{Y}_t = \dot{C}_t = \mu \dot{\Pi}_t \] (29)

\[ \dot{W}_t^R = \ddot{z}_t = -\dot{\mu}_t = (\eta + \sigma \mu) \dot{N}_t = \frac{\eta + \sigma \mu}{\mu} x_t \] (30)

\[ \dot{\Pi}_t = \beta E_t \dot{\Pi}_{t+1} + \frac{\theta - 1}{\phi_p} \ddot{z}_t = \beta E_t \dot{\Pi}_{t+1} + \frac{\theta - 1}{\phi_p} \frac{\eta + \sigma \mu}{\mu} x_t \] (31)

Define \( \kappa \) as the slope of the Phillips curve in Equation (31).

6. To derive Equations (4) - (6) in the main text, combine the results from Equations (29) - (31) and Equations (25) - (26). In addition, use lowercase variables to denote log variables and normalize steady state output \( Y \) and consumption \( C \) to equal one. For clarity of exposition, I omit the two additional covariance terms in Equations (25) and (26) which are related to the specific exogenous process for the preference shocks in the main text. The coefficients on these terms are very small and they do not provide any additional intuition. Hence, I replace the equality sign with the approximately equal sign in the main text to reflect these omissions.
B  Numerical Solution Method

To analyze the impact of uncertainty on the effectiveness of forward guidance, I solve the model using the policy function iteration method of Coleman (1990). This global approximation method allows me to model the occasionally-binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows a price-level targeting rule of Eggertsson and Woodford (2003). The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: \( \{a_t \times P_{t-1}\} \)

2. Conjecture initial guesses for the policy functions of the model \( N_t = N(a_t, P_{t-1}) \), \( \Pi_t = \Pi(a_t, P_{t-1}) \), \( R_t = R(a_t, P_{t-1}) \), and \( R_t^R = R^R(a_t, P_{t-1}) \).

3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variable \( a_t \) to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space. The solution method enforces the zero lower bound for each point in the state space and in expectation.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

I implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. When monetary policy follows an alternative specification, the state variables and Euler equations are adjusted appropriately.

C  Optimal Monetary Policy Under Commitment

The optimal monetary policy maker under commitment aims to maximize the representative household’s utility subject to the constraints of the economy. Some of the constraints include expectations of future variables. Following Khan, King and Wolman (2003), I introduce lagged Lagrange multipliers to make the solutions time-invariant. The augmented Lagrangian for the optimal policy problem under commitment
can be written as follows:

\[
L = \min_{\{\omega_{t+s}\}_{s=0}^{\infty} \{d_{t+s}\}_{s=0}^{\infty}} \max_{\mathbb{E}_t} \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{a_{t+s}}{a_t} \right) \left( C_{t+s}^\eta \left( 1 - N_{t+s} \right)^{1-\eta} \right)^{1-\sigma} \right. \\
+ \omega_{1t+s} \left( Y_{t+s} - C_{t+s} - \frac{\phi_p}{2} \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right)^2 Y_{t+s} \right) \\
+ \omega_{2t+s} \left( N_{t+s} - \Phi - Y_{t+s} \right) \\
+ \omega_{3t+s} \left( W_{t+s}^R - \frac{1-\eta}{\eta} C_{t+s} \left( 1 - N_{t+s} \right)^{-1} \right) \\
+ \omega_{4t+s} \left( (\theta - 1) - \theta W_t^R + \phi_p \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+s}}{\Pi} \right) \left( a_{t+s} C_{t+s}^\eta \left( 1 - N_{t+s} \right)^{1-\eta} \right)^{1-\sigma} Y_{t+s} \right) \\
- \omega_{4t+s-1} \left( \phi_p \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+s}}{\Pi} \right) \left( a_{t+s} C_{t+s}^\eta \left( 1 - N_{t+s} \right)^{1-\eta} \right)^{1-\sigma} \Pi_{t+s}^{-1} \right) \\
+ \omega_{5t+s} \left( a_{t+s} C_{t+s}^\eta \left( 1 - N_{t+s} \right)^{1-\eta} \Pi_{t+s}^{-1} \right) \\
- \omega_{5t+s-1} \left( a_{t+s} C_{t+s}^\eta \left( 1 - N_{t+s} \right)^{1-\eta} \Pi_{t+s}^{-1} \right) \\
+ \omega_{6t+s} \left( R_{t+s} - 1 \right) \right\},
\]

where \( d_t = \{Y_t, C_t, N_t, W_t^R, \Pi_t, R_t\} \) is the set of decision variables and \( \omega_t = \{\omega_{1t}, \omega_{2t}, \omega_{3t}, \omega_{4t}, \omega_{5t}, \omega_{6t}\} \) is the vector of Lagrange multipliers. The final constraint imposes the zero lower bound constraint since the gross nominal policy rate \( R_t \) must be greater than or equal to one. After solving for the first-order conditions, the optimal policy problem is solved using the algorithm outlined in Appendix B. To determine the equilibrium real interest rate \( R_t^R \), I also include the Euler equation for a zero net supply real bond as well. The algorithm solves for the policy functions for \( N_t = N(a_t, \omega_{4t-1}, \omega_{5t-1}), \Pi_t = \Pi(a_t, \omega_{4t-1}, \omega_{5t-1}), R_t = R(a_t, \omega_{4t-1}, \omega_{5t-1}), R_t^R = R_t^R(a_t, \omega_{4t-1}, \omega_{5t-1}),\omega_{4t} = \omega(a_t, \omega_{4t-1}, \omega_{5t-1}), \) and \( \omega_{5t} = \omega_5(a_t, \omega_{4t-1}, \omega_{5t-1}) \) on a discretized state space for \( \{a_t \times \omega_{4t-1} \times \omega_{5t-1}\} \).
References


Blanchard, Olivier, Giovanni Dell’Ariccia, and Paolo Mauro. 2010. “Rethinking Macroeconomic Policy.” International Monetary Fund Staff Position Note.


Table 1: Calibration of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
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<tr>
<td>$\beta$</td>
<td>Household Discount Factor</td>
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<td>$\phi_P$</td>
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<td>$\Pi$</td>
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<td>$\sigma$</td>
<td>Parameter Affecting Household Risk Aversion</td>
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<td>$\eta$</td>
<td>Consumption Share in Period Utility Function</td>
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<td>$\theta$</td>
<td>Elasticity of Substitution Intermediate Goods</td>
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<td>$\rho_a$</td>
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<td>Low Uncertainty Preference Shock Volatility</td>
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<td>$\sigma^a$</td>
<td>High Uncertainty Preference Shock Volatility</td>
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Figure 1: Precautionary Labor Supply Intuition

![Precautionary Labor Supply Intuition](image-url)
Figure 2: Model Responses to Zero Lower Bound Episode under Price-Level Targeting

Note: The output gap and markup impulse responses are plotted as percent deviations. The nominal and real interest rates and inflation are plotted in annualized percent. The price level plotted in percent deviation from its pre-shock level.
Figure 3: Model Simulations After Encountering Zero Lower Bound under Price-Level Targeting

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent.
Figure 4: Expected Distributions After Encountering Zero Lower Bound under Price-Level Targeting

Note: The output gap is plotted in percent deviation and inflation is plotted in annualized percent.
Figure 5: Model Responses to Zero Lower Bound Episode under Optimal Policy

Note: The output gap and markup impulse responses are plotted as percent deviations. The nominal and real interest rates and inflation are plotted in annualized percent. The price level plotted in percent deviation from its pre-shock level.
Figure 6: Model Simulations After Encountering Zero Lower Bound under Optimal Policy

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent.
Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent.
Figure 8: Model Simulations and Recent Macroeconomic Data

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent. See main text for data sources.
Figure 9: Model Simulations and Recent Macroeconomic Data Without Zero Lower Bound

High Uncertainty Without Zero Lower Bound

Output Gap

Inflation

Nominal Interest Rate

High Uncertainty With Zero Lower Bound

Output Gap

Inflation

Nominal Interest Rate

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent. See main text for data sources.
Figure 10: Nominal Interest Rate Distribution with Zero Lower Bound Constraint

Figure 11: Simple Monetary Policy Rules & Fisher Relation with Zero Lower Bound Constraint
Figure 12: Model Simulations Under Two and Four Percent Inflation Targets

High Uncertainty With 2% Inflation Target

High Uncertainty With 4% Inflation Target

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent. Monetary policy is conducted optimally under commitment.