Department of Economics
Seminar Series

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“The Adverse Effects of Asset Means-Testing Income Support”

Monday,
January 27, 2014
3:45 p.m.
S016 Memorial Union
The Adverse Effects of Asset Means-Testing Income Support

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November 5, 2013

Abstract

This paper uses a life-cycle model with uninsurable idiosyncratic earnings risk to study the welfare consequences of two cost neutral reforms which abolish the asset means-test from US income support programs. A reform which decreases allotments proportionally for all households is undesirable for a yet unborn household. The welfare costs arise because the reform reallocates transfers away from households with low innate earning abilities. A reform without reallocation implies substantial welfare gains. Means-testing carries welfare costs because households with low innate abilities reduce precautionary and retirement savings. Consequently, they suffer from high consumption volatility and a sharp drop of average consumption upon retirement.

Keywords: Incomplete markets, Means-tested programs, Public insurance

JEL: D91, I38, J26

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1 Introduction

Income support programs aim to provide insurance to households in need. To identify needs, many programs employ an asset means-test; they grant benefits only to low income households whose wealth is also below certain thresholds. A policy maker faces a nontrivial trade-off when deciding about the means-test. On the positive side, for a given amount of government expenditures, the means-test allows allocating relatively high allotments to those who have no private means to cover their needs and need the support the most—a desirable insurance property. In addition, households with low assets tend to be young and have low innate earning abilities. Therefore, the means-test allows redistribution of resources to lower earnings households. On the negative side, means-testing imposes an implicit tax on savings and thus distorts households’ asset accumulation decisions. Households trade off precautionary and retirement savings against the eligibility to the income support programs. This raises concerns that households impoverish themselves and miss private means to finance consumption during retirement or after poor labor-market outcomes.

To determine the quantitative importance of the distortion to savings incentives and to quantify which of the above effects dominates in terms of welfare, I use a structural, small open economy model with incomplete markets. Households’ earnings are subject to idiosyncratic shocks that differ in their persistence. At labor-market entry, households draw their permanent innate earnings ability. Earnings grow as households age until they reach retirement age, at which point they drop sharply. During working life, households face persistent shocks to their labor-market opportunities. In addition, they are subject to large but transitory earning shocks that arise from unemployment. The government provides temporary insurance against unemployment, and, at all ages, it provides asset means-tested transfers to households with income below certain thresholds.

Using data from the Survey of Income and Program Participation for the years 1996 to 2007, I show that on average about 15% of US households receive transfers from these means-tested income support programs. I use the asset thresholds and income-dependent transfers from these programs to parameterize the model. To disentangle the different welfare consequences of the means-test, I consider two different expenditure-neutral reforms that eliminate the asset means-test and keep the income thresholds in place. The first reform decreases allotments proportionally for all households to keep the total expenditures fixed. As in Conesa et al. (2009), I evaluate welfare effect based on an yet unborn household. I find that such a household is willing to forgo 0.29% of lifetime consumption to keep the means-test. The reform has two effects which decrease social welfare. First, conditional on a household’s earnings and age, it does not allocate transfers to only those households in most need. Second, it redistributes transfers away from young
households and households with low innate abilities because these are households with the lowest asset holdings. Such a reallocation decreases social welfare by construction in this paper’s environment. The second expenditure-neutral reform eliminates this latter effect by keeping total spending conditional on households’ age and earnings constant. Now, an unborn household is willing to forgo 0.74% of lifetime consumption to abolish the means-test.\footnote{In an extension, I show that also the currently living gain from abolishing the means-test.} Put differently, the undesirable incentive effects of the means-test outweigh its desirable insurance property in terms of social welfare.

The present paper adds to the literature on the implications from asset means-testing. The existing literature focuses on the response of labor supply decisions on the means-test. French and Jones (2011) estimate the effects of health insurance programs on retirement decisions, among them the means-tested \textit{Medicaid}. Rendahl (2012) uses an environment where households can permanently escape unemployment to study the effects of means-tested unemployment benefits on search intensity.\footnote{Koehne and Kuhn (2012) extend his analysis to a framework with multiple unemployment spells.} Golosov and Tsyvinski (2006) study the incentives to claim disability insurance in the presence of an asset means-test. Similar to Rendahl (2012), the only uncertainty is about an absorbing state (disability) and the focus is only on the life before retirement. In contrast, the present paper abstracts from the labor supply decision.\footnote{There is a large literature which focuses on the labor supply distortion without a means-test. For example, Hansen and Imrohoroglu (1992) study the trade-off between the insurance effect of unemployment benefits and their adverse effect on search intensity. Contrary to this literature, my framework is silent about the optimal size of transfers.} Instead, the paper studies the welfare implications of means-testing which arise from fluctuating earnings, instead of an absorbing state, and retirement savings behavior.

In the model, as a response to the means-test, some of the poor households choose to save at most the imposed asset limit. Therefore, they forgo the opportunity to accumulate wealth for financial self-insurance and retirement. Because of forward-looking behavior, the means-test also affects households which currently do not receive transfers. These make today’s consumption decisions knowing that their earnings risk. Moreover, once households reach retirement and age further, their incentives to hold retirement savings decreases, which increases the incentives to enter into the means-tested program. To prepare for these possible future events, households consume more today to pass the means-test in the future. Households with low innate abilities are the most likely to pass the earnings-test. Therefore, they have relatively strong incentives to accumulate little wealth. In fact, my model implies that a significant fraction of these households holds almost no wealth throughout their life.

As a result, mostly households with low innate abilities forgo the opportunity of self-insurance. They have to reduce consumption strongly after poor labor-market outcomes.
Without the means-test, consumption is more smooth during working life for these households. Moreover, similar to Hubbard et al. (1994, 1995), some of these households hold almost no assets at retirement. Despite perfect foresight about the time of retirement, the average consumption of households with the lowest earnings ability drops by 14 percent when their labor income is replaced by lower retirement income.

Since the means-test affects households of different innate abilities differently, households have heterogeneous preferences regarding the policy. Resulting from the higher consumption volatility and the relatively low consumption during retirement under means-testing, households of low innate abilities favor the distributional-neutral reform that abolishes the means-test. Households with high innate ability oppose this reform. Their savings behavior is barely distorted by the program, and they participate in the income support program only after a series of poor labor-market outcomes. In that case, they gain from the relatively high allotments under means-testing which decreases their consumption volatility. In sum, for households with high innate earnings ability the desirable insurance properties of means-testing, and for households with low ability the adverse incentives effects dominate.

Do the data support the adverse incentive effects created by means-testing? The model almost perfectly matches the wealth inequality in the data. Furthermore, these data show that households with low savings tend to be of low earnings ability (see Hubbard et al. (1994)). Blundell et al. (2008) report that households of low earnings ability are less insured against shocks to income than households with high ability. The model matches the relative insurance coefficient they obtain. Finally, Hurst (2008) summarizes evidence that the lowest 20 percent of the wealth distribution decrease their consumption by 20 to 32 percent upon retirement, which compares well with the consumption decline for the lowest earnings group in the model.\(^4\)

The remainder of the paper is structured as follows. The next section puts the paper in the context of the wider literature. Section 2 present the model. Section 3 characterizes the solution to household behavior analytically and provides intuition for the main mechanisms. The section thereafter discusses the calibration of the model. Section 5 shows that the mechanisms of the model find support from well-known facts from the data. Section 6 conducts the welfare analysis, and the final section conclude.

\(^4\)In the data, the measured drop in consumption upon retirement occurs for a much larger group. Yet, lower work related expenses and more home production can explain most of the drop for higher wealth quintiles.
Related Literature

There is a large empirical literature suggesting that a significant fraction of households has insufficient savings to smooth earnings shocks or to finance consumption during retirement. Regarding the latter, this includes Bernheim et al. (2001), Hurd and Rohwedder (2003), Aguiar and Hurst (2005), Scholz et al. (2006), Hurst (2006) and Ameriks et al. (2007). Concerning self-insurance of poor labor-market outcomes, Gruber (1997) finds that 35 percent of households hold no wealth when entering unemployment. Carroll et al. (2003) shows that this trend to be households with low life-time income. Dynan et al. (2004) show that households with high innate abilities have higher saving rates in general. My structural model links these phenomena to the presence of the means-test. Gruber (1999) and Ziliak (2003) use reduced form approaches and argue that households indeed hold lower wealth when faced by a means-test.

I am not the first who links the presence of the means-test to low wealth holdings of households. Hubbard et al. (1994, 1995) show that augmenting a life-cycle model with a consumption floor for households without wealth allows the model to match the share of low wealth households in the data. The consumption floor acts similar to a means-tested program by penalizing all asset holdings above zero. The present paper uses a similar framework to ask a normative question. It weights the costs created by the means-test against the gains arising from it. I extend the framework of Hubbard et al. (1994, 1995) in several directions which make this normative approach feasible. First, I differentiate between unemployment risk and persistent shocks to earnings because households may find it easier to insure against transitory shocks. Second, households may also receive transfers with small positive savings which grants them a more reasonable amount of self-insurance against earnings shocks. Third, households have a bequest motive which implies a more reasonable amount of households with low wealth towards their end of life.

A related literature which uses structural life-cycle models without a means-test finds that households are quite successful in self-insuring against transitory earnings shocks. For example, similar to my set-up, Low et al. (2010) differentiate between the temporary shocks of unemployment and persistent shocks to earnings. They find that households value more the insurance against permanent shocks. The present paper shows that households of low innate abilities fail to self-insure against transitory income shocks once income support programs feature an asset means-test as in the data. Moreover, it shows an interesting heterogeneity in insurance. While households of low innate ability have much smaller consumption responses after becoming unemployed without the means-test, households with high innate abilities are actually better insured against short-term unemployment risk under means-testing than without the means-test.
2 A Model of Life-Cycle Savings

This section specifies the model in which households of heterogeneous innate abilities make consumption decisions under risk of unemployment and persistent shocks to earnings opportunities. The government provides insurance against the unemployment risk and means-tested transfers to households with low earnings.

2.1 The Household Problem

The economy is populated by a unit mass of households. A household dies in quarter \( t \) with probability \( \iota_t \) and dies with certainty after \( T \) quarters. When a household dies, it is replaced by a newborn household. During the first \( T_W \) quarters, the household works and retires with certainty in quarter \( T_W + 1 \). The household takes as given initial beginning of period assets \( a_1 \), its employment status and the laws of motion for labor-market earnings. It chooses each period \( t \) consumption \( c_t \) and implied end of period assets \( k_t \), which pay certain return from the world capital market \((1+r)\).

When employed, the household faces the risk of unemployment with probability \( \delta \). When unemployed, it finds a new job with probability \( \lambda \). Unemployment insurance is supposed to mimic legislation in the US where benefits \( b_t \) replace a constant fraction of previous labor-market earnings \( w_{t-1} \) subject to a cap of \( b_{max} \):

\[
b_t = \min\{\nu w_{t-1}, b_{max}\}.
\]

The scheme reflects the fact that insurance is paid only temporarily, i.e., the period after the job loss.

Except when unemployed, households’ log earnings depend additively on a deterministic component \( \mu_t \) and a stochastic component \( \varphi_t \). The deterministic component, \( \mu_t \), captures the predictable part of households’ earnings. It evolves according to a function \( \mathcal{F} \) which depends on the innate ability draw and time: \( \mu_t = \mathcal{F}(\mu_1, t) \). The stochastic component follows an exogenous mean-zero Markov process during working life. The vector of values is denoted by \( \varphi^v \). Transition probabilities are common among households:

\[
\pi_{j,k} = \text{prob}[\varphi_t = \varphi^k|\varphi_{t-1} = \varphi^j].
\]

The process is intended to capture the uncertainty from changes in households’ labor-market possibilities. During retirement, the household receives a constant fraction of its

\textsuperscript{5}The focus of this paper are the saving decisions of the relative poor which hold little of the country’s capital stock. Therefore, changes in their savings behavior are unlikely to have major impacts on the equilibrium interest rate.
last earnings possibility.\textsuperscript{6} Thus,

$$\ln(w_t^i(\varphi_t, \mu_1)) = \begin{cases} \mathcal{F}(\mu_1, t) + \varphi_t & \text{if } t \leq T_W \text{ and employed} \\ \mathcal{K}(\mu_{T_W}, \varphi_{T_W}) & \text{if } t > T_W. \end{cases}$$

In the case that unemployment benefits, labor-market earnings and retirement income are sufficiently low, a household may receive end of period means-tested governmental transfers $TR(k_t, w_t, b_t, t)$. The eligibility depends on the households’ end of period asset choice $k_t$. These choices must come from the feasibility correspondence:

$$\Gamma(a, w, b, t) = \begin{cases} a + w_t + b_t + TR(k_t, w_t, b_t, t) \\ a_{t+1} = (1 + r)k_t + TR(k_t, w_t, b_t, t) \\ a_{t+1} \geq 0. \end{cases}$$

Households can save at most their beginning of period assets plus their income and possible end of period transfers. They must satisfy a zero borrowing constraint for beginning of period assets.\textsuperscript{7} The motivation for a zero borrowing constraint is that those most affected by the income support programs have low credit ratings; therefore, their access to credit is strongly limited. Moreover, the borrowing constraint by itself is of little importance for the quantitative welfare implications of the means-test.\textsuperscript{8} Instead, what matters is the difference between the borrowing constraint and the maximum of assets allowed by the means-test. Appendix E shows that the results are quite robust to variations in this difference.

I state the household problem recursively. A retired household of age $t$ with asset position $a$, innate earnings ability $\mu_1$ and transitory component $\varphi_{T_W}$ solves:

$$V_t(a, \varphi_{T_W}, \mu_1, R) = \max_{k \in \Gamma(a, w, 0, t)} \left\{ \frac{(a + w - k)^{1-\gamma}}{1 - \gamma} \right. \\
\left. + \mathbb{E}_t \left\{ (1 - \iota_t) \beta V_{t+1}(\phi(k), \varphi_{T_W}, \mu_1, R) + \iota_t \theta b \bar{V}(\varphi_{T_W}, \mu_1, k) \right\} \right\} \quad (1)$$

\textsuperscript{6}In reality, replacement rates depend on a workers entire earnings history. An additional state variable makes the numerical approximation infeasible. The current modeling choice makes earnings shocks towards retirement very persistent; hence, leads to too strong consumption adjustments before retirement. It is unclear whether this makes means-testing more or less attractive. On the one hand, means-testing implies additional insurance in case of very poor outcomes before retirement. On the other hand, households hold fewer assets and cannot react to these poor outcomes by means of self-insurance.

\textsuperscript{7}The assumption is that households can borrow against end of period means-tested transfers.

\textsuperscript{8}For example, Kaplan and Violante (2010) use a very similar model environment and show that households have almost the same amount of self-insurance against persistent earnings shocks with a zero borrowing constraint or a natural borrowing constraint. Their Table 5 shows that this result is true as long as the autocorrelation of earnings is sufficiently close to one which will be true in my calibration.
\[ \phi(k) = (1 + r)k + TR(k, w, 0, t), \tag{2} \]

where \( E_t \) is the expectation operator. Households consume \( a + w - k \) and have a risk-aversion parameter of \( \gamma \). In case they do not die, they start next period with assets \( \phi(k) \). In case of death, they value bequests according to \( \bar{V} \). I assume that households care about the utility of their offspring. Several studies, e.g., Cameron and Heckman (1998) and Brant Abbott and Violante (2013), suggest that there exists a strong link between parents’ and children’s’ innate abilities. I capture this persistence in earnings possibilities by a function \( \Pi(\mu_1, \varphi_{T_w}) \). Let \( V_1(a, \varphi, \mu_1, E) \) be the value function of a newborn that starts employed and let \( V_1(a, \varphi, \mu_1, U) \) be the corresponding value function of an unemployed. The value of bequests is given by: 9

\[
\bar{V}(\varphi_{T_W}, \mu_1, k) = G \left( V_1((1 + r)k, \varphi_{T_w}, \cdot, E), V_1((1 + r)k, \varphi_{T_w}, \cdot, U), \Pi(\mu_1, \varphi_{T_w}) \right).
\]

The function \( G \) computes the expected value function of the offspring according to the exogenous probabilities for the initial employment state and the transition function for abilities. For tractability, I assume that households use a linear quadratic approximation to the true expected value functions of their offspring. Therefore, \( \bar{V} \) is concave and everywhere differentiable, even though \( V_1 \) will not satisfy these requirements. Appendix A shows that the approximation is close; the \( R^2 \) is above 0.994 for each earnings state.

Before households reach the last period of their working life, they face shocks to their earnings possibilities and employment. An employed household solves:

\[
V_t(a, \varphi, \mu_1, E) = \max_{k \in \Gamma(a, w, 0, t)} \left\{ \frac{(a + w - k)^{1-\gamma}}{1 - \gamma} \right. \\
\left. + E_t \{ (1 - \iota_t) \beta (1 - \delta) V_{t+1}(\phi(k), \varphi', \mu_1, E) + \delta V_{t+1}(\phi(k), \varphi', \mu_1, U) \} \right\} \tag{3}
\]

\[
\phi(k) = (1 + r)k + TR(k, w, 0, t). \tag{4}
\]

\( \bar{V}(\varphi_{T_W}, \mu_1, k) = (1 + r)k + TR(k, w, 0, t) \), 9

I assume that the persistent component of earnings possibilities is perfectly passed to the offspring. This is an arbitrary choice, and my results are robust to other assumptions.
state $Ub$. This value function solves:

$$V_t(a, \varphi, \mu_1, Ub) = \max_{k \in \Gamma(a,0,b,t)} \left\{ \frac{(a + b - k)^{1-\gamma}}{1 - \gamma} \right\} \epsilon_t \left\{ (1 - \iota_t) \beta \left[ (1 - \lambda) V_{t+1}(\phi(k), \varphi', \mu_1, U) + \lambda V_{t+1}(\phi(k), \varphi', \mu_1, E) \right] \right\} (5)$$

$$\phi(k) = (1 + r)k + TR(k, 0, b, t). \quad (6)$$

In case the household does not find a job, which occurs with probability $(1 - \lambda)$, he moves into the state $U$ where it does not receive unemployment insurance any longer. This value function solves:

$$V_t(a, \varphi, \mu_1, U) = \max_{k \in \Gamma(a,0,0,t)} \left\{ \frac{(a - k)^{1-\gamma}}{1 - \gamma} \right\} \epsilon_t \left\{ (1 - \iota_t) \beta \left[ (1 - \lambda) V_{t+1}(\phi(k), \varphi', \mu_1, U) + \lambda V_{t+1}(\phi(k), \varphi', \mu_1, E) \right] \right\} (7)$$

$$\phi(k) = (1 + r)k + TR(k, 0, 0, t). \quad (8)$$

Note that I omit any means of financing for the governmental programs, which is mainly to keep notation simple. Having proportional labor taxation to finance the program would leave my results unchanged because there is no employment decision, and I use a welfare measure which is independent of the scale of consumption.

### 2.2 Means-Tested Transfers

The design of the means-tested program is intended to mirror important features from the programs in place in the US at the beginning of this century. I make five simplifying assumptions. First, I assume that there is a 100% pick-up rate. Second, I calculate the dollar value of all in-kind transfers.\(^{10}\) Third, I abstract from household composition issues.\(^{11}\) The representative household has four members during working life and two members during retirement. Fourth, I assume that the government can perfectly observe savings $k_t$.\(^{12}\) Finally, I assume a common asset and earnings threshold for all programs.\(^{12}\) I set the

\(^{10}\) Reassuring, all programs provide benefits that are quick to access and serve everyday basic needs.

\(^{11}\) Appendix D show that the key mechanisms from Section 3 will prevail, if households can hide a fixed amount of savings. Moreover, the results will be identical to the present set-up for a range of parametrization, if the technology for hiding savings is probabilistic.

\(^{12}\) All programs are initiated by the Federal Government. However, the Federal Government only provides a general framework and carries part of the total costs. The individual states are free to design
earnings threshold $w^t$ to 130% of poverty income and the asset limit $\bar{a}$ to $3000$, both common limits across the different programs. Regarding the latter, some states allow allowances for a households' car and housing value. These assets might be less suited to insure against earnings shocks, but they may be important to finance consumption during retirement. Appendix E shows that my qualitative results are robust to much higher values of $\bar{a}$. However, the welfare costs of the means-test become smaller.

To reduce the computational burden, I assume that each household which becomes unemployed and satisfies the eligibility criterion has a common amount of benefits $\bar{b}$ which are used to compute the amount of transfers.\footnote{This allows me to compute (5) without the prior wage being a state variable. Instead, I can treat this wage dependent transfer as a lump-sum transfer to the household, similar to Low et al. (2010), and only need to condition on the appropriate feasibility correspondence.} I compute this as the mean benefits received by unemployed households in the data conditional on receiving positive benefits. Define total gross earnings as:

$$w^{\text{gross}}_t = w_t + \bar{b}$$

Following US federal legislation for, e.g., the Supplemental Nutrition Assistance Program, the amount of net earnings used to determine the amount of receivable transfers is:

$$w^{\text{net}}_t = 0.3(0.8w^{\text{gross}}_t - d),$$

where $d$ is a cash deductible.

I now turn to the transfer programs which are available to households at different stages of their life-cycle.\footnote{Medicaid is a major means-tested program which is not included in my analysis. The program is only available to single parent households. Moreover, it is less suitable as insurance against income shocks but provides medical support in case of bad health.} For a more detailed review see Moffitt (2003). The Supplemental Nutrition Assistance Program provides households with vouchers for food. The goal of the program is to make high quality nutrition food available to low income households. These transfers are available to households at all stages of their life-cycle. When eligible, the maximum amount of benefits is $TR_F$. Another program available to households at all stages of their life-cycle is the Low Income Home Energy Assistance Program which provides energy assistance. Eligibility is usually guaranteed when a household participates in another welfare program, and I find little correlation between income and the amount of benefits in the data. Therefore, I assume that each eligible household receives a common amount of benefits which differs among working and retired households: $TR^W_H$ and $TR^R_H$.

During working life, the household may receive Temporary Assistance to Needy Fam-
ilies\textsuperscript{15} which provides income support to families with children under 19 years of age.\textsuperscript{16} The program provides both cash and in-kind transfers. The latter serves basic needs such as child care, education and transportation. Denote by $\mathcal{T}T_T$ the maximum amount of receivable benefits. Females who are pregnant or have children less than five years of age may be eligible to the Special Supplemental Nutrition Program for Women, Infants and Children. The eligibility criteria are somewhat weaker than for the other programs and my data suggests that there is almost no correlation between income and benefits of those households receiving transfers. However, almost no household older than 38 years participates in the program. Therefore, I assume that all households under age of 38 and satisfying the income and asset test receive the same amount of benefits $\mathcal{T}W$.

The total amount of benefits an eligible household receives during working life is, thus:

$$TR^W = \max\{\mathcal{T}F + \mathcal{T}T - w^\text{net} t, 0\} + \mathcal{T}W_H + \mathcal{T}W I_{<38},$$

where $I_{<38}$ is an indicator variable which is one when the household is younger than 38 years.

During retirement, the household may receive benefits from Supplemental Security Income.\textsuperscript{17} This program deducts income more strongly from the maximum allotment $\mathcal{T}S$:

$$w^{SSI} t = 0.8 w^\text{gross} t - d.$$ 

Total transfers to an eligible households are, thus:

$$TR^R = \max\{\mathcal{T}F - w^\text{net} t, 0\} + \max\{\mathcal{T}S - w^{SSI} t, 0\} + \mathcal{T}R_H.$$

Summarizing the above yields:\textsuperscript{18}

$$TR(k_t, w_t, b_t, t) = \begin{cases} 0 & \text{if } k_t > \frac{\bar{a}}{1+r} \text{ or } w_t > w^{t\text{ elig}} \text{ or } b_t > w^{t\text{ elig}} \\ TR^W & \text{if } k_t \leq \frac{\bar{a}}{1+r} \text{ and } w_t \leq w^{t\text{ elig}} \text{ and } b_t \leq w^{t\text{ elig}} \text{ and } t \leq T_W \\ TR^R & \text{if } k_t \leq \frac{\bar{a}}{1+r} \text{ and } w_t \leq w^{t\text{ elig}} \text{ and } t > T_W. \end{cases}$$

\textsuperscript{15}Formerly, Aid to Families with Dependent Children.

\textsuperscript{16}Looking at data from the SIPP suggests that at all ages of working life a non-trivial fraction of households receives transfers. Different from the other programs, it is designed to promote labor force participation. Household members must either be working or prove to be actively searching for employment.

\textsuperscript{17}The legislation also allows non-retired disabled and blind children to participate which I abstract from.

\textsuperscript{18}Some readers may want to compare my specification to the one put forward by Hubbard et al. (1995). Abstracting from medical expenses, which they have in their model, and ignoring unemployment, which they have not as distinctive state, they specify $TR(k_t, w_t) = \max\{0, \bar{C} - [(1+r)k_t + w_t]\}$ where $\bar{C}$ is a guaranteed consumption floor. In this set-up, all households participating in the program choose $k_t = 0.$
3 Distortions from the Means-Test

To understand the welfare implications of asset means-testing, it is crucial to understand the effects it has on optimal choices. This section first characterizes regions of the state space where households fail to equate expected marginal utility across periods because the means-test distorts choices. Thereafter, I characterize regions where households are not affected by the means-test at all; therefore, their choices satisfy first order conditions. Finally, I trace out regions where households satisfy first order conditions, but their choices are still distorted by the means-test because of forward looking behavior. I relegate all proofs to Appendix C.

Let me begin by introducing some notation. Summarize the states $X = (\varphi, \mu_1, Z)$, where $Z$ is the employment state. Let $k_t(a, X)$ be the optimal policy for end of period assets induced by the state vector in period $t$. Likewise, let $a_{t+1}(a, X)$ be the optimal policy for next period assets.$^{19}$ Last, define the interval with length $\epsilon$ and center $k^0$ as $B_\epsilon(k^0)$.

Given this notation, I can characterize optimal policy. The first lemma shows that optimal choices imply that a small change in the asset position does not lead to large changes in the value function, even though the law of motion of the endogenous state variable is not continuous.$^{20}$ Moreover, it shows weak monotonicity of the policy $k_t(\cdot, X)$ which is a direct result from the strictly concave period utility function.

**Lemma 1.** $V_t(\cdot, X)$ is strictly increasing and continuous $\forall a$. The policy $k_t(\cdot, X)$ is increasing.

Even though the policy satisfies weakly monotonicity, the following Lemma establishes that for agents with $w_t \leq w_{t, \text{elig}}$ the policy $k_t(a, X)$ is flat with choice $k_t = \frac{a}{1+r}$ for a range of $(a, X)$. However, it becomes optimal to choose $k_t > \frac{a}{1+r}$ for a large enough. Intuitively, the household weights the utility gains from consumption smoothing against the forgone transfer from choosing $k_t > \frac{a}{1+r}$.

**Lemma 2.** $\forall w_t \leq w_{t, \text{elig}}$ there may exist $k_t(a, X) \in B_\epsilon(k^0_t(a^0, X))$ with $k^0_t = \frac{a}{1+r}$. Yet, $\exists \tilde{a}_t(X)$ s.th. $\tilde{k}_t(\tilde{a}_t, X) > \frac{a}{1+r}$ $\forall a > \tilde{a}_t(X)$.

Equipped with these Lemmas, I can state my theorems which are about conditions for first order conditions to be either necessary or sufficient. The first two theorems characterize regions where households’ choices are not affected by the means-test.

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$^{19}$These correspondences are not necessarily single valued for a range of the state space given the problem stated in (1), (3), (5) and (7). I assume that the household chooses the larger $k_t$ when it is indifferent between choices. I show why non-uniqueness can arise and show that it is of little practical relevance.

$^{20}$Note the importance of defining $a_t$ as total assets, including $TR$ and defining $k_t$ as the choice excluding $TR$. Defining the state as assets excluding $TR$ would lead to a downward jump in $V_t$. 

11
Theorem 1. Let \( t > T_W \) and \( w_t > w^t_{\text{elig}} \). Then \( \frac{\partial V_t(\cdot, \phi, \mu_1, R)}{\partial k_t} \) exists and \( \frac{\partial V_t(\cdot, \phi, \mu_1, R)}{\partial k_t} = 0 \) is sufficient for an optimum \( \forall \, a_{t+1}(a, X) > 0 \) and \( \forall \, t \in \{T_W + 1, T\} \).

The result follows from the assumptions placed on the bequest function and on the assumption that earnings are fixed during retirement. Therefore, \( \forall \, a_{t+1} \neq 0 \) and \( \forall \, t \in \{T_W + 1, T\} \).

Theorem 2. Let \( \dot{a}_t(X) \) and \( \ddot{a}_t(X) \) be defined as above. Then \( \frac{\partial V_t(\cdot, X)}{\partial k_t} \) exists and \( \frac{\partial V_t(\cdot, X)}{\partial k_t} = 0 \) is sufficient for an optimum \( \forall \, a_{t+1}(a, X) > 0 \) and \( a_t > \dot{a}_t(X) \). The same holds \( \forall \, a \) s.th. \( a_{t+1}(a, X) > 0 \) and \( a \leq \ddot{a}_t(X) \).\(^{21}\)

To summarize household behavior characterized by the two theorems above, Figure I (Panel A) shows the value and policy function of an eligible household in period \( T - 1 \). To the very left, Theorem 2 establishes that households satisfy first order conditions at non-binding choices. In the area \( B \), households save exactly the maximum to still satisfy the means-test. The value function becomes relatively concave because of decreasing returns to this period consumption. This behavior inflicts a cost on social welfare. The social planner prefers that households equate the expected marginal utility of consumption. To provide a better understanding for the trade-off the household faces between the income effect and the consumption smoothing effect (Lemma 1 and 2), let me define the following objective function at \( (\tilde{a}_{T-1}(X), X) \):

\[
W_{T-1}(K_{T-1}, \tilde{a}_{T-1}, X) = U(\tilde{a}_{T-1} + w_t - K_{T-1})
+ \beta V_T((1 + r)K_{T-1} + TR(K_{T-1}, w_t, 0, T - 1), \phi, \mu_1, R)
\]

\[
K_{T-1} \leq \tilde{a}_{T-1} + w_t + TR(K_{T-1}, w_t, 0, T - 1).
\]

Panel B, depicts the function. The first local maximum is the choice \( K_{T-1} = \frac{\tilde{a}}{1+r} \). Choices just above this point lead to lower returns because the negative income effect dominates the additional consumption smoothing effect. Larger choices lead to additional consumption smoothing gains, which are largest at the second local maximum, where households satisfy first order conditions. The value function becomes steeper again leading to a downward kink at \( (\tilde{a}_{T-1}, X) \). Theorem 2 establishes that choices are again non-distorted to the right.

\(^{21}\)These points are known when solving the problem recursively, implying large computational gains.
The two theorems above characterize regions where households are not affected by the means-test at all. My last theorem characterizes the region where households may currently not participate in the program but are still affected by it. These are choices which are to the right of $\tilde{a}_t(X)$ but to the left of $\hat{a}_t(X)$. First order conditions are still necessary for an optimum in this region. I demonstrate that these points must be downward kinks. Clausen and Strub (2012) show that these cannot be optimal choices; therefore, the function is differentiable at all optimal choices. Standard variation arguments then lead to the necessity of first order conditions.

**Theorem 3.** $\frac{\partial V_t(c, X)}{\partial k_t} = 0$ is a necessary condition for $k_t(a, X)$ to solve (3), (5) and (7) $\forall a \geq \hat{a}_t(X)$ and $a_{t+1}(a, X) > 0$.

To understand how the means-test distorts choices in this region, note that the life-cycle dimension and stochastic earnings imply that households possibly attach positive probability to a state where they want to claim benefits in the future. They adjust their savings decisions already today to fulfill the asset requirements in this case.

Let me first elaborate on the role of the life-cycle dimension. Consider a household in period $T - 2$ who has income $w_t \leq w_{t}^{elig}$. Following the above reasoning, the policy function makes a jump at point $\tilde{a}_{T-2}(X)$. The policy makes a second jump to the right of $\tilde{a}_{T-2}(X)$. Consider the point $\tilde{a}_{T-2}(X)$ s.th. $a_{T-1}(\tilde{a}_{T-2}(X), X) > \hat{a}_{T-1}(X)$. Define the
objective function:

\[ W_{T-2}(K_{T-2}, \tilde{a}_{T-2}(X), X) = U(\tilde{a}_{T-2}(X) + w_t - K_{T-2}) + \beta V_{T-1}((1 + r)K_{T-2} + TR(K_{T-2}, w_t, 0, T - 2), \varphi, \mu_1, R). \]

Figure II (Panel A) shows the two local maxima of this objective function. The first maximum is the choice where the household satisfies the first order conditions by choosing to the left of \( \tilde{a}_{T-1}(X) \) and receives means-tested transfers at end of period \( T - 1 \). Yet, choosing asset choices which lead to next period choices in area \( B \) of Figure I is relatively unattractive because \( V_{T-1} \) is relatively concave there leading to a decreasing objective function. However, there is a second local maximum where the household satisfies the first order conditions again by choosing to the right of \( \tilde{a}_{T-1}(X) \) and never participates in the means-tested program. The policy function makes a second jump at \( \tilde{a}_{T-2}(X) \), the savings function becomes steeper, and the value function becomes non-differentiable at this point. I highlight this graphically in Panel B.

Uncertain earnings have a similar effect, but households with \( w_t > w^t_{\text{elig}} \) also become affected. These households place positive probability on becoming eligible for means-testing in the future. Consequently, they adjust their savings behavior today to smooth consumption better intertemporally.\(^{23}\)

\(^{22}\)The figure highlights that non-uniqueness in \( k_t(a, X) \) can arise when the household is exactly indifferent between choosing to the left and the right of a non-differentiability.

\(^{23}\)This leads to a rapid increase in the number of non-differentiabilities in the value function because any path of the state variables which makes the household at any point in the future eligible to means-testing has to be considered.
4 Data Description and Calibration

This section introduces the data set I am using and explains my calibration strategy. The main idea is to use parameters of the earnings process and employment transitions to mirror the uncertainty households face in the data, and to use preference parameters to match averages of households’ assets holdings. Table 1 summarizes the calibration.

4.1 Data Description

My analysis requires longitudinal household data on components of earnings, assets and means-tested transfers. The dataset which best meets these requirements is the Survey of Income and Program Participation (SIPP). The SIPP is a representative sample of the non-institutionalized civilian US population maintained by the US Census Bureau. I use the 1996 (1996-1999), 2001 (2001-2003) and 2004 (2004-2007) samples.\textsuperscript{24} I deflate all data with the CPI and convert it to 2007 nominal values. The sample period features a stable institutional setting. A major reform replaced the Aid to Families with Dependent Children program by the Temporary Assistance to Needy Families, as of 1996. Moreover, it allowed states to harmonize eligibility criteria for major income support programs (categorical eligibility). After the sample period, the 2008 Farm Bill exempts all tax preferred retirement accounts from the means-test for the Supplemental Nutrition Assistance Program from October 2008 onwards. I find it unlikely that households in my data adjusted their savings in anticipation of this bill given that retirement savings are long-term investment decisions.

The SIPP samples provide monthly information on earnings, transfers from different means-tested programs and household affiliation. The decision period in the model is one quarter; thus, I aggregate the data to quarterly frequency. The data provides detailed information on different liabilities and asset holdings.\textsuperscript{25} My data counterpart to households’ savings in the model is the sum of all assets and liabilities the household has.\textsuperscript{26}

The model is about savings behavior at the household level. I define a household as a group of persons living at a common address,\textsuperscript{27} and I define the head as the person in whose name the place is owned or rented.\textsuperscript{28} I aggregate earnings, retirement income and...

\begin{itemize}
\item \textsuperscript{24}The 1996 panel oversamples households close to poverty. I use household weights provided by the SIPP in all samples to correct for this issue.
\item \textsuperscript{25}The 1996 sample provides four times data on assets and liabilities, the 2001 sample three times and the 2006 panel twice.
\item \textsuperscript{26}Savings may not reflect precautionary saving motives or retirement saving decisions, but necessary business equity which a household holds resulting from incomplete markets for business financing. I drop all households holding business equity to account for this latter concern.
\item \textsuperscript{27}See the SIPP User Guide for detailed information about the definition of an address.
\item \textsuperscript{28}I change the head of a household when the default head lives non-married in a household together with his parents who have higher earnings and are younger than 67. Moreover, I define a new household...
\end{itemize}
asset data of head and spouse to mimic the within household insurance present in the model. I assume that a household enters the labor-market with age 25 and its economic life ends with age 82.\textsuperscript{29} In the model, households work for 43 years and, afterward, live for 15 more years in retirement. To focus on the part of the population which is affected by the means-tested scheme, I drop all households which have average earnings in excess of four times the federal poverty limit which eliminates about 20\% of households.\textsuperscript{30}

4.2 Earnings Process, Employment Transitions and Retirement Income

The model is intended to capture consumption decisions given an exogenous earnings process. I take as data counterpart households’ earnings in the labor-market.\textsuperscript{31} To estimate the earnings process, I restrict the sample to employed\textsuperscript{32} households with prime aged heads \((25 - 50)\). I postulate the following yearly log earnings process for household \(i\) in the data:

\[
\ln(w_{i,t}) = \phi_i + z_{i,t} + \iota_{i,t},
\]

\[
z_{i,t} = \rho z_{i,t-1} + \epsilon_{i,t},
\]

where \(\epsilon_{i,t} \sim N(0, \sigma^2), \ i_{i,t} \sim N(0, \sigma_i^2)\). In a first step, I obtain residuals \((\tilde{w}_{i,t})\) from regressing individual household’s log earnings on time dummies, two race dummies (white, non-white), a gender dummy, a dummy for being disabled, age dummies, four education dummies (less than high school, high school, some college, college) and an interaction between education and age dummies, each using the head of the household. The cross sectional dimension far exceeds the time dimension in the data set and the panel is not balanced. Therefore, I opt to identify \(\rho\) and \(\sigma\) by matching cross sectional moments of the age distribution, as in Storesletten et al. (2004), instead of matching moments of the autocorrelation function. Cross sectional earnings dispersion evolves over the life-cycle according to:

\[
\text{Var}(\ln(w_{i,t})) = \sigma_\phi^2 + \sigma_i^2 + \sigma^2 \sum_{s=0}^{t-1} \rho^{2s}.
\]

\(\rho\) controls the curvature of the profile and \(\sigma\) the increase over time. I match these moments by minimizing the area between the theoretical moment (11) and the earnings
Figure III: Earnings Process

(A) Residual Earnings Dispersion

(B) Earnings Growth over the Life-cycle

Notes: Panel A displays the cross sectional earnings dispersion over age from the data and the theoretical moment (11) for the optimal choice of autocorrelation and variance of earnings shocks. Panel B shows the estimated earnings growth of households over the life-cycle for four education groups. L HS: less than high school, HS: high school, S C: some college, C: completed college.


residuals estimated from the data:

\[
\min_{\rho, \sigma} \left\{ \sum_{1}^{50} \left| \text{Var} \left( \ln(w_{i,t}(\rho, \sigma)) \right) - \text{Var} \left( \ln(\tilde{w}_{i,t}) \right) \right| \right\}.
\]

Figure III (Panel A) plots the data and the resulting profile with \( \rho = 0.95 \) and \( \sigma^2 = 0.013 \). Assuming that the true process is quarterly implies \( \rho = 0.987 \) and \( \sigma = 0.0034 \).

To match the stochastic component of earnings in the model, I use the Markov process with \( N = 5 \) states. Following Tauchen (1986), I use the entries of the vector of values and the transition matrix to match the moments of the AR(1) process.\(^{33}\) In the first period, each household draws a stochastic component from the ergodic distribution.

I use the predictable component of earnings, \( F \), to match average earning profiles of different education groups. The reduced form regression provides profiles for earnings growth over the life-cycle for each education group. I allow households to have four different initial permanent earnings states, \( \mu_1 \), and use these to match the intercepts of the four education groups. I approximate the life-cycle profiles by a second order polynomial, impose monotonicity and assume that earnings do not grow after the age of 50. Panel B shows the resulting profiles. Finally, I calibrate the transitions between the employment states by matching the mean rates I observe in the data.\(^{34}\)

Let me now describe the parametrization of earnings during retirement. The amount of

\(^{33}\)The reason for the relatively low number of earnings states is the computational burden.

\(^{34}\)To be counted as unemployed, the household may not work major hours in the quarter. In that sense, my unemployment process captures rather long-lasting unemployment spells. The stochastic component of earnings opportunities captures shorter unemployment spells.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0.04$</td>
<td>4% Yearly interest rate</td>
</tr>
<tr>
<td>$\beta = 0.994$</td>
<td>Median wealth to earnings ratio of 13.2</td>
</tr>
<tr>
<td>$\theta_\nu = 0.038$</td>
<td>Mean wealth at age 82</td>
</tr>
<tr>
<td>$\gamma = 1.5$</td>
<td>Bell and Miller (2002)</td>
</tr>
<tr>
<td>$\lambda_1(\cdot, \varphi, \mu_1, Z)$</td>
<td>Density of asset holdings of 25 years old</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>Initial differences in education groups</td>
</tr>
<tr>
<td>$\mathcal{F}(\mu_1, t)$</td>
<td>Age-earnings profile of education groups</td>
</tr>
<tr>
<td>$N = 5$</td>
<td></td>
</tr>
<tr>
<td>$\Pi^W, \varphi$</td>
<td>$\rho = 0.987, \sigma = 0.058$</td>
</tr>
<tr>
<td>$K(w^i_W)$</td>
<td>Social Security Administration (2004)</td>
</tr>
<tr>
<td>$\lambda = 0.33$</td>
<td>UE transition in data</td>
</tr>
<tr>
<td>$\delta = 0.02$</td>
<td>EU transition in data</td>
</tr>
<tr>
<td>$\nu = 0.56$</td>
<td>Mean reported by Meyer (2002)</td>
</tr>
<tr>
<td>$b^{max} = 6624$</td>
<td>Mean reported by Meyer (2002)</td>
</tr>
<tr>
<td>$\mathcal{TR}_F = 1413$</td>
<td>Trenkamp and Wiseman (2007)</td>
</tr>
<tr>
<td>$\mathcal{TR}_T = 1308$</td>
<td>Mean reported by Kassabian et al. (2011)</td>
</tr>
<tr>
<td>$\mathcal{TR}_H^W = 1428$</td>
<td>Mean in the Data</td>
</tr>
<tr>
<td>$\mathcal{TR}_H^R = 677$</td>
<td>Mean in the Data</td>
</tr>
<tr>
<td>$\mathcal{TR}_W = 466$</td>
<td>Mean in the Data</td>
</tr>
<tr>
<td>$\mathcal{TR}_S = 2592$</td>
<td>Trenkamp and Wiseman (2007)</td>
</tr>
</tbody>
</table>

Notes: The left column states the calibrated variable with its value and the second states the relevant moment. 'Data' refers to my SIPP sample. Dollar values are expressed in 2007$. 
households’ savings for retirement, and hence their incentive to participate in the means-tested program, depends on their replacement rate during retirement. Retirement savings can be exempt from the means-test when they are not readily available.\footnote{The individual states have some freedom in determining which savings are readily available.} As a result, all promises obtained from social security, most defined pension plans, and retirement plans managed by the employer (401k plans) are not counted towards the asset limit. However, individual retirement plans (IRA) and retirement plans of the self-employed (KEOGH) usually do count. Moreover, 401k plans are transferred under some conditions into an IRA account in the case of unemployment. Henceforth, I define retirement income as the sum of social security income and pensions from former employers, unions or the government.\footnote{I define a household as retired when the head is currently not working or looking for work, head or spouse report to have retired before, the household has positive retirement income, is older than 61 and does not receive earnings in excess of 2.8.} I exclude from retirement income payments received from IRA and KEOGH accounts and 401k plans. Using data from Social Security Administration (2004), I compute the distribution of the retirement replacement rate in my population. Appendix B provides the procedure and the distribution.

### 4.3 Preferences, Demographics and Initial Distribution

Consistent with Siegel (2002), I set the yearly world interest rate at 4%. I use the two factors in the utility function, $\beta$ and $\theta_b$, to match moments of average wealth holdings. The ability of households to smooth consumption across periods depends on their asset holdings. Therefore, I use the discount factor $\beta$ to match the median wealth to earnings ratio in the working population, which is 13.2 in my sample. The incentives to participate in the means-tested program later in life are affected by the desire of leaving bequests. Therefore, I use $\theta_b$ to match mean wealth holdings of households at age 82. I set $\gamma = 1.5$.

I still need to parametrize survival probabilities and conditions at labor-market entry. I use annual survival probabilities reported in Bell and Miller (2002) and assume that within a year households face a constant death probability for each quarter. To calibrate the initial distribution, I use data on households of 25 years of age. I calibrate to an unemployment rate of 6.9 percent at labor-market entry.\footnote{A household is considered unemployed when it works less than 96 hours a quarter, or earnings are less than 2.8, is not yet retired and reported to be searching at some point during the quarter.} For the employed, I match the densities of wealth holdings for the four education groups in the data. The unemployed, start with the mean amount of assets which I observe in the data for that group.\footnote{Due to the small sample of unemployed, I retain from computing a distribution of asset holdings.} Note that this calibration does not necessarily imply that the amount of bequests left from dying households is the same as households’ initial wealth. Average bequests are higher under non-means-testing; therefore, distributing these additional bequests implies that...
my welfare analysis is biased in favor of means-testing.

5 Comparing Implications of the Model with the Data

The calibration targets the earnings uncertainty present in the data and averages of asset holdings. Both the decision to participate in the transfer program and the distribution of asset holdings are non-targeted and arise endogenously from households’ life-cycle consumption choices. This section compares participation in the means-tested program, moments of cross sectional wealth holdings and life-cycle consumption behavior between the model and the data. It puts a particular emphasis on how the model maps the exogenous earnings process into savings decisions, and thus the decision to participate in the means-tested program, the amount of available self-insurance and the amount of life-cycle savings. The selection of moments is guided by the findings in the next section: Asset means-testing imposes welfare costs by distorting consumption decisions of households with low innate earnings abilities. This section presents well-known facts from the data which are consistent with the distortions present in the model: A substantial fraction of low ability households holds almost no wealth, their consumption decreases substantially at retirement and low ability households adjust strongly consumption upon poor labor-market outcomes.

Figure IV (Panel A) displays the share of households which would be eligible for the program due to sufficient low earnings in the model. Moreover, it shows the share of households which choose to actually participate in the program both in the model and data. The model matches the overall mean and the decreasing profile over the working life. However, it predicts somewhat too few households receiving transfers just before retirement. Once retirement is reached, the model predicts that the share rises again, but it decreases in the data. There are three possible explanations for the latter. First, it might be a data issue, because the SIPP has no information on households in nursing homes. Second, facing certain death at age $T$ is likely to lead to too strong wealth decumulation incentives during the last years of life. Third, my model may understatethe inter-generational mobility; therefore, it provides too little incentives for the earnings-poor to leave bequests. To address this issue, I also solve the model under an alternative where all households have the same gains from leaving bequests. The welfare effects are quantitatively almost identical, even somewhat larger, to the present set-up, but imply a counterfactual steep increase in wealth of high school dropouts during the last two years.

Panel A suggests that endogenous saving decisions, and not changes in earnings, are the main driver behind the decreasing share of households in means-testing over the life-cycle. Panel B compares the fraction of households with relatively few assets, less than
Figure IV: Comparing Model and Data

(A) Participation in Means-Testing
(B) Share of Households with low Wealth

(C) Lorenz Curve of Wealth
(D) Average Consumption of < High School

Notes: Panel A displays the fraction of households which participates in means-tested programs. Data refers to SIPP households which participate in any of the programs outlined in Section 2.2. MT refers to the model. The dotted line shows the fraction of households which has earnings below the eligibility threshold in the model. Panel B shows the fraction of households with less than $10000. Panel C displays the Lorenz curve of wealth for the non-retired population in the model and the data. Panel D displays the average consumption profiles over the life-cycle for high school dropouts in the model. Consumption and asset units are expressed in 2007 $. $10000, in the model to the data. The model replicates the overall mean and the downward sloping profile over the life-cycle. As households age, they build up retirement savings and assets for bequests. Yet, a large fraction of households holds little assets throughout all stages of the life-cycle. Hubbard et al. (1995) show that this tend to be households of low earnings ability in US data. The model replicates this share because of the strong incentive created by means-testing for earnings-poor households to hold relatively little wealth. Again, the model implies too strong consumption incentives during the last years of life compared to the data.

Households with higher innate earnings ability find it more unlikely to ever pass the earnings test of the insurance program. Consequently, they do not adjust their assets as
Table 2: Insurance against Earnings Shocks

<table>
<thead>
<tr>
<th></th>
<th>% of unemployed with low wealth</th>
<th>$\Delta \log(c_t)$ upon unemployment</th>
<th>Insurance of &lt; high school relative to college graduates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>25</td>
<td>-6.08</td>
<td>2.86</td>
</tr>
<tr>
<td>Data</td>
<td>35</td>
<td>-6.8</td>
<td>2.24</td>
</tr>
</tbody>
</table>

Notes: Column one compares the share of households just becoming unemployed and hold less than $3000 of wealth in the model to the SIPP data. Column two displays the change in log consumption in the first period of unemployment. Data refers to Gruber (1997). Column three displays the consumption responses to a non-predictable shock to households’ income. It displays the ratio of households with less than high school and households with a college degree. Data refers to estimates for income shocks from Blundell et al. (2008).

strongly as a response to the means-test. The result is a large wealth inequality. Panel C shows the Lorenz curve of wealth of the non-retired in the data and the model.\(^\text{39}\) The model comes close in matching the overall inequality, not only at the bottom, but throughout the distribution.

Panel D shows how these savings decisions aggregate into average consumption of high school dropouts, i.e., the 11 percent of households with the lowest innate ability. In the model, it shows a discontinuous decline of 14% around retirement. Consumption drops because a significant fraction of households holds few assets when entering retirement because they want to participate in the means-tested program prior to it. When earnings drop due to retirement, these agents must adjust consumption downwards. A large literature finds that consumption drops substantially upon retirement, a finding often referred to as the 'retirement consumption puzzle’. Hurst (2008) summarizes evidence that for middle to rich households this can largely be explained by decreases in work related expenditures and home production of food. At the same time, the lowest 20% of the wealth distribution cannot sustain their consumption during retirement. The summarized evidence finds that consumption of these households declines between 20 and 32 percent upon retirement, too much to be accounted for by work related expenses and home production.

The average consumption profile shows one more notable features which is also unique to the lowest innate ability state. Consumption declines at the beginning of the life-cycle. Households dissave their initial assets to become eligible to the means-tested program. Fernández-Villaverde and Krueger (2007) show that consumption expenditures of the low educated indeed falls early in life; however, the decline is smaller than in the model.

The next section shows that, besides too low consumption during retirement, the costs of asset means-testing transpire through necessary consumption adjustments in face of poor labor-market outcomes. Table 2 compares the consumption behavior of house-

\(^{39}\)The wealth data is top-truncated, leading to too little wealth inequality.
holds facing negative shocks to their earnings. In the data, 35% of households entering unemployment have wealth below $3000. Carroll et al. (2003) show that particularly households with low innate abilities suffer from low precautionary savings in US data. In the model, 25% of households have such low wealth holdings upon unemployment, and these are mostly households with low education. Because of the low wealth holdings, households decrease their consumption upon unemployment on average by 6.1%, which is close to the 6.8% of food consumption reported by Gruber (1997). Unemployment is just one type of shock to households’ earnings. Blundell et al. (2008) report the amount of consumption changes resulting from permanent shocks to household income. One of their findings is that households with college education are twice as good insured against permanent income shocks compared to households without college education. Comparing high school dropouts to college graduates, the model almost matches this statistic. However, the model implies more consumption insurance compared to the data in both cases, possibly reflecting the assumption that shocks to earnings are not permanent.

6 Welfare Analysis

This section discusses the welfare implications of the asset means-test. I compare the current system to an alternative where there is no asset means-test but all income thresholds stay in place. The model environment implies that full insurance is optimal from a perspective of an unborn household. Therefore, the reform keeps the total value or resources needed to finance the system constant. Put differently, I do not address the question whether the level of current governmental insurance is optimal, but whether abolishing asset means-testing increases social welfare given the same amount of expenditures. There is no unique way to achieve an expenditure-neutral reform. In the following, I discuss two alternative reforms. The first decreases allotments proportional to all households. This reform implies redistribution of allotments across household types. The second reform eliminates this redistribution by introducing state contingent transfers.

6.1 Proportional Decrease in Allotments

Denote by $\mathcal{T}_t(a, X)$ the end of period transfer which an household with the state $(t, a, X)$ receives given his optimal policy under means-testing. Because abolishing the means-test implies that more households receive the transfer, I need to lower the amount of transfers

---

40See Gruber (2001) for similar results. A difference between the model and the data is that a substantial larger fraction of households holds zero wealth in the data.

41To make the results comparable, I use shocks based on non-predictable household income, instead of households’ earnings.
each household receives to keep total expenditures constant. In a first step, I analyze a reform which achieves this by proportionally decreasing average allotments:

$$\sum_{t=1}^{T} TR_t(a, X) d\lambda_t(a, X) = \sum_{t=1}^{T} (1 - \psi) TR_t(a, X) d\hat{\lambda}_t(a, X),$$

where $\lambda_t(a, X)$ and $\hat{\lambda}_t(a, X)$ are the distribution functions of households with age $t$ under each regime, and $\psi$ gives the required proportional decrease in transfers.

My welfare measure is based on the willingness to pay of expected lifetime consumption of an yet unborn household. Denote by $c_t(a, X)$ the optimal consumption function of an agent under the regime with the means-tested program and let $\hat{c}_t(a, X)$ be the corresponding function without means-testing. The fraction of lifetime consumption which makes the average household of age $t$ indifferent between the two regimes, $\omega_t^U$, solves:

$$\int E_t \sum_{s=t}^{T} \beta^{s-t} U([1 + \omega_t^U] c_s(a, X)) d\lambda_s(a, X)$$

$$= \int E_t \sum_{s=t}^{T} \beta^{s-t} U(\hat{c}_s(a, X)) d\hat{\lambda}_s(a, X). \quad (12)$$

The top row of Table 3 shows that an unborn is willing to pay 0.29 percent of lifetime consumption to keep the means-test. The table also displays the welfare gains by households’ education. Households with the lowest and highest innate abilities loose from abolishing the means-test. I will elaborate on households with high abilities below. To understand the welfare loses of households with low abilities, note that the means-test allocates relatively many transfers to young households and those with low innate abilities because they hold relatively little wealth. Young households hold little wealth because they had no time yet to accumulate assets for retirement and bequests. Households with low innate abilities hold little wealth because they are the most likely to pass the earnings-test; therefore, they have the strongest incentives to change their savings behavior. Panel A of Figure V visualizes this point. It shows that high school dropouts reduce their wealth much more than college graduates as a reaction to the means-test. Hence, abolishing the means-test redistributes transfers to older households and those with higher innate abilities, which is deteriorating welfare by construction in the present environment. Put differently, the considered reform has two effects which deteriorate welfare: First, conditional on the total transfers to household of type $(t, X)$, it does not allocate transfers to those households most in need, the insurance effect. Second, it reallocates resources away from households which are young and have low innate abilities.
Table 3: Welfare Effects of Abolishing the Means-Test

With reallocation of transfers

\[ \omega_U^{t_1} = -0.29 \% \]

<table>
<thead>
<tr>
<th>&lt; High school</th>
<th>High school</th>
<th>Some college</th>
<th>College graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \omega_U^{t_1}(\mu_1) ]</td>
<td>-4.38%</td>
<td>1.01%</td>
<td>0.73%</td>
</tr>
</tbody>
</table>

State dependent transfers

\[ \omega_U^{t_1} = 0.74 \% \]

<table>
<thead>
<tr>
<th>&lt; High school</th>
<th>High school</th>
<th>Some college</th>
<th>College graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \omega_U^{t_1}(\mu_1) ]</td>
<td>4.93%</td>
<td>1.02%</td>
<td>-0.2%</td>
</tr>
</tbody>
</table>

Notes: The table displays the average willingness to pay of an unborn from moving to a non means-tested regime \[ \omega_U^{t_1} \]. It also displays the average willingness to pay of an unborn conditional on knowing the innate ability, \[ \omega_U^{t_1}(\mu_1) \]. The top panel displays the statistic for a reform where allotments are decreased proportionally for all households. The reform displayed in the bottom panel controls for redistribution of transfers across household types.

### 6.2 State Contingent Transfers

To disentangle the insurance effect from the redistribution effect, I consider an alternative reform. Again, the reform abolishes the asset means-test, keeps the income thresholds in place and is expenditure-neutral. However, instead of decreasing transfers proportional, it introduces state dependent transfers. I compute for the state vector \((t, X)\) the amount of total transfers that all households in this state receive under means-testing: \[ B_t(a, X) = \int TR_t(a, X) d\lambda_t(a, X). \] The non-means-tested regime provides for each state \((t, X)\) the same amount of total transfers:

\[
\int B_t(a, X) d\lambda_t(a, X) = \int A_t(X) d\hat{\lambda}_t(a, X),
\]

The means-tested policy still has the advantage that it allocates relatively high allotments to those households most in need conditional on any state \((t, X)\). Nevertheless, the bottom panel of Table 3 shows that an unborn household is now willing to pay 0.74 percent of life-time consumption to abolish the means-test. Appendix F shows that also those

---

42One can think of the reform as the government promising all households to receive the same amount of total transfers conditional on earnings and age.
generations which live before the new steady state is reached gain from the reform. Put differently, the welfare costs from abolishing means-testing discussed above arise because the resulting program features less redistribution of resources towards young households and households with low innate abilities. Once the government controls for distributional effects, there are welfare gains from abolishing the means-test because the undesirable incentive effects of the means-test outweigh its desirable insurance property.

To understand the welfare costs of means-testing, Table 3 displays the welfare change for the four different permanent earnings states. Now, households with low permanent earnings are the ones suffering from means-testing. More specifically, high school dropouts drive most of the adverse effects of means-testing. For these households the welfare costs of means-testing can be summarized in three broad categories: Average consumption is lower, average consumption is not smooth over the life-cycle and individual consumption is more volatile. Regarding the first, note that average wealth is lower under means-testing. This reflects both households not saving to pass the means-test today, and households which currently not participate but still reduce savings because of their forward looking behavior. Consequently, income from asset holdings and average consumption are higher without asset means-testing.

The second type of welfare costs for high school dropouts comes from their choice to consume relatively little during retirement under means-testing. Panel B compares the life-cycle consumption profile under means-testing and non-means-testing. Without means-testing, households of low earnings ability have a much smoother average consumption profile. More specifically, they do not suffer from the discontinuous drop of consumption at retirement.

Finally, Panel C displays the variance of log consumption over the life-cycle under means-testing relative to the regime without the means-test. Throughout working life, high school dropouts have much higher cross-sectional dispersion in consumption under means-testing. They experience lower consumption dispersion upon retirement. The reason for the latter is that a large fraction of households started to decumulate wealth before retirement to become eligible for the transfer program. Once retirement occurs, they see their consumption decline and consume their period retirement income plus the transfer each period.

Why is consumption dispersion among low earning households high during working life? These households strongly adjust their saving decisions and reduce precautionary and retirement savings. Consequently, they have no means to insure against poor labor-market outcomes and behave similar to hand-to-mouth consumers. Table 4 shows average wealth holdings and changes in consumption after earning shocks. For high school dropouts, 14.32% of a negative shock to the persistent earnings component pass through to con-
Figure V: Comparing Means-Testing to Non-Means-Testing

(A) Relative Wealth Holdings
College and < High School

(B) Average Consumption Profile
< High School

(C) Consumption Dispersion
for Education Groups

(D) Share of Households
with low Wealth

Notes: Panel A displays the average wealth holdings under means-testing, MT, relative to the regime without the means-test, No MT, for two education groups. L HS: less than high school, C: completed college. Panel B displays the average consumption of high school dropouts under means-testing and no means-testing. Panel C shows the variance of cross-sectional log consumption over the life-cycle under means-testing relative to the variance of cross-sectional consumption over the life-cycle under a regime without the asset means-test. Panel D shows the share of households with wealth below $10000. Consumption and asset units are expressed in 2007$.

Consumption under means-testing. Moreover, consumption declines by 16.34% when these households become unemployed. The last column shows that consumption is down by 36.93% relative to the last employment quarter after three quarters of unemployment. In contrast, these households are successful to smooth consumption after poor labor-market outcomes without the means-test. The consumption drop after three periods in unemployment is almost 5 times smaller, and only 3.61% of a negative shock to the persistent component of earnings passes through to consumption. The reason is that high school dropouts enter unemployment with four times more wealth on average which allows them to almost sustain their consumption level despite lower allotments. This fact can also be seen in Panel D of Figure V which plots the share of households with asset holdings less
Table 4: Wealth Levels and Consumption Response after Earnings Shocks

<table>
<thead>
<tr>
<th>Persistent Earnings Shock</th>
<th>Unemployment</th>
<th>3 Quarters of Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% passed to c</td>
<td>Wealth in 1000$</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>14.32</td>
<td>14</td>
</tr>
<tr>
<td>Means-tested ( \mu_2 )</td>
<td>-0.42</td>
<td>183</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>3.42</td>
<td>356</td>
</tr>
<tr>
<td>( \mu_4 )</td>
<td>5.19</td>
<td>709</td>
</tr>
<tr>
<td>Non-means-tested ( \mu_1 )</td>
<td>3.61</td>
<td>55</td>
</tr>
<tr>
<td>means-tested ( \mu_2 )</td>
<td>9.7</td>
<td>219</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>21.24</td>
<td>330</td>
</tr>
<tr>
<td>( \mu_4 )</td>
<td>15.37</td>
<td>750</td>
</tr>
</tbody>
</table>

Notes: The table displays the beginning of period mean wealth and consumption adjustments after poor labor-market outcomes for the four innate ability states, \( \mu_1 \). The top panel displays the results under the means-tested regime, and the lower panel shows the same for the non-means-tested regime. The cases are: The stochastic component of earnings falls from the medium to second worst state, the household becomes unemployed and the household is unemployed for three consecutive quarters. The third column displays the percent of earnings shock passed through to consumption and columns five and seven show the percentage drop in consumption relative to the pre-shock period. Wealth units are expressed in 2007$.

than $10000. After the age of 40, almost no households belong to this category when there is no means-testing. Contrary, at each stage of the life-cycle, a significant fraction holds little wealth in the presence of asset means-testing.

High school graduates’ consumption is less affected by persistent earnings shocks under the means-tested regime. In fact, households slightly increase consumption after a negative shock under means-testing. One has to be careful with interpreting this as a pure insurance effect. For some of these households, a drop in earnings makes it attractive to increase today’s consumption to become eligible to the transfer program and to consume less in the future. The last column shows that these households do not hold sufficient assets to finance long spells of unemployment under means-testing leading to a 2.5 times larger drop in consumption after three quarters in unemployment. Moreover, their welfare is reduced by increasing relative consumption dispersion over the retirement period. Each period, it becomes attractive for some of these households to strongly dissave and enter into the means-tested program. Also, as for all education groups, average consumption is somewhat lower for them under means-testing. In total, these households still lose from the means-test, but much less than high school dropouts.
Upper earnings groups experience welfare gains from means-testing, but these are comparably small. These households need to be in a relatively low earnings state to become eligible. Therefore, their savings are less affected by the means-test and they deplete their savings only in the case of repeatedly poor labor-market outcomes. In this case, they gain from the relatively generous transfers.\textsuperscript{43} This can also be seen in Table 4. Households with a completed college degree reduce consumption on average by 4.86\% when becoming unemployed in the non-means-tested regime, but they reduce it only by 4.36\% under means-testing. The insurance effect for persistent shocks to earnings is even larger. This occurs, despite that these households hold lower average savings under means-testing. Only in case of long unemployment spells, do these lower savings translate into stronger consumption declines. Particularly early in life these households gain from the means-test. The higher insurance under means-testing imply an additional welfare gain because households need to build up less precautionary savings early in life which brings their consumption closer to the social planner solution.

7 Conclusion

The rationale behind asset means-testing income support programs is to help those households most in need. This paper compares the asset means-tested US system to two expenditure-neutral reforms which abolish the means-test but keep income thresholds in place. To achieve expenditure-neutrality, the first reform decreases allotments proportionally for all households. I show that a yet unborn household opposes this reform because it reallocates resources away from young households and those with low innate abilities. In case of the second reform, the government eliminates the distributional aspect. Now, an unborn household is willing to pay 0.74 percent of life-time consumption to live under a regime without asset means-testing. I show that those households with low innate earnings ability, households with no college education, are in favor of abolishing means-testing. Contrary, households with higher earnings ability, those with at least some college education, gain from the means-test.

The welfare costs for the low earning households arise from their distorted saving incentives. To fulfill the means-test, these households reduce precautionary and retirement savings. Consequently, they have to adjust consumption downward when they reach retirement. Moreover, their consumption drops more under means-testing after poor labor-market outcomes, because they have little private means of insurance. Households with higher earnings ability face a lower likelihood to fulfill the earnings test of these programs.

\textsuperscript{43}Note, if the government could condition on permanent earnings instead of overall earnings, it could increase social welfare by introducing the means-test only for high permanent earnings states.
Therefore, their savings choices are less distorted and they benefit from relatively high allotments in case of a negative earnings shock. Consequently, these households are able to better smooth their consumption under means-testing.

There are several possible avenues of extending the analysis presented in this paper. First, one may widen the set of policies available to the government. Particularly the welfare costs of reduced consumption during retirement may be mitigated by allowing the household to save in a ‘retirement asset’ that is not subject to the means-test. Components of the 2008 Farm Bill provide a step in that direction. Another natural extension is the inclusion of health uncertainty together with the means-tested Medicaid. Braun et al. (2013) show that the means-test of Medicaid in old age has different welfare implications than the one of the programs studied here because it provides insurance against large but rare expenditure shocks which strongly depend on age. This reduces the distortions on savings decisions during working life relative to the income support programs studied in the present paper. Regarding the exogeneity assumption of earnings, there may be important interactions between the asset means-test and employment choices. Households with low wealth and low labor-market earnings may select themselves into unemployment, but high wealth households may choose to continue to work.

The present paper also has several implications for the value of insurance programs which have been the subject of analysis in the literature, but which I abstract from, or take as given, in the present framework. Conesa and Krueger (2006) study the welfare implications of progressive income taxation. They weight the gains of redistribution towards poor households against the adverse incentive effects of high earning households. The means-test implies a similar redistribution, and the government may want to weight the different welfare costs against each other. Finally, my analysis shows that in the presence of a means-test we should expect to see many households of low earnings close to their borrowing constraint. Therefore, households may value insurance against temporary earnings shocks, e.g., unemployment, more than suggested for example in Low et al. (2010). Similarly, the optimal replacement rate in unemployment will be much higher when social assistance is means-tested in the framework of Pavoni and Violante (2007).
A  The Bequest Motive

Households have an intrinsic bequest motive given by the function:

\[ \bar{V}(\varphi, \mu_1, k) = \mathcal{G}\left( V_1((1 + r)k, \varphi_{T_w}, \cdot, E), V_1((1 + r)k, \varphi_{T_w}, \cdot, U), \Pi(\mu_1, \varphi_{T_w}) \right), \]

where \( V_1((1 + r)k, \varphi_{T_w}, \cdot, E) \) is the value function of a newborn who starts employed, \( V_1((1 + r)k, \varphi_{T_w}, \cdot, U) \) is the corresponding function for an unemployed, and \( \Pi(\mu_1, \varphi_{T_w}) \) is a transition function that maps the permanent component of earnings of the parents into those of the offspring. The idea is that the household cares about its offspring and takes into account that the newborn may start employed, or unemployed which are given by \( \varpi \) and \( 1 - \varpi \), respectively. Define:

\[ EV(a, \varphi_{T_w}, \mu_1) = \sum_{k=1}^{4} Pr(\mu_1 = \mu^k_1 | \mu_1) \left[ \varpi V_1((1 + r)k, \varphi_{T_w}, \mu^k_1, E) + (1 - \varpi) V_1((1 + r)k, \varphi_{T_w}, \mu^k_1, U) \right] \]

For tractability, I assume that households approximate \( EV \) by means of the following linear regression:

\[ EV(\cdot, \varphi_{T_w}, \mu_1) = \kappa_0 + \kappa_1 a + \kappa_2 a^2 + \varsigma, \]

where \( \varsigma \) is the approximation error. Table 5 shows that this error is small.

Table 5: Accuracy of the Bequest Function

<table>
<thead>
<tr>
<th>Persistent earnings state (( \varphi ))</th>
<th>( \mu^1_1 )</th>
<th>( \mu^2_1 )</th>
<th>( \mu^3_1 )</th>
<th>( \mu^4_1 )</th>
<th>( \mu^5_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu^1_1 )</td>
<td>0.9965</td>
<td>0.9964</td>
<td>0.9961</td>
<td>0.9958</td>
<td>0.9952</td>
</tr>
<tr>
<td>( \mu^2_1 )</td>
<td>0.9958</td>
<td>0.9955</td>
<td>0.9951</td>
<td>0.9947</td>
<td>0.9945</td>
</tr>
<tr>
<td>( \mu^3_1 )</td>
<td>0.9968</td>
<td>0.9966</td>
<td>0.9964</td>
<td>0.9963</td>
<td>0.9983</td>
</tr>
<tr>
<td>( \mu^4_1 )</td>
<td>0.9974</td>
<td>0.9976</td>
<td>0.9981</td>
<td>0.9983</td>
<td>0.9985</td>
</tr>
</tbody>
</table>

Notes: The table displays the \( R^2 \) of the linear regression specified in the text. \( \mu^1_1 \) is the innate ability of the household.

B  Computing Retirement Replacement Rates

This section provides further information on the way I compute the retirement replacement rate. Social Security Administration (2004) provides information on earnings during retirement from different sources conditional on pre-retirement income. To obtain the entire distribution, I use linear interpolation between points. In the model, retirement income
is paid unconditional on assets and the source, i.e., the retirement wealth does not count against the asset limit either. Therefore, reflecting US legislation, I include all income from Social Security and pensions. However, I exclude payments received from IRA and KEOGH accounts and 401k plans. The wealth in these plans do count against asset limits in several US states and I decide to include them; therefore, in households’ wealth. Based on this definition, I use data from Social Security Administration (2004) to compute the retirement replacement rate. Figure VI displays the result. Replacement rates decrease in households’ pre-retirement earnings, mainly reflecting that Social Security replacement rates are decreasing in earnings.

Figure VI: Retirement Replacement Rate

Notes: The graph displays the replacement rate provided by retirement income conditional on earnings during working life.

C Proofs

This section provides proves for the theories laid out in the main part of the paper. To keep notation simple, I sometimes focus on the current state of being employed. Moreover, to make the notation more compact define conditional on the employment state, e.g., $E$:

$$V_{t+1}(\phi(k), \varphi', \mu_1) = E_t\{(1 - \iota_t)(1 - \delta)V_{t+1}(\phi(k), \varphi', \mu_1, E) + \delta V_{t+1}(\phi(k), \varphi', \mu_1, Ub) + \iota_t \tilde{V}((\varphi', \mu_1, k))\}.$$

**Proof of Lemma 1:** Let $k_t(a^j, X)$ be the optimal policy and let $a^k > a^j$. By the definition of $\Gamma$, I have $\Gamma^j \subset \Gamma^k$. Thus, $k_t(a^j, X)$ is an admissible policy for $a^k$ with strictly larger current consume this period. Because $U$ is increasing in current consumption
and \( k_t(a, X) \) maximizes \( V_t, V_t(a^k, X) > V_t(a^j, X) \). To prove the third part, assume that the optimal policy is such that \( k_t(a^j, X) > k_t(a^k, X) \). It directly follows for the transfer induced by point \( a^k \): \( TR^k \geq TR^j \). First, assume they are equal. Because \( V(\cdot, X) \) is strictly increasing, I have \( \nabla_{t+1}(\phi(k_t(a^j, X)), \varphi', \mu_1) > \nabla_{t+1}(\phi(k_t(a^k, X)), \varphi', \mu_1) \). Resulting from the concavity of \( U, k_t(a^k, X) \) cannot be optimal given the optimality of \( k_t(a^j, X) \). Assume now \( TR^k > TR^j \). Hence, \( TR^j = 0 \) and \( TR^k > 0 \). This again contradicts the concavity of \( U \) because the marginal gain from consuming more today are larger for the lower asset position. Finally, I prove that \( V_{T-1}(\cdot, X) \) is continuous. Once established, the same logic carries through for all periods. Consider all \( a^T \_1 \) s.th. \( k_{T-1}(a^T \_1, X) > \frac{a}{1+r} \). Then

\[
TR(k_{T-1}(a^T \_1, X), w_{T-1}, 0, T - 1) = 0,
\]

and there exists a smallest point for which this condition still holds, which I call \( \hat{a}^T \_1 \). Moreover, both \( U \) and \( V_T \) are continuous (The continuity of \( V_T \) follows from the fact that the bequest function is continuous and \( \phi(k) \) is a constant.). Therefore, I maximize a continuous function over a continuous correspondence, and by Berge’s theorem of the maximum the resulting value function is continuous. By the same logic, I can establish continuity at all points \( a^T \_1 \) s.th. \( k_{T-1}(a^T \_1, X) \leq \frac{a}{1+r} \). Consequently, I only need to establish continuity at the switching point. When \( V_{T-1}(\cdot, X) \) would not be continuous, it features either an upward or downward jump at \( \hat{a}^T \_1 \). We already established that it cannot have a downward jump. Now assume it would have an upward jump. Then by the continuity of \( U \) and \( \Gamma \), it must be that \( k_{T-1}(\hat{a}^T \_1 - \epsilon, X) > \frac{a}{1+r} \) because such a policy would bring the implied value arbitrary close to the upward jump. But this contradicts the fact that \( \hat{a}^T \_1 \) is the least such point.

**Proof of Lemma 2:** The proof goes by contradiction. \( k_t(a_t, X) \) would be strictly increasing when \( \exists \) a point \( (\hat{a}_t - \epsilon, X) \) s.th. \( k_t(\hat{a}_t - \epsilon, X) = \frac{a}{1+r} \) and \( \forall \epsilon \) the point \( (\hat{a}_t, X) \) leads to \( k_t(\hat{a}_t, X) > \frac{a}{1+r} \). Moreover, \( TR(k_t(\hat{a}_t - \epsilon, X), w_t^0, 0, t) > 0 = TR^+ \) and \( TR(k_t(\hat{a}_t, X), w_t, 0, t) = 0 \). I now show that for this case \( k_t(\hat{a}_t, X) > \frac{a}{1+r} \) cannot be an optimal policy \( \forall \epsilon \). The policy \( \tilde{k}_t(\hat{a}_t, X) = \frac{a}{1+r} \) was preferred iff \( \exists \) an \( \epsilon \) s.th.

\[
U(\hat{a}_t + w_t - k_t(\hat{a}_t, X)) - U(\hat{a}_t + w_t - \tilde{k}_t(\hat{a}_t, X) + \epsilon) < \beta[\nabla_{t+1}((1 + r)[k_t - \epsilon] + TR^+, \varphi', \mu_1) - \nabla_{t+1}((1 + r)k_t, \varphi', \mu_1)],
\]

where the inequality on the right hand side comes from the fact that \( V_{t+1} \) is increasing in \( a \) and \( TR^+ > 0 \). For the second part of the Lemma, assume \( \forall a, k_t(a_t, X) \leq \frac{a}{1+r} \).
For expositional reasons, I assume the equality holds. Moreover, assume $w_t \leq w^t_{elag}$. The result for all other $w$ follow trivially. Now consider the alternative policy $k_t(a^0, X) = \frac{\bar{a} + x}{1 + r}$ for some state $(a, X)$ and $x > TR(k_t, w_t, b_t, t)$. This alternative policy is better iff the following inequality holds:

$$U(a + w_t - \frac{\bar{a}}{1 + r}) - U(a + w_t - \frac{\bar{a} + x}{1 + r}) \to 0 \text{ for a large enough }$$

$$\beta[\nabla_{t+1}(\bar{a} + x, \varphi', \mu_1) - \nabla_{t+1}(\bar{a} + TR(k_t, w_t, b_t, t), \varphi', \mu_1)] > 0.$$ 

The convergence to 0 of the left hand side results from the concavity of $U$ and the inequality on the right hand side results from Lemma 1. The second part of the Lemma results from the monotonicity of the value function.

**Proof of Theorem 1:** Call a typical element from $\Gamma(a, w, b, t) \bar{a}_t$. By assumption

$$\phi(k_t(\bar{a}_t, X)) = (1 + r)k_t(\bar{a}_t, X),$$

which is a continuous function. Both the feasibility correspondence and the law of motion are concave. Moreover, the inside of (3), (5) and (7) are just the sum of concave functions and hence concave itself. Thus, $V_t$ is concave in this range. To proof the Theorem, I apply the Benveniste and Scheinkman (1979) Lemma. Let $\bar{k}_t(\bar{a}_t, X)$ solve (3). Now define $A_t \in B_{\epsilon}(\bar{a}_t)$ where $\epsilon$ is chosen s.t.h. $\bar{k}_t$ is still feasible $\forall A_t$. Define the function

$$W(A_t, X) = U(A_t + w_t - \bar{k}_t) + \beta \nabla_{t+1}((1 + r)\bar{k}_t, \varphi, \mu_1).$$

Note that $W(\cdot, \varphi, \mu_1)$ is continuous and concave because $U$ is continuous and concave and $\beta \nabla_{t+1}((1 + r)\bar{k}_t, \varphi, \mu_1)$ is a constant. It follows that $W(A_t, \varphi, \mu_1) \leq V_t(A_t, \varphi, \mu_1)$ with equality at $\bar{a}_t \in A_t$. Thus, the Benveniste and Scheinkman Lemma establishes differentiability of $V_t(A_t, \varphi, \mu_1)$. Because the function is concave and, by assumption, the borrowing constraint is slack, the first order conditions are sufficient for a maximum.

**Proof of Theorem 2:** By assumption $TR(k_t(\bar{a}_s, X), X) = 0$ $\forall t \geq s$ and by Lemma 1 this holds $\forall a_s > \bar{a}_s(X)$. Call a typical element from this later set $\bar{a}_t$. Thus, for $\bar{a}_t$:

$$\phi(k_t(\bar{a}_t), X) = (1 + r)k_t(\bar{a}_t, X),$$

which is a continuous function. Consequently, the same logic as in Theorem 1 applies. For the second part, it is sufficient to show that $V_t(\cdot, X)$ is concave. Then the result follows
by the same logic as before. Call the set of \( a \) satisfying the above conditions \( A_t \). Because 
\[
k_t(a, \mathbf{X}) < \frac{a}{1+r}
\] I have that \( \forall a \in A_t \) \( \phi(k_t(a, \mathbf{X})) = (1 + r)k_t + TR(k_t, w_t, b_t, t) \). Moreover, by assumption for each induced \( a_s \), \( \phi(k_s(a, \mathbf{X})) = (1 + r)k_s + TR(k_s, w_s, b_s, s) \) \( \forall s > t \). Hence, \( \phi(A_s) \) is a concave function and \( V_{t+1} \) is just the sum of concave functions, which is concave. Therefore, the function inside the max operator in (1), (3), (5) and (7) is concave and the constraints are concave, assuring concavity of \( V_t(A_t, \mathbf{X}) \).

**Proof of Theorem 3:** Theorem 2 establishes the result \( \forall \dot{a}_s(\mathbf{X}) \); hence, I focus here on all other points. Clausen and Strub (2012) show that non-differentiable points can be classified into upward, the function is not sub-differentiable, and downward kinks, the function is not superdifferentiable. As they demonstrate, choosing \( k_t \) at a downward kink cannot be optimal because the slope of \( V_t^Z(\cdot, \phi, \mu) \) is increasing to the right. Therefore, it is sufficient for me to show that all points of discontinuity of \( V_t(\cdot, \mathbf{X}) \) are downward kinks or equivalently that \( V_t \) is sub-differentiable. Following the notation of Clausen and Strub (2012), call \( \partial D V_t(a^0, \mathbf{X}) \) the sub-differentiable of \( V_t \) at \( a^0 \):

\[
\partial D V_t(a^0, \mathbf{X}) = \left\{ m \in \mathbb{R} : \limsup_{\Delta a^0 \to 0^-} \left\{ \frac{V_t(a^0 + \Delta a^0, \mathbf{X}) - V_t(a^0, \mathbf{X})}{\Delta a^0} \right\} \leq m \leq \liminf_{\Delta a^0 \to 0^+} \left\{ \frac{V_t(a^0 + \Delta a^0, \mathbf{X}) - V_t(a^0, \mathbf{X})}{\Delta a^0} \right\} \right\}.
\] (13)

\( V_t(a^0, \mathbf{X}) \) is sub-differentiable at \( a^0 \) iff \( \partial D V_t(a^0, \mathbf{X}) \) is non-empty. Intuitively, a function is sub-differentiable at a point when its slope approaching the point from the right is larger than the slope approaching from the left.

I first show that the upward jump in the policy function at \( \tilde{a}_t(\mathbf{X}) \) leads to \( V_t \) being still sub-differentiable. For the ease of presentation, I omit the dependence of \( \tilde{a}_t \) on the exogenous state vector \( \mathbf{X} \) from here on. Lemma 2 establishes that \( k_t(\tilde{a}_t, \mathbf{X}) = k_t(\tilde{a}_t - \epsilon, \mathbf{X}) = \tilde{k} \). Therefore, the first part of (13) simplifies to

\[
\limsup_{\Delta \tilde{a}_t \to 0^-} \left\{ \frac{U(\tilde{a}_t + \Delta \tilde{a}_t + w_t - \tilde{k}) - U(\tilde{a}_t + w_t - \tilde{k})}{\Delta \tilde{a}_t} \right\}.
\] (14)
The second part of (13) becomes
\[
\liminf_{\Delta \tilde{a}_t \to 0^+} \left\{ \frac{U(\tilde{a}_t + \Delta \tilde{a}_t + w_t - k_t(\tilde{a}_t + \Delta \tilde{a}_t, X))}{\Delta \tilde{a}_t} \right. \\
- \frac{U(\tilde{a}_t + w_t - \bar{k})}{\Delta \tilde{a}_t} \\
+ \beta \left[ \frac{\mathcal{V}_{t+1}((1 + r)k_t(\tilde{a}_t + \Delta \tilde{a}_t, X), \varphi', \mu_1)}{\Delta \tilde{a}_t} \right] \\
- \left. \frac{\mathcal{V}_{t+1}((1 + r)\bar{k}, \varphi', \mu_1)}{\Delta \tilde{a}_t} \right\}. \tag{15}
\]

Because \(k_t(\tilde{a}_t + \Delta \tilde{a}_t, X)\) is optimal, it must be that
\[
U(\tilde{a}_t + \Delta \tilde{a}_t + w_t - k_t(\tilde{a}_t + \Delta \tilde{a}_t, X)) \\
+ \beta \mathcal{V}_{t+1}((1 + r)k_t(\tilde{a}_t + \Delta \tilde{a}_t, X), \varphi', \mu_1) \geq \\
U(\tilde{a}_t + \Delta \tilde{a}_t + w_t - \bar{k}) + \beta \mathcal{V}_{t+1}((1 + r)\bar{k}, \varphi', \mu_1).
\]

Together with the fact that \(k_t(\cdot, X)\) is weakly increasing and \(\mathcal{V}_{t+1}(\cdot, \varphi', \mu_1)\) is strictly increasing implies (15) \(\geq\) (14) as was to be shown.

I still need to show that \(V_t\) is sub-differentiable, given that \(\mathcal{V}_{t+1}\) is sub-differentiable. Clausen and Strub (2012) show that kinks do not cancel out under addition. Hence, it is sufficient to show that the upper envelope of a sub-differentiable function is sub-differentiable.\(^{44}\) When \(V_t(\cdot, \varphi, \mu_1)\) is the upper envelope of some sub-differentiable function, \(f(a, K)\), with \(V_t(a^0, \varphi, \mu_1) = f(a^0, k)\):
\[
f(a + \Delta a, k) - f(a, k) \leq V_t(a^0 + \Delta a, \varphi, \mu_1) - V_t(a^0, \varphi, \mu_1).
\]
It follows that \(\partial_D f \in \partial_D V_t(\cdot, X)\) and consequently \(V_t(\cdot, X)\) is sub-differentiable. The desired result follows directly: All non-differentiable points cannot be a solution to (1), (3), (5), (7).

\section*{D \ Hidden Savings}

This section relaxes the assumption that the government can perfectly observe savings \(k_t\).\(^{45}\) A full characterization of the household problem is beyond the aim of this paper. Instead, I provide intuition for some specifications of particular interest. I first show that a specification in which households can hide a fixed amount of savings does not alter the

\(^{44}\)My proof follows their Lemma 4 where I replace the derivative with the sub-differential.

\(^{45}\)The section uses notation and refers to results from Section 3 and I advise to read that section first.
Figure VII: Policy with Hidden Savings

(A) Case I

(B) Case II

Notes: Panel A displays the policy function of a household where the probability to successfully hide savings below the point \( \tilde{a}_t(X) \) is almost one and zero for all higher savings. For comparison, it also plots the policy function from my baseline model. Panel B shows the policy function when the ability to hide savings is decreasing slowly along the asset dimension.

main mechanisms of the model. I can construct examples in which savings behavior is significantly different from my baseline model when the government observes hidden savings only with a certain probability. Nevertheless, a significant range of parameterization implies the same household behavior as in my baseline model even in that case.

Consider the following modification for the means-tested transfer:

\[
TR(k_t, w_t, b_t, t) = \begin{cases} 
0 & \text{if } 1 - P(k_t) \text{ or } w_t > w_{\text{elig}} \text{ or } b_t > w_{\text{elig}} \\
TR_W & \text{if } P(k_t) \text{ and } w_t \leq w_{\text{elig}} \text{ and } b_t \leq w_{\text{elig}} \text{ and working} \\
TR_R & \text{if } P(k_t) \text{ and } w_t \leq w_{\text{elig}} \text{ and retired,}
\end{cases}
\]

where \( 1 - P(k_t) \) is the probability that the government observes that the household has savings exceeding \( \frac{\tilde{a}}{1+r} \). It is straightforward to see that the logics of Lemma 2 still apply \( \forall P(k_t) < 1 \). \( \forall w_t \leq w_{\text{elig}} \) the policy function is flat in a range of the asset state and makes a jump at some \( \tilde{a}(X) \).

Consider now a special case in which households can hide savings \( \hat{k}_t \). So \( P(k_t) = 1 \forall k_t \leq \hat{k}_t \) and zero thereafter. In this simple case, the proofs from Section 3 still apply. The only modification is that the flat region and the jump point characterized by Lemma 2 are to the right in the asset state compared to my baseline model.

Now consider the general case with an arbitrary \( P(k_t) \). It is obvious that I can find a schedule \( s.t.h. \) the solution with hidden savings coincides exactly with the solution of my main model. Crucial for this result is that \( P(k_t) \) is sufficiently small close to \( \tilde{a}_t(X) \). To see this point take an extreme case in which \( P(k_t(\cdot, X)) = 0.99 \forall \frac{\tilde{a}}{1+r} < k_t < \tilde{a}_t(X) \) and \( P(k_t(\tilde{a}_t(X), X) + \epsilon) = 0 \). Figure VII Panel A plots the policy function in \( T-1 \) together
with the policy function from my baseline model. Note that the region characterized
by Lemma 2 becomes quite small because taking the risk of increasing savings becomes
attractive quickly. Moreover, the policy function becomes flat in a second region of the
asset state. Agents choose the maximum savings that have positive probability of not
being detected in this region. To get an intuition for the robustness of my results to a
more general specification of hidden savings, consider the following parameterization:

\[
P(k_t) = \begin{cases} 
1 & \text{if } k_t \leq \frac{\bar{a}}{1+r} \\
\max(1 - \frac{k_t - \bar{a}}{\alpha}, 0) & \text{if } k_t > \frac{\bar{a}}{1+r}
\end{cases}
\]

\(\alpha\) controls the ability of the government to accurately observe savings. My baseline model
is the limit case with \(\alpha \to 0\). Figure VII Panel B plots the value and policy function of
an eligible household in \(T - 1\) for two different values of \(\alpha\). With \(\alpha = 27.8\) the households
attach positive probability of successfully hiding assets in the range \(14.7 \leq k_{T-1} \leq 42.0\)
and the resulting policy function is identical to my baseline model. The range expands to
\(14.7 \leq k_{T-1} \leq 47.5\) with \(\alpha = 27.8\) and households policy starts to deviate slightly from
my baseline specification.

E Changing the Asset Threshold

The baseline calibration allows households to hold at most 3000$ to be eligible to the
means-tested program. While this was representative for many states during my sample
period, some of these states allowed allowances for a car and a households’ house. These
assets may be ill suited to smooth consumption after poor earnings shocks which motivated
my baseline calibration. However, households may use these assets to finance consumption
during retirement. This section recalibrates the model with two alternative thresholds:
\(\bar{a} = 10000\) and \(\bar{a} = 30000\). With these calibrations, I perform the policy experiment and
compute the same welfare measures as in Section 6.

Table 6 compares the welfare effects from abolishing the means-test from my baseline
to the two alternative specifications. Welfare is always higher when the means-test is
abolished. The welfare gain decreases from 0.74% of life-time consumption to 0.67% with
\(\bar{a} = 10000\) and to 0.57% with \(\bar{a} = 30000\). This should come at little surprise because
abolishing means-testing implies setting \(\bar{a} = \infty\). Also gains and losses are distributed
similar to my baseline calibration. High school dropouts are the households most suffering
from the means-test across calibrations. Households with at least some college education
gain from the means-test.
Table 6: Welfare with Alternative Asset Thresholds

<table>
<thead>
<tr>
<th>$\bar{a} = 3000$ (Baseline)</th>
<th>$\omega^U = 0.74%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; \text{High school}$</td>
<td>High school</td>
</tr>
<tr>
<td>$\omega^U_1(\mu_1)$</td>
<td>4.93%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\bar{a} = 10000$</th>
<th>$\omega^U = 0.67%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; \text{High school}$</td>
<td>High school</td>
</tr>
<tr>
<td>$\omega^U_1(\mu_1)$</td>
<td>4.51%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\bar{a} = 30000$</th>
<th>$\omega^U = 0.57%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; \text{High school}$</td>
<td>High school</td>
</tr>
<tr>
<td>$\omega^U_1(\mu_1)$</td>
<td>3.87%</td>
</tr>
</tbody>
</table>

Notes: The table displays the average willingness to pay of an unborn from moving to a non means-tested regime $\omega^U$. It also displays the average willingness to pay of an unborn conditional on knowing the innate ability, $\omega^U_1(\mu_1)$. The policy experiment assures no redistribution of transfers among household types. The table shows the results for three different parameterizations of the asset threshold in the means-test.

F Welfare with Transition Dynamics

The main text shows that welfare is higher in a world without asset means-testing when comparing steady states. However, this does not directly imply that society is better off from abolishing means-testing. Agents living under means-testing hold relatively little assets and the asset poor receive less transfers under the alternative regime forcing them to lower consumption. These temporary welfare costs may well outweigh the long-term gains from abolishing means-testing.

Because a change in the regime has no general equilibrium price effects, all yet unborn households gain $\omega^U$, independent of their time of birth. Therefore, it is sufficient to show that the currently living generation has welfare gains from abolishing means-testing. The
Notes: The graph displays the welfare gains from abolishing asset means-testing. It shows for each generation living at the date of the reform their average willingness to pay of remaining life-time consumption.

Currently living with age $t$ are distributed according to $\lambda_t(a, X)$. Hence, the willingness to pay of remaining life-time consumption of a generation with age $t$ is given by:

$$\omega^U_{t,\text{tran}} = \left[ \frac{\int V_t(a, X) d\lambda_t(a, X)}{\int V_t(a, X) d\lambda_t(a, X)} \right]^{\frac{1}{1+\gamma}} - 1$$

Figure VIII displays $\omega^U_{t,\text{tran}}$. Each age group gains from abolishing the asset means-test. The gains are particularly high during retirement. To understand this fact, recall that these gains are not expressed in total life-time consumption but in remaining consumption from age $t$ onwards.

**G Numerical Algorithm**

My algorithm depends on the state the household is currently in. *Theorems 1 and 2 characterize regions where the solution to first order conditions are unique. I solve for the policy functions in that region by the endogenous grid method proposed by Caroll (2006). In other parts of the state space, first order conditions are not necessarily unique. In theory, one knows all non-differentiabilities in the expected value function when computing the problem backwards and could compare all candidate points to the choice $k = \frac{a}{1+\gamma}$. *Theorem 3 establishes that the choice leading to the highest value is indeed the global optimum. However, uncertain earnings increases the number of non-differentiabilities quickly making it computationally expensive to allow for off-grid choices. As a compromise, I allow for a very fine asset grid for low asset choices and make it coarser towards higher
asset states. In total, I allow for almost 9000 asset choices. Solving the household problem with Fortran 90 and a computing cluster with twenty workers takes about 1 hour.

Following the computation of optimal policies, I update the initial value functions and the value of leaving bequests. I iterate on the value function until convergence. Thereafter, I compute the median wealth to earnings ratio by simulating a history of 100000 households. I update the discount factor using bisection search. Finally, to reduce the simulation error when computing welfare measures, I compute the stationary distribution \( \lambda \) using distribution function iteration.
References


