Signaling through Price and Quality to Consumers with Fairness Concerns

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August 2015

Abstract

Consumers with inequity aversion experience some psychological disutility when buying products at unfair prices. Empirical evidence and behavioral research suggest that consumers may perceive a firm’s price as unfair when its profit margin is too high relative to consumers’ surplus. We develop a game-theoretic model to investigate the effects of the consumer’s inequity aversion on a firm’s pricing and quality decisions. We highlight several interesting findings. First, because of the consumer’s uncertainty about the firm’s cost, the firm’s optimal quality may be non-monotone with respect to the degree of the consumer’s inequity aversion. Second, stronger inequity aversion makes an inefficient firm worse off, but may benefit an efficient firm. Third, we show that stronger inequity aversion by the consumer can actually lower the consumer’s monetary payoff (economic surplus) because the firm may reduce its quality to a greater extent than it reduces its price. Lastly, as the expected cost-efficiency in the market decreases, both the expected quality and the social welfare may increase rather than decrease.

Key words: behavioral economics, fairness, inequity aversion, asymmetric information, signaling, quality, pricing, search goods

* Xiaomeng Guo is a PhD student and Baojun Jiang is Assistant Professor of Marketing at Olin Business School at Washington University in St. Louis. Both authors contributed equally. The authors thank Vineet Kumar, Noah Lim, Elie Ofek, Subrata K. Sen, Jiwoong Shin, K. Sudhir, and participants at Summer Institute in Competitive Strategy (SICS) and the INFORMS Marketing Science Conference, and seminar participants at Johns Hopkins University, Yale University, University of Wisconsin-Madison, Carnegie Mellon University, Washington University in St. Louis, Fudan University, Shanghai University of Finance and Economics, Peking University, Nanjing University, Shanghai Jiaotong University, and Cheung Kong Graduate School of Business for their helpful comments.
1. Introduction

Consumers’ fairness concerns have been well documented in the marketing and economics literature (e.g., Rabin 1993, Xia et al. 2004, Anderson and Simester 2008, Ho and Su 2009, Li and Jain 2015). Empirical evidence and behavioral research suggest that the consumer tends to perceive a firm’s price as unfair when its profit margin from the transaction is too high relative to the consumer’s surplus. Bearden et al. (2003) show that the consumer perceived the fairness of a price offer to be significantly higher when the consumer was shown a high invoice amount (i.e., a high seller cost) than a low invoice amount. Thaler (1985), Kahneman et al. (1986) and Bolton et al. (2003) find that the consumers’ perception of price fairness is affected by their perception of the seller’s cost. Similarly, our survey of 170 students shows that even though they derive $100 of use benefits from a product priced at $95, their satisfaction with the purchase tends to be higher when they know that the firm has a higher unit cost. Our study also finds that, when the participants know that the firm’s unit cost is $20 or $30, more than 60% of them decide not to buy the product even though a purchase could give them a $5 economic surplus. The empirical, experimental, and anecdotal evidence all suggest that the consumer’s willingness-to-pay, i.e., her overall utility from a product, depends on not only her economic/monetary surplus but also the firm’s cost or profit margin.

Many firms made the mistake of neglecting the consumer’s fairness concerns in their pricing decisions. For example, in 2011, Netflix lost 800,000 members and much good will from customers when it unbundled its physical DVD rental and online movie-streaming operations and increased the subscription fees, without clearly communicating its high and rising costs. Its fee changes were widely viewed as a way to increase profits at the customers’ expense and hence triggered a large exodus of customers (Wingfield and Stelter 2011). Consumers may often be uncertain of the fairness of a firm’s price offer since they typically do not know the firm’s actual cost. For example, on Internet forums, some consumers discussed whether Google’s new Nexus 6 smartphone was fairly price based on the inferred costs and the quality of its parts. In a market with inequity-averse consumers, a high-cost firm needs to convince consumers that its prices are based on its high costs rather than unfairly high profit goals.

Consumer fairness concerns may also influence a firm’s quality decision. Note that even if a firm is de facto a monopoly, the consumer’s fairness concern can still be critical to the firm’s pricing and product decisions. For example, many popular, high-demand books are first released in a hardcover edition and several months later will the cheaper paperback edition come out. The popular Harry Potter novels, for instance, were typically released first in hardcover editions, and later in paperback editions at about half the price. One might argue that this is a traditional price discrimination story because some readers have strong preferences for the durability of the hardcover edition whereas others might not care as much. However,

1 See comments in the discussion forum following the article: http://www.extremetech.com/computing/192047-google-announces-the-nexus-6-a-phablet-monster-running-android-5-0-lollipop
this does not explain well why publishers do not release both editions at the same time and often delay the release of paperback editions by months. Our conversation with a consultant at a major publisher reveals that introduction of hardcover and paperback editions is for price discrimination reasons, but often not because readers love the hardcover itself, but because for popular genre of books (such as Harry Potter novels), some readers have strong desires to read the books right away. If a month later the publisher drops the price of the same book by half to target price-sensitive consumers, the earlier readers or fans will more likely be upset about the publisher’s “exploitation” of their strong preferences. Hence, by releasing the slightly different (lower quality) version at a later time, the publisher reduces the likelihood of early buyers feeling unfair about its large price drop. As Krugman (2000) also points out, dynamic pricing (of the same product) can feel unfair to consumers, and the later-released paperback is a little cheaper to produce but it is mainly to target price-sensitive customers after the impatient customers have been exploited. Similarly, online gaming firms often first release many in-game items at high prices and later introduce “lower quality” versions at lower prices. For example, Dota 2 usually releases its in-game “Genuine” weapons several months before the “Auspicious” versions, which have the same functions as “Genuine” versions (with only minor appearance and naming differences) but have much lower prices.

In addition, our experiment with 96 MBA students (acting as managers) shows that managers will choose different quality levels under different cost structures (between subjects). Under the same cost structure (within subjects), with consumers not knowing the firm’s cost, 75% of the managers choose a different quality level when consumers have fairness concerns than when they have no fairness concerns. The average quality chosen under a high-cost structure is significantly lower than under a low-cost structure. In short, our experiment shows that managers may strategically choose product quality when facing consumers with fairness concerns.

Our research centers on the question: given the well-established consumers’ fairness behaviors, how should a firm optimally and strategically choose its product quality and prices? We develop a game-theoretic model to study the effects of consumer fairness concerns on a firm’s pricing and quality decisions and on firm profitability, consumer surplus, and social welfare.

This paper contributes to two streams of literature. First, it contributes to the behavioral economics literature, which has incorporated human agents’ psychological and behavioral aspects into their economic

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2 Here is another well-known example showing that dynamic pricing can make consumers feel strongly unfair. Many customers voiced strong complaints about Apple reducing the price of iPhone by $200 two months after its release in 2007; Apple apologized in an open letter, and in an attempt to appease the anger offered $100 credit to customers who bought iPhone at the original price. http://www.nbcnews.com/id/20624042/#.VUUhYPTcx31

3 Consumers’ fairness behavior can also be interpreted as a result of an anticipated negotiation or bargaining situation. If the seller has a very high profit margin (or low cost), the consumer may be less likely to accept a high-price offer or less happy even when accepting the offer, for perhaps she has not negotiated a better deal or “fairer” price.
decision making. A comprehensive review of such literature is beyond the scope of this paper; please refer to Goldfarb et al. (2012) and Ho et al. (2006) for such reviews. We will discuss related works on consumers’ or firms’ social/psychological preferences. Fehr and Schmidt (1999) propose an inequity aversion model in which a player has a disutility of receiving a payoff that is different from those of the other players. Charness and Rabin (2002) extend the inequity aversion model to incorporate reciprocity in the utility function. Bolton and Ockenfels (2000) propose the Equity-Reciprocity-Competition model, in which each agent’s utility function depends on both her absolute payoff and her relative share of the total payoff. In addition, the consumer’s utility may depend on her social needs (e.g., Amaldoss and Jain 2005) or the context of her product choice set (Narasimhan and Turut 2013). Amaldoss and Jain (2008) show that the presence of reference group effects can lead to firms’ addition of costly features that provide consumers little or no functional benefit. Cui et al. (2007) find that fairness concerns in a conventional channel may help achieve better coordination for profit maximization in the channel. Ho and Zhang (2008) conduct experiments and use reference-dependent utility functions to examine the effects of the presentation of pricing contracts on the channel outcome. Ho and Su (2009) study peer-induced fairness when agents engage in social comparison and are averse to receiving less than their peers.

Our work closely relates to Guo (2015), which examines a firm pricing strategies when consumers have inequity aversion; but our model has a clear conceptual difference. Guo (2015) studies only pricing decisions and assumes that firms of different cost types (i.e., different cost efficiencies) must all produce the same product of identical quality (with the same consumer valuation). Hence, his model is in essence a model of a commodity product, whose quality and other valuation attributes are exactly the same for all firms regardless of their very different cost efficiencies. If the product is not a commodity, then it will be unreasonable to assume that firms of vastly different cost structures or efficiencies will all produce the same product, as done in Guo (2015). For a non-commodity product, the firm’s optimal choice of product quality depends on its cost structure and efficiency, regardless of whether there is consumer inequity aversion or information asymmetry (as our experiment shows). Thus, when consumers with fairness concerns do not know the firm’s cost, they should make rational inferences about the firm’s cost from its observed (quality and price) strategies, e.g., a firm with a very high cost for marginal quality will not make any positive profit producing a very high quality and hence should not be believed to produce such quality. In contrast with Guo (2015), which studies pricing of a commodity product, our paper examines a firm’s optimal product quality and pricing decisions of non-commodity product. We explicitly allow for and examine the firm’s incentive to produce different quality levels that reflect its own cost and profit incentives, from which consumers will make rational inferences about fairness. From a theoretical point of view, our model entails a two-dimensional signal for the firm’s cost—the firm’s quality and price together rather than the firm’s price alone can signal the firm’s cost. Our richer, more general framework, which incorporates the firm’s
product quality decision, yields many interesting substantive results that cannot be obtained from the Guo’s framework.

Our research also contributes to the signaling literature (e.g., Desai and Srinivasan 1995, Moorthy and Srinivasan 1995, Simester 1995, Desai 2000, Balachander 2001, Soberman 2003, Shin 2005, and Jiang et al. 2011). Jiang et al. (2014) examine a two-dimensional asymmetric information game involving credence goods, for which consumers may not know the quality or level of service even after the service has been performed. Jiang and Yang (2015) study two-dimensional asymmetric information involving experience goods when early consumers observe neither the firm’s quality choice nor its cost, but they can learn the product quality after purchase and will share the quality information with later consumers. In contrast, we focus on search goods—whose features, characteristics or quality levels are readily evaluated by the consumer before purchase—or those experience goods with publicly available user generated content (e.g., consumer reviews) that reveals the product quality. Thus, the firm’s quality and price serve as a two-dimensional signal for its cost type, which influences the inequity-averse consumer’s willingness-to-pay.

We highlight four interesting findings. First, the firm’s optimal quality is non-monotone with respect to the consumer’s inequity aversion, which contradicts the intuition that stronger inequity aversion reduces the consumer’s willingness-to-pay and will hence reduce the firm’s equilibrium quality. For any given quality, inequity-averse consumers are more willing to accept a high price if they believe the firm’s cost is higher. So, stronger inequity aversion gives the cost-efficient firm more incentives to mimic the inefficient firm, and may make it too costly for the inefficient firm to credibly signal its type. This can lead to a pooling outcome, in which the inefficient firm raises its quality and the efficient firm reduces its quality to the same level, resulting in higher expected quality in the market. This contrasts with the case when the firm’s cost is observed, where the firm chooses a quality level independent of the consumer’s inequity aversion but adjusts its price to accommodate any change in inequity aversion.

Second, stronger inequity aversion reduces an inefficient firm’s profit but can benefit an efficient firm. When the consumer’s inequity aversion increases, the high signaling cost may induce the inefficient firm to give up trying to separate from the efficient firm, leading to a pooling outcome, where the efficient firm will reduce its quality to the inefficient firm’s pooling quality. In essence, the efficient firm can benefit from the consumer’s higher willingness to pay for quality for an average firm than for itself; as the inequity aversion gets stronger, that benefit can become large enough for the efficient firm to forgo its first-best quality and pricing to pool with the inefficient firm.

Third, though stronger inequity aversion gives consumers a higher monetary payoff when the firm’s cost is common knowledge, it may actually lower their monetary payoff when the firm’s cost is not observed. This is because the firm may reduce its quality to a greater extent than it reduces its price, resulting in a lower monetary payoff especially for the consumers with high valuations for quality. This result suggests
that, because of the firm’s strategic quality decision, consumers may not want to pretend to have stronger fairness concerns. Fourth, interestingly, because of the information asymmetry and the efficient firm’s incentive to mimic the inefficient firm, both the expected quality and the social welfare may *increase* when the expected cost-efficiency in the market decreases.

2. Model

Consider a monopoly market in which a profit-maximizing firm sells a product to consumers, each of whom demands at most one unit of the product. The firm charges a take-it-or-leave-it price \( p \) for each unit of the product. A consumer’s monetary or material payoff from purchasing the product is \( \theta q - p \), where \( q \) is the quality of the product and \( \theta \) represents the consumer’s willingness-to-pay for quality.\(^4\) Consumers are heterogeneous in \( \theta \). A fraction \( \lambda \) of consumers are \( H \)-type with \( \theta = \theta_H \) and a fraction \( 1 - \lambda \) of consumers are \( L \)-type with \( \theta = \theta_L \), where \( \theta_H > \theta_L > 0 \).\(^5\) Let \( M_t(q,p) = \theta_t q - p \) denote a \( t \)-type consumer’s monetary payoff from purchasing the product, where \( t \in \{H,L\} \). Without loss of generality, we normalize the total number of consumers to one. The firm has two types with respect to its cost efficiency—an efficient type (\( E \)-type) and an inefficient type (\( I \)-type). The \( j \)-type firm has a constant unit production cost of \( c_j(q) = k_j q^2 \), where \( j \in \{I,E\} \) and \( k_j \) represents the firm’s cost efficiency and \( k_I > k_E > 0 \). The firm is an \( I \)-type with probability \( \gamma \) and an \( E \)-type with probability \( 1 - \gamma \). The \( j \)-type firm’s profit margin is \( p - c_j(q) \) for each unit sold. For the base model, the firm’s fixed cost is assumed sunk or zero; the firm’s profit is thus \( \pi_j(p,q) = m[p - c_j(q)] \), where \( m \) is the market demand.

The consumer may care about not only her monetary payoff but also the “fairness” of the firm’s offer. Much research in behavioral and experimental economics shows that consumers’ utility is affected by their relative rather than absolute levels of performance or payoffs. Our survey results also suggest that, if the firm’s profit margin is too high relative to the consumer’s surplus, the consumer may consider it “unfair” and feel that the firm is extracting too much surplus from her. That is, all else being equal, a consumer with such fairness concerns has a lower utility than otherwise. In other words, the consumer’s overall utility consists of two parts—her monetary payoff and the potential disutility from her fairness concerns. We model this as follows. The \( t \)-type consumer, if knowing the firm’s true cost (i.e., \( k_j \)), derives an overall utility of

\[
U_t(q,p,k_j) = \theta_t q - p - \alpha \cdot \max\{(p - k_j q^2) - (\theta_t q - p), 0\},
\]

\( \alpha \) is a parameter representing the relative importance of fairness concerns.

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\(^4\) In this paper, we distinguish a consumer’s monetary or material payoff from her overall utility, which may include some disutility because of fairness concerns about the firm’s offer.

\(^5\) Our main results remain qualitatively the same even when \( \theta \) is continuously distributed.
where $\alpha \in [0,1]$ represents the strength of the consumer’s fairness concerns (i.e., the degree of her inequity aversion). This utility specification captures the consumer’s disutility resulting from her perception of “unfairness” when the firm’s profit margin is higher than her own monetary payoff. The higher this difference, the higher the consumer’s disutility from inequity aversion. In our base model, all consumers have the same degree of inequity aversion ($\alpha$). We extend the base model to examine the effect of heterogeneity in consumer inequity aversion in Section 4.

This paper focuses on search goods—whose features, characteristics or quality levels are readily evaluated by the consumer before purchase—or those experience goods with publicly available reviews that reveal their true quality. Though consumers observe the firm’s product quality and price, they may not directly observe the firm’s cost or cost efficiency. This reflects the consumer’s limited knowledge in reality about the firm’s actual production cost. The consumer knows only the prior distribution of the firm’s cost type, i.e., $k = k_I$ with probability $\gamma$ and $k = k_E$ with probability $1 - \gamma$. Note that $\alpha = 0$ corresponds to the standard economics model where the consumer has no fairness concerns and makes purchase decisions based only on the firm’s price and quality, not on its cost.

More formally, the game proceeds as follows. Nature decides the firm’s type ($j$); with probability $\gamma$, the firm is $I$-type and with probability $1 - \gamma$ it is $E$-type. The firm privately observes its own type whereas the consumer knows only the prior distribution. The firm chooses its product quality $q_I$ and price $p_I$. After observing the firm’s quality and price, consumers update their belief about the firm’s type and decide, based on their posterior expected utility, whether to buy the product. We normalize the consumer’s utility from the outside option to zero, thus a consumer will buy the product if she derives a positive expected utility from the purchase. Without loss of generality, we use the following tie-breaking rule: if a consumer is indifferent between buying and not buying the product, she will buy; if the firm is indifferent between targeting only $H$-type consumers and targeting both types, it will target only $H$-type. Finally, we assume that the firm cannot price discriminate based on the consumer’s type; in other words, the firm knows the distribution of consumer types but does not directly observe each individual consumer’s willingness-to-pay for quality. For convenience, Table 1 provides a list of key notations used in this paper.

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6 Fehr and Schmidt (1999) use a similar utility function but with both advantageous and disadvantageous inequity aversion. For succinctness, we assume that the parameter for advantageous inequity aversion is zero; the equilibrium outcomes are actually not affected even if we include the advantageous inequity aversion. Experimental research shows that indeed a buyer perceives the unfairness to be much less severe when the inequity is biased to the buyer’s advantage than when it is to the buyer’s disadvantage (see Ho and Su 2009).
TABLE 1  Key Notations

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
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<tbody>
<tr>
<td>$t$</td>
<td>The consumer’s type, $t \in {H,L}$.</td>
</tr>
<tr>
<td>$j$</td>
<td>The firm’s type, $j \in J \equiv {I,E}$.</td>
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<tr>
<td>$\theta_t$</td>
<td>The $t$-type consumer’s willingness-to-pay for quality, where $\theta_H &gt; \theta_L$.</td>
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<tr>
<td>$M_t$</td>
<td>The $t$-type consumer’s monetary payoff, where $t \in {H,L}$.</td>
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<tr>
<td>$\lambda$</td>
<td>The fraction of the $H$-type consumers.</td>
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<tr>
<td>$k_j$</td>
<td>The $j$-type firm’s cost efficiency, where $j \in {I,E}$ and $k_I &gt; k_E$.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The prior probability that the firm is $I$-type.</td>
</tr>
<tr>
<td>$p$</td>
<td>The price of the product.</td>
</tr>
<tr>
<td>$q$</td>
<td>The quality of the product.</td>
</tr>
<tr>
<td>$c_j(q)$</td>
<td>The unit variable production cost of the type-$j$ firm, $c_j(q) = k_j q^2$.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The degree of the consumer’s inequity aversion, $\alpha \geq 0$.</td>
</tr>
<tr>
<td>$U_t$</td>
<td>The $t$-type consumer’s utility, where $t \in {H,L}$.</td>
</tr>
<tr>
<td>$\pi_j$</td>
<td>The $j$-type firm’s profit.</td>
</tr>
<tr>
<td>$\mu(I</td>
<td>q,p)$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>The fraction of consumers with fairness concerns.</td>
</tr>
<tr>
<td>$\overline{\overline{\text{sym}}}$</td>
<td>The upper bar indicates the symmetric information outcome.</td>
</tr>
<tr>
<td>“sep”</td>
<td>This subscript indicates the separating outcome.</td>
</tr>
<tr>
<td>“pool”</td>
<td>This subscript indicates the pooling outcome.</td>
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3. Analysis

3.1. Symmetric Information

We first analyze the symmetric-information benchmark, in which consumers observe the firm’s cost (or equivalently, its efficiency). We use “$\overline{\overline{\text{sym}}}$” over a variable to indicate the case of symmetric information. Proposition 1 shows that when the consumer observes the firm’s cost type, stronger inequity aversion will not affect the firm’s optimal quality decision, but will lead to a lower price. When consumers know the firm’s true cost, it is more efficient for the firm to respond to the consumer’s inequity aversion by lowering its price rather than changing its quality from the optimal first-best level. We will show later that, when consumers do not observe the firm’s cost, the firm’s quality may serve as a signal for its cost, and hence can depend on the consumer’s inequity aversion. Proposition 1 also finds that, as we may intuit, stronger inequity aversion leads to a higher monetary surplus for consumers since the same product quality is offered at a lower price; the firm clearly becomes worse off as the consumer inequity aversion gets stronger.

PROPOSITION 1. If the firm’s type $j \in \{I,E\}$ is common knowledge,

(a) its optimal quality $\bar{q}_j^*$, price $\bar{p}_j^*(\bar{q}_j^*)$ and the corresponding profit $\bar{\pi}_j^*$ are
\[
\tilde{q}_j^* = \begin{cases} 
\frac{\theta_H}{2k_j}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
\frac{\theta_L}{2k_j}, & \text{otherwise }
\end{cases}, \quad \tilde{p}_j^*(\tilde{q}_j^*) = \begin{cases} 
\frac{(1+\alpha)\theta_H\tilde{q}_j^* + \alpha k_j\tilde{q}_j^2}{1+2\alpha}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2} \\
\frac{(1+\alpha)\theta_L\tilde{q}_j^* + \alpha k_j\tilde{q}_j^2}{1+2\alpha}, & \text{otherwise }
\end{cases}
\]

and \( \bar{\pi}_j^* = \begin{cases} 
\frac{\lambda \theta_H^2 (1+\alpha)}{4k_j(1+2\alpha)}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2} \\
\frac{\theta_L^2 (1+\alpha)}{4k_j(1+2\alpha)}, & \text{otherwise }
\end{cases} \).

(b) \( \tilde{p}_j^*(\tilde{q}_j^*) \) and \( \bar{\pi}_j^* \) both decrease in \( \alpha \), while \( \tilde{q}_j^* \) is independent of \( \alpha \);

(c) the consumer’s monetary payoff \( M_t(\tilde{q}_j^*, \tilde{p}_j^*) \) increases in \( \alpha \), where \( t \in \{H, L\} \).

3.2. Asymmetric Information

In practice, consumers usually do not have perfect knowledge about a firm’s cost. How does such information asymmetry affect the market outcome? Now we analyze our focal case in which consumers do not directly observe the firm’s cost efficiency \( k_j \).

If consumers observe the firm’s type, one can easily see from (1) that for any given quality, the consumer’s utility is higher when she knows the firm’s cost is higher. Thus, if consumers do not observe the firm’s cost, the \( E \)-type firm has an incentive to mimic the \( I \)-type and the \( I \)-type has an incentive to reveal its true type. Upon observing the firm’s quality \( r \) and price \( p \), consumers will update their belief about the firm’s cost type. Let \( \mu(I|q, p) \) denote the consumer’s posterior belief about the firm being \( I \)-type. A \( t \)-type consumer’s expected utility, where \( t \in \{H, L\} \), is then computed as

\[
EU_t(q, p) = \mu(I|q, p)U_t(q, p, k_I) + (1 - \mu(I|q, p))U_t(q, p, k_E),
\]

(2)

3.2.1. Equilibrium Refinement

In the following sections, we analyze the Perfect Bayesian Equilibria (PBE) in the asymmetric case. There are two types of Perfect Bayesian Equilibria (PBE): separating equilibrium and pooling equilibrium. At a separating equilibrium, the two types of firms choose different pair of qualities and prices, from which consumers can infer the firm’s type. While at a pooling equilibrium, both types of firm choose the same quality and price and hence the consumer’s posterior belief is the same as her prior. There is a continuum of PBE depending on the consumer’s off-equilibrium-path beliefs about the firm’s type. We apply the Undefeated Equilibrium refinement tool that is formalized by Mailath et al. (1993) to select the reasonable equilibrium.\(^7\)

Before providing the formal definition of the undefeated equilibrium in our setting, we first introduce some notations. A firm’s pure strategy is denoted by \( \omega : J \to S \), where \( J = \{I, E\} \) is the set of the firm’s type and \( S = \{(q, p) : q \geq 0, p \geq 0\} \) is the set of the firm’s strategy. Consumers’ pure strategy is denoted by

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\(^7\) Another widely used refinement tool is the intuitive criterion (Cho and Kreps 1987). However, many economists have pointed out the limitations of the intuitive criterion. For example, in some situations (as is the case in our setting), the intuitive criterion eliminates plausible pooling equilibria and selects a separating outcome supported by “unreasonable” off-equilibrium beliefs. Please see Mailath et al. (1993) for detailed discussion and examples.
\( \varphi: M \to R \), where \( R = \{ \{ \text{Only } H\text{-type buys the product}\}, \{ \text{Only } L\text{-type buys the product}\}, \{ \text{Both types buy the product}\}, \{ \text{No type buys the product}\} \) is the consumers’ strategy set. Let

\[
BR((q, p), \mu(l|q, p)) = \arg\max_{\tau \in \{H, L\}} \left[ \mu(l|q, p)U_t(q, p, k_t) + (1 - \mu(l|q, p))U_t(q, p, k_E) \right]
\]

be the consumers’ best response to strategy \((q, p)\) given their belief \(\mu(l|q, p)\). An equilibrium is defined as \(\sigma = (\omega, \varphi, \mu)\). Let \(\pi_j(\sigma)\) denote the \(j\)-type firm’s profit at equilibrium \(\sigma\). The definition of the undefeated equilibrium applied to our model setting is provided below.

**DEFINITION 1.** A pure-strategy sequential equilibrium \(\sigma = (\omega, \varphi, \mu)\) defeats another pure-strategy sequential equilibrium \(\sigma' = (\omega', \varphi', \mu')\) if \(\exists (q, p) \in S\) such that:

1. For all \(j \in J\): \(\omega'(l) \neq (q, p)\), and \(K \equiv \{ j \in J | \omega(j) = (q, p) \} \neq \emptyset\);
2. For all \(j \in K\): \(\pi_j(\sigma) \geq \pi_j(\sigma')\), and there exists \(j \in K\): \(\pi_j(\sigma) > \pi_j(\sigma')\);
3. For any \(j \in K\): \(\mu'(l|q, p) \neq \frac{\gamma \delta(l)}{\gamma \delta(l) + (1 - \gamma) \delta(E)}\) for any \(\delta(j): J \to [0, 1]\) satisfying
   a. \(\delta(j) = 1\) if \(j \in K\) and \(\pi_j(\sigma) > \pi_j(\sigma')\), and
   b. \(\delta(j) = 0\) if \(j \notin K\).

**DEFINITION 2.** A pure-strategy sequential equilibrium \(\sigma = (\omega, \varphi, \mu)\) is undefeated if there does not exist another pure-strategy sequential equilibrium \(\sigma'\) that defeats \(\sigma\).

Mailath et al. (1993) formalize two related refinement concepts—the lexicographically maximum sequential equilibrium (LMSE) and the undefeated equilibrium. The undefeated equilibrium refinement in our setting is equivalent to LMSE and essentially selects the most profitable equilibrium outcome among all equilibrium outcomes from the perspective of the firm that wants its type revealed (i.e., the \(I\)-type). In the Technical Appendix, we show that the only separating equilibrium outcome that could be undefeated is the least-cost separating outcome (from the \(I\)-type firm’s perspective) and that the only pooling equilibrium outcome that could be undefeated must be the \(I\)-type’s most profitable pooling among all pooling equilibria. We show that the least-cost separating equilibrium defeats the most-efficient pooling equilibrium only when it provides a higher profit for the \(I\)-type firm than the most-efficient pooling equilibrium (i.e., when the probability of the firm being \(I\)-type is below some threshold \(\gamma^*\)), otherwise the most-efficient pooling defeats the least-cost separating. We prove that the least-cost separating equilibrium exists and is the unique undefeated outcome when \(\gamma < \gamma^*\), and that the most-efficient pooling equilibrium exists and is the unique undefeated outcome when \(\gamma > \gamma^*\).
After applying the undefeated equilibrium refinement, we can select the unique “reasonable” equilibrium for any given parameters. We report the equilibrium outcome in Proposition 2.

**PROPOSITION 2.** There exists a unique \( \gamma^* \in [0,1] \) such that

(a) if \( \gamma < \gamma^* \), the separating equilibrium \( (q_{j, \text{sep}}, p_{j, \text{sep}}, \pi_{j, \text{sep}}) \) is the unique undefeated equilibrium outcome, where \( j \in \{I, E\} \), \( q_{E, \text{sep}} = \bar{q}_E \), \( p_{E, \text{sep}} = \bar{p}_E \), \( q_{I, \text{sep}} = \begin{cases} \bar{q}_I, & \text{if } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \\ \bar{q}, & \text{otherwise} \end{cases} \), \( p_{I, \text{sep}} = \begin{cases} \bar{p}_I, & \text{if } \lambda \geq \frac{\theta_I^2}{\theta_H^2}, \text{and } Q_{\text{sep}} = \frac{k_E(1+\alpha) - \sqrt{(1+\alpha)k_E(k_I-k_E)}}{2k_E[(1+2\alpha)k_E-ak_I]} \\ \bar{p}_I, & \text{otherwise} \end{cases} \).

(b) if \( \gamma > \gamma^* \), the pooling equilibrium \( (q_{\text{pool}}, p_{\text{pool}}, \pi_{j, \text{pool}}) \) is the unique undefeated equilibrium outcome, where \( j \in \{I, E\} \), \( q_{\text{pool}} = \begin{cases} Q_{\text{pool}} \theta_H, & \text{if } \lambda \geq \frac{\theta_I^2}{\theta_H^2}, \text{and } Q_{\text{sep}} = \frac{(1+\alpha)^2[(2+4\alpha-\gamma\theta)k_I-\alpha(1-\gamma)k_E]}{4(1+2\alpha)[(1+2\alpha-\gamma)k_I-\alpha(1-\gamma)k_E]^2} \\ Q_{\text{pool}} \theta_L, & \text{if } \lambda < \frac{\theta_I^2}{\theta_H^2}, \text{and } Q_{\text{pool}} = \frac{(1+\alpha)^2[(2+4\alpha-\gamma\theta)k_I-\alpha(1-\gamma)k_E]}{4(1+2\alpha)[(1+2\alpha-\gamma)k_I-\alpha(1-\gamma)k_E]^2} \end{cases} \).

*Figure 1 Equilibrium Outcomes*

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Figure 1 depicts the equilibrium market outcome in different \( \left( \frac{k_E}{k_I}, \gamma \right) \) parameter regions. If the \( E \)-type firm is much more efficient than the \( I \)-type firm (i.e., \( \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \)), the \( E \)-type’s opportunity cost to mimic the \( I \)-type will be too high. That is, it is much more profitable for the \( E \)-type to produce a high-quality product and deal with the consumer’s inequity aversion through pricing than to mimic the \( I \)-type’s low first-best quality \( (\bar{q}_I^*) \). Under this condition, the \( I \)-type firm is able to separate itself from the \( E \)-type by choosing \( \bar{q}_I^* \) and \( \bar{p}_I^* \) just as under symmetric information; thus, we call this type of equilibrium “costless separating.” Note that when \( \frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha} \), the unique undefeated equilibrium is the costless-separating for any \( \gamma \in [0,1] \).
In contrast, if the cost efficiencies of the two types of firms are relatively close (i.e., \( \frac{k_E}{k_I} \geq \frac{1+\alpha}{1+2\alpha} \)), the \( I \)-type firm can either incur a signaling cost to credibly separate itself from the \( E \)-type firm (we call such an equilibrium “costly separating”) or pool with the \( E \)-type. Note that as \( \frac{k_E}{k_I} \) increases or equivalently as \( \gamma \) increases, the \( I \)-type firm will find the pooling equilibrium (in which the consumer’s utility and hence purchase decisions will be based on the expected cost efficiency \( \gamma k_I + (1-\gamma)k_E \)) increasingly more appealing than the separating equilibrium since the expected cost efficiency becomes closer to the \( I \)-type’s own efficiency \( k_I \). As we see in Figure 1, when \( \gamma > \gamma^* \), the equilibrium market outcome is pooling. When \( \gamma < \gamma^* \), the \( I \)-type firm will separate itself from the \( E \)-type by choosing lower-than-first-best quality (\( q_{I,sep}^* < \hat{q}_I^* \)) and a correspondingly lower price (\( p_{I,sep}^* < \tilde{p}_I^* \)) to ensure that the \( E \)-type firm will find it unprofitable to mimic, giving rise to a costly-separating equilibrium.

We can show that \( \gamma^* \) is strictly decreasing in consumers’ inequity aversion \( \alpha \) when \( \frac{k_E}{k_I} \geq \frac{1+\alpha}{1+2\alpha} \). Therefore, Proposition 2 can also be stated in terms of the consumer’s inequity aversion, i.e., there exists \( \alpha^* \) such that if \( \alpha > \alpha^* \), the equilibrium market outcome is pooling, and otherwise separating.\(^8\) As \( \alpha \) increases, the pooling equilibrium outcome becomes more likely and the costless-separating outcome less likely. This is intuitive since as consumers become more inequity-averse, the \( E \)-type firm has an increased incentive to mimic the \( I \)-type firm, to which the consumer has a higher willingness-to-pay for any given quality level, and thus leads to higher separating cost for the \( I \)-type.

Next we compare the equilibrium outcome in the asymmetric information with the symmetric information and report the result in Corollary 1.

**Corollary 1.** In the costless-separating parameter region (i.e., \( \frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha} \)), \( q_{I,sep}^* = \hat{q}_I^* \), \( p_{I,sep}^* = \hat{p}_I^* \) and \( \pi_{I,sep}^* = \hat{\pi}_I^* \); in the costly-separating parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma < \gamma^* \)), \( q_{I,sep}^* < \tilde{q}_I^* \), \( p_{I,sep}^* < \hat{p}_I^* \) and \( \pi_{I,sep}^* < \hat{\pi}_I^* \); in the pooling parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma > \gamma^* \)), \( q_{pool}^* < \hat{q}_E^* \), \( p_{pool}^* < \hat{p}_E^* \), \( \pi_{pool}^* < \hat{\pi}_I^* \) and \( \pi_{E,pool}^* > \hat{\pi}_E^* \).

Note that at the (both costly and costless) separating parameter region, the \( E \)-type’s equilibrium quality, price and profit are the same as in the symmetric-information case (as described in Proposition 1), i.e., \( q_{E,sep}^* = \hat{q}_E^* \), \( p_{E,sep}^* = \hat{p}_E^* \) and \( \pi_{E,sep}^* = \hat{\pi}_E^* \).

In the “costly separating” equilibrium region, the \( I \)-type firm achieves least-cost separation by choosing lower-than-first-best quality (i.e., \( q_{I,sep}^* < \hat{q}_I^* \)) and a correspondingly lower price (i.e., \( p_{I,sep}^* < \tilde{p}_I^* \)) to

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\(^8\) The formal proof is given in the proof of Lemma 1(c) (in the online Technical Appendix).
ensure that the $E$-type firm finds it unprofitable to mimic.\footnote{When $\gamma < \gamma^*$, the unique undefeated equilibrium is the least-cost separating equilibrium and one can show that this equilibrium also survives the intuitive criterion (the formal proof is available from the authors upon request).} In equilibrium, though price and quality as a pair form a signal, the firm’s quality choice plays the more critical signaling role in the sense that the $I$-type firm distorts its quality downward from its first-best quality but its price is actually optimal (i.e., first-best) conditional on that quality, i.e., $p^*_{I,\text{sep}} = \bar{p}^*_I(q^*_{I,\text{sep}})$. Though a large price reduction alone can also prevent mimicry from the $E$-type firm, it is too costly for the $I$-type firm in our case since its profit margin is already low. Note that the same amount of downward distortion in quality (from the $I$-type firm’s first-best quality) will reduce the $I$-type’s profit to a much lesser extent than it will reduce the $E$-type’s profit. That is, reducing quality is less costly for the $I$-type firm than for the $E$-type firm and hence quality distortion can be a more effective way than price distortion for the $I$-type firm to keep the $E$-type firm from mimicking.

In the pooling parameter region, the unique undefeated equilibrium outcome is the most-efficient (for the $I$-type) pooling equilibrium, and both the $I$-type and the $E$-type firm strictly prefer the pooling outcome to any separating outcome. The pooling-equilibrium quality is lower than any type of firm’s optimal quality under symmetric information (i.e., $q^*_{pooled} < \bar{q}^*_I < \bar{q}^*_E$). Intuitively, this is because for the $I$-type firm, consumers’ willingness-to-pay is lower when they are uncertain about the firm’s type than when they know the firm is $I$-type, and hence, the $I$-type firm has less incentive to invest in quality when its type is not observed. The pooling price is also lower than the $I$-type firm’s first-best price conditional on the pooling quality, i.e., $p^*_{pooled} < \bar{p}^*_I(q^*_{pooled}) < \bar{p}^*_E$. The downward price distortion in the pooling equilibrium is due to both the consumer’s lower willingness-to-pay and the firm’s lower quality.

Note that the inequity-averse consumers’ willingness-to-pay for any given quality is increasing in the firm’s cost. Does the firm therefore want to intentionally increase its cost through wasteful use of resources? The answer is no. Although with fairness concerns the consumer’s willingness-to-pay increases when the expected cost of production increases, it does not imply that a firm has any incentive to intentionally decrease its cost efficiency (by, as an extreme example, intentionally and publicly destroying one unit of the product for every $n$ units it produces). In fact, in both the separating and pooling outcomes the firm of either type will make a strictly lower profit if its efficiency decreases. That is, even if consumers with fairness concerns do not resent the firm’s intentional wasteful use of resources, no firm can profit by raising its cost while keeping its quality unchanged. We can also show that in both separating and pooling equilibria, the $E$-type firm makes a higher profit than the $I$-type firm—the $E$-type firm has an incentive to mimic the $I$-type firm’s price and quality strategy to benefit from the consumers’ higher willingness-to-pay, but it never wants to raise its actual cost. In other words, there is no “efficiency curse”; other things being equal, the more efficient a firm is (i.e., the smaller its cost coefficient), the higher its profit. These results are
summarized in Proposition 3. One can easily show that these results also hold under the symmetric-information case. Since no firm wants to intentionally raise its cost through wasteful use of resources even under the most favorable assumption about consumers’ attitudes about fairness, even less will any firm want to waste resources to raise costs if consumers have some resentment about a firm’s intentional waste.

PROPOSITION 3. (a) The j-type firm’s profit is always decreasing in $k_j$, i.e., $\frac{d\pi^*_j}{dk_j} < 0$ and $\frac{d\pi^*_{j,\text{pool}}}{dk_j} < 0$, for $j \in \{I, E\}$; (b) the E-type firm’s profit is always higher than the I-type firm, i.e., $\pi^*_E > \pi^*_{I,\text{sep}}$ when $\gamma < \gamma^*$ and $\pi^*_{E,\text{pool}} > \pi^*_{I,\text{pool}}$ when $\gamma > \gamma^*$.

3.2.3. Effects of Consumer Inequity Aversion and Cost-Information Asymmetry

In this section, we examine the effects of consumer’s inequity aversion level ($\alpha$) and the probability of firm being inefficient ($\gamma$) on the equilibrium outcome. First, we analyze how the quality and price will change as $\alpha$ or $\gamma$ changes.

LEMMA 1. (a) In the costly-separating parameter region (i.e., $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma < \gamma^*$), the I-type firm’s quality $q^*_{i,\text{sep}}$ and price $p^*_{i,\text{sep}}$ decrease in $\alpha$ and are independent of $\gamma$; (b) in the pooling parameter region (i.e., $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma > \gamma^*$), the equilibrium quality $q^*_{\text{pool}}$ and price $p^*_{\text{pool}}$ decrease in $\alpha$ and increase in $\gamma$; (c) when $\gamma = \gamma^*$ or $\alpha = \alpha^*$, $q^*_{i,\text{sep}} < q^*_{\text{pool}} < q^*_{i,\text{sep}}$ and $q^*_{\text{pool}} > q^*_{\text{sep}} + (1-\gamma)q^*_{i,\text{sep}}$.

At the costly- or costless-separating equilibrium, the E-type firm’s quality ($q^*_{E,\text{sep}}$), price ($p^*_{E,\text{sep}}$) and profit ($\pi^*_{E,\text{sep}}$) are the same as in the symmetric-information case (as described in Proposition 1), i.e., $q^*_{E,\text{sep}} = \bar{q}^*_E$, $p^*_{E,\text{sep}} = \bar{p}^*_E$ and $\pi^*_{E,\text{sep}} = \pi^*_E$. Thus the E-type firm’s price and profit decrease in $\alpha$ and its quality is independent of $\alpha$. Note that in the costless-separating parameter region (i.e., $\frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha}$), the I-type firm’s quality, price and profit are also the same as in the symmetric-information case. By contrast, in costly-separating parameter region (i.e., $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma < \gamma^*$), as Lemma 1 shows, the I-type firm’s separating-equilibrium quality, price and profit all decrease in $\alpha$. As $\alpha$ increases, the E-type firm’s incentive to mimic the I-type increases, thus the I-type has to incur a higher signaling cost by lowering its quality (and price) even further to ensure that the E-type’s mimicking is unprofitable. Moreover, as expected, neither firm’s separating-equilibrium strategy depends on the probability of the firm being inefficient ($\gamma$).

In the pooling parameter region, both equilibrium quality and price will decrease as $\alpha$ increases. This intuition lies mainly in the fact that the I-type firm’s quality choice dictates the pooling-equilibrium quality since it is the I-type that has an incentive to reveal its identity. The I-type firm’s optimal quality under pooling is based on the consumer’s expectation of both firms’ costs, which is lower than the I-type’s true
cost. As $\alpha$ increases, the consumer’s willingness-to-pay for any given quality by the average type of firm decreases, so the $I$-type firm’s incentive to invest on product quality also decreases. Thus, as $\alpha$ increases, the pooling-equilibrium quality and price will both decrease.

Further, as $\gamma$ increases in the pooling parameter region (i.e., $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma > \gamma^*$), for any given quality the consumer’s willingness-to-pay to the expected type of the firm increases, although her willingness-to-pay to each specific type of firm does not change (assuming the firm’s type is known). Thus, as $\gamma$ increases, the $I$-type firm has more incentive to produce higher quality and also charge a higher price to achieve optimal profits. Since the pooling-equilibrium quality is dictated by the $I$-type firm’s quality choice, the pooling-equilibrium quality increases in $\gamma$.

Figure 2  Equilibrium Quality under Asymmetric Information

Note. The equilibrium outcome is separating if $\gamma < \gamma^*$ or $\alpha < \alpha^*$, and pooling otherwise.

Figure 2(a) illustrates the case of $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and Figure 2(b) illustrates the case of $k_E > \frac{k_I}{2}$.

A higher $\gamma$ implies that the “average” firm in the market is less cost-efficient at producing any given quality. Thus we say a market with a larger $\gamma$ is a less cost-efficient market since the expected cost is higher for any given quality. The ex-ante expected quality in the market is defined as $\gamma q_I + (1 - \gamma)q_E$. One may intuit that, since the expected cost efficiency in the market decreases as $\gamma$ increases, the expected quality in the market should decrease. In the separating regime ($\gamma < \gamma^*$), as Lemma 1(a) shows, a lower expected cost efficiency in the market (i.e., a larger $\gamma$) will lead to a lower expected quality. However, in the pooling parameter region ($\gamma > \gamma^*$), the expected quality (i.e., $q_{pool}$) increases in $\gamma$ as shown in Lemma 1(b). In addition, as illustrated in Lemma 1(c), there exists a discrete jump in the expected quality at the transition point $\gamma = \gamma^*$ or $\alpha = \alpha^*$, where the equilibrium regime switches from separating to pooling (see Figure 2). This jump occurs because the $I$-type firm no longer needs to distort its quality downward to separate when
\( \gamma \) increases from below \( \gamma^* \) to above \( \gamma^* \), or equivalently when \( \alpha \) increases from below \( \alpha^* \) to above \( \alpha^* \). Proposition 4 shows the counter-intuitive result that the expected quality may be higher when \( \gamma \) is higher (i.e., in a less cost-efficient market) or when \( \alpha \) is higher.

PROPOSITION 4. (a) The ex-ante expected quality is decreasing in \( \gamma \) when \( \gamma < \gamma^* \) and increasing in \( \gamma \) otherwise. (b) The ex-ante expected quality may be higher when consumers have stronger inequity aversion.

Next we analyze how the firm’s profit is affected by \( \gamma \) and \( \alpha \). Neither type of firm’s profit is affected by \( \gamma \) in the separating parameter region (\( \gamma < \gamma^* \)). Lemma 2 shows that, in the pooling region (\( \gamma > \gamma^* \)), both types of firms benefit from a higher \( \gamma \) since the consumer’s willingness-to-pay for quality increases leading to a higher optimal quality by the \( I \)-type firm, whom the \( E \)-type firm will mimic with less quality distortion from its first-best quality. By contrast, in both parameter regions, the profit for both types of firms decreases in \( \alpha \) since the consumer’s overall willingness-to-pay is lower as \( \alpha \) increases.

LEMMA 2. (a) In the costly-separating parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma < \gamma^* \)), the \( I \)-type firm’s profit \( \pi^*_{I,sep} \) decreases in \( \alpha \) and is independent of \( \gamma \); (b) in the pool parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma > \gamma^* \)), the \( j \)-type firm’s profit \( \pi^*_{j,pool} \) decreases in \( \alpha \) and increases in \( \gamma \), for \( j \in \{I, E\} \); (c) when \( \gamma = \gamma^* \) or \( \alpha = \alpha^* \), \( \pi^*_{I,sep} = \pi^*_{I,pool} \) and \( \pi^*_{E,sep} < \pi^*_{E,pool} \).

![Figure 3 Firm’s Profit](image)

| Note. | The equilibrium outcome is costless-separating if \( 0 \leq \alpha < \frac{k_I-k_E}{2k_E-k_I} \), costly-separating if \( \alpha < \alpha^* \), and pooling otherwise. Figure 3 illustrates the case of \( \frac{k_E}{k_I} > \frac{k_I}{2} \). |

One may intuit that the firm’s profit should always decrease in the consumer’s inequity aversion, where all types of the firm have the same exogenous quality. However, in our setting with the firm making strategic quality decisions, Lemma 2 (c) shows that at the transition point \( \alpha = \alpha^* \), there exists a discrete jump for the \( E \)-type firm’s profit. When the level of inequity aversion increases from just below \( \alpha^* \) to above \( \alpha^* \) (i.e.,
the equilibrium switches from costly separating to pooling), the E-type firm will prefer mimicking the I-
type firm rather than resorting to its first-best (separating) outcome. Thus, when $\alpha$ increases from just below
to above $\alpha^*$, the E-type firm will reduce its quality to the pooling quality of the I-type firm, but its profit
can increase from its first-best level as long as the increase in $\alpha$ is not too large, as illustrated in Figure 3.
We report this counter-intuitive result that stronger inequity aversion can benefit the E-type firm in
Proposition 5.

PROPOSITION 5. Stronger inequity aversion ($\alpha$) may increase the E-type firm’s profit.

Let us examine the consumer’s monetary payoff. One may intuit that the consumer’s stronger inequity
aversion will lead to a higher monetary payoff for the consumer since the firm will charge a lower price
than otherwise. This intuition is correct when the firm’s cost is known to consumers or in the case where
the consumer does not know the firm’s cost but her inequity aversion is very small (i.e., $\frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha}$, or
equivalently $\alpha < \frac{k_I-k_E}{2k_E-k_I}$ and $k_E > \frac{k_I}{2}$). In the latter case, the equilibrium outcome is costless-separating—
the same as the outcome under symmetric information. Thus in those two cases, as Proposition 1 shows,
both types of the firm keep their respective first-best quality and will reduce their prices as the inequity
aversion increases. However, when consumers do not observe the firm’s cost and have strong inequity
aversion, Proposition 6 shows that under some conditions the $H$-type consumer’s monetary payoff may
decrease as $\alpha$ increases.

PROPOSITION 6. Stronger inequity aversion may reduce H-type consumers’ monetary payoff.

Figure 4  Monetary under Asymmetric

Note. The equilibrium outcome is costless separating if $0 \leq \alpha < \frac{k_I-k_E}{2k_E-k_I}$, costly separating if $\alpha < \alpha^*$, and pooling
otherwise. Figure 4 illustrates the case of $k_I = 1, k_E = 0.9, \theta_H = 1, \theta_L = 0.03, \gamma = 0.9$ and $\lambda \in (0, \frac{\theta_L^2}{\theta_H^2})$.

Figure 4 provides an example to show that in both the costly-separating and the pooling parameter
regions, $H$-type consumers’ monetary payoff may decrease in $\alpha$. The main reason for lowered consumer
monetary payoff is that the firm may reduce its quality to a greater extent than it reduces its price. $H$-type
consumers are more negatively affected by the quality drop (than the L-type consumers) leading to a net reduction in monetary payoff despite the decrease in price. This implies that, interestingly, if the consumer’s inequity aversion ($\alpha$) is not observed by the firm, consumers might not want to pretend to have strong inequity aversion to pressure the firm into reducing its price because the firm may strategically reduce its quality. Note that Proposition 6 holds in the whole costly-separating or pooling region (i.e., $k_E > 1+\alpha$ and $\alpha < \alpha^*$, or $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\alpha > \alpha^*$) rather than the transaction point $\alpha^*$.

**PROPOSITION 7.** (a) A less cost-efficient market (i.e., a larger $\gamma$) may have higher expected social welfare; (b) Stronger inequity aversion (i.e., a larger $\alpha$) may lead to higher expected social welfare.

![Figure 5 Social Welfare](image)

*Note.* The equilibrium outcome is separating if $\gamma < \gamma^*$ or $\alpha < \alpha^*$, and pooling otherwise.

Figure 5(a) illustrates the case of $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and Figure 5(b) illustrates the case of $k_E > \frac{k_I}{2}$.

Lastly we examine the implications on the social welfare, defined as the sum of the firm’s profit and all consumers’ monetary payoffs. Intuitively, one may expect that since the social welfare under the I-type firm is lower than that under the E-type firm, the *ex-ante* (expected) social welfare will decrease in $\gamma$. Proposition 7 shows that, counter-intuitively, the expected social welfare may be higher when $\gamma$ is higher or when $\alpha$ is higher (see Figure 5). The increase in social welfare comes from the fact that when a larger $\gamma$ or $\alpha$ shifts the market from a separating regime to a pooling regime, the expected quality in the market will rise (see Figure 2). In both separating and pooling parameter regions, the expected quality level is lower than the socially optimal quality level. Thus the social welfare will increase as the expected quality increases leading to a lower level of quality distortion in the market.
4. Heterogeneity in Inequity Aversion

Thus far, we have assumed that consumers have the same degree of inequity aversion ($\alpha$). In practice, they may have different degrees of inequity aversion. Indeed, our survey shows that some consumers have strong fairness concerns while others have little concerns and focus only on their own absolute surplus. In this section, we extend our model to consider heterogeneity in the consumer’s inequity aversion. Though our main results from the base model remain qualitatively the same, our analysis in this section shows additional insights. First, the presence of different market segments based on the consumer’s inequity aversion provides the $I$-type firm with another dimension to separate itself from the $E$-type firm. We show that, if the fraction of highly inequity-averse consumers is in the middle range, the $I$-type firm may prefer separating itself from the $E$-type firm using a higher-than-first-best price (to target only consumers without strong fairness concerns). This contrasts with our earlier finding that the $I$-type firm uses less-than-first-best quality to credibly signal its cost type, a result based on the assumption that consumers have the same inequity aversion. Second, we find that under some conditions, consumer heterogeneity in inequity aversion may influence the equilibrium market coverage. Third, although in both the base model and the current extension, the consumer’s monetary payoff may decrease in inequity aversion, the underlying reason can be different. In the base model, this result (Proposition 6) arises from a decrease in equilibrium quality. By contrast, in the current extension, we obtain the result because the firm may increase its price to target only consumers without fairness concerns, thereby both types of consumers will get a lower monetary payoff.

In this extension, we consider two types of consumers with respect to their fairness concerns—$F$-type consumers have stronger inequity aversion than $N$-type consumers. In particular, the $N$-type consumers have zero inequity aversion whereas the $F$-type consumers have inequity aversion of degree $\alpha > 0$. A fraction $\tau$ of all consumers are $F$-type, and a fraction $1 - \tau$ of consumers are $N$-type. Our analysis shows that heterogeneity in the consumer’s willingness-to-pay for quality (i.e., $\theta$) does not provide any additional insights and only makes the analysis more cumbersome. Thus, we assume in this section that consumers have the same willingness-to-pay for quality, i.e., $\theta_L = \theta_H = \theta$.

4.1. Symmetric Information

With some abuse of notation, for the current case with consumer heterogeneity in inequity aversion, we again use the symbols $\bar{q}_j^*, \bar{p}_j^*$ and $\bar{\pi}_j^*$ to denote the $j$-type firm’s optimal quality, price and profit, for $j \in \{I, E\}$. By similar analyses to those in Section 3.1, we can obtain

$$\bar{q}_j^* = \frac{\theta}{2k_j},$$

(3)
\[ p_j^*(q_j^*) = \begin{cases} 
\frac{(2+3\alpha)\theta^2}{4k_j(1+2\alpha)}, & \text{if } \tau \geq \frac{\alpha}{1+2\alpha} \\
\frac{\theta^2}{2k_j}, & \text{otherwise} 
\end{cases} \] (4)

\[ \pi_j^* = \begin{cases} 
\frac{(1+\alpha)\theta^2}{4k_j(1+2\alpha)}, & \text{if } \tau \geq \frac{\alpha}{1+2\alpha} \\
\frac{(1-\tau)^2}{4k_j}, & \text{otherwise} 
\end{cases} \] (5)

From (3), we see that the firm's optimal quality is independent of the fraction of \( F \)-type consumers \( (\tau) \) or their degree of inequity aversion \( (\alpha) \). From (4) and (5), we can see that when the fraction of inequity-averse \( (F \)-type) consumers is high (i.e., \( \tau \geq \frac{\alpha}{1+2\alpha} \)), the firm will choose a low price to serve both types of consumers, otherwise it will choose a high price to serve only the \( N \)-type consumers. One can verify that, under symmetric information, the firm’s optimal price \( \bar{p}_j^* \) and profit \( \bar{\pi}_j^* \) both decrease in the \( F \)-type consumer’s inequity aversion \( \alpha \). This is consistent with Proposition 1, which also shows that when all consumers have the same inequity aversion level, the consumers’ monetary payoff will increase with their inequity aversion. By contrast, with consumer heterogeneity in inequity aversion, as the \( F \)-type consumer’s inequity aversion increases, the firm may switch from serving both types of consumers to serving only \( N \)-type consumers by increasing its price, leading to a decrease in both types of consumers’ monetary payoff.

4.2. Asymmetric Information

With similar analyses to those in Section 3.2.2, we obtain the following proposition.

**PROPOSITION 8.** There exists a unique \( \gamma^* \in [0,1] \) such that if \( \gamma < \gamma^* \), the separating equilibrium \((q_{j,sep}^*, p_{j,sep}^*, \pi_{j,sep}^*)\) is the unique undefeated equilibrium outcome, where \( q_{E,sep}^* = q_E^*, p_{E,sep}^* = p_E^* \), \( q_{l,sep}^* = \begin{cases} 
\overline{q}, \text{if } \tau > \tau^* \text{ and } \frac{k_E}{k_l} > \frac{1+\alpha}{1+2\alpha} 
\end{cases} \) and \( p_{l,sep}^* = \begin{cases} 
\overline{p}_l, \text{if } \tau \leq \frac{\alpha}{1+2\alpha} \text{ or } \frac{k_E}{k_l} \leq \frac{1+\alpha}{1+2\alpha} 
\end{cases} \)

in which \( \overline{q} \equiv \frac{k_E(1+\alpha)\theta - \theta\sqrt{\alpha(1+\alpha)k_E(k_l-k_E)}}{2k_E(1+2\alpha)k_l-\alpha k_l} \), if \( \gamma > \gamma^* \), the pooling equilibrium \((q_{pool}^*, p_{pool}^*, \pi_{j,pool}^*)\) is the unique undefeated equilibrium outcome.\(^\text{10}\)

The equilibrium outcomes for the case of \( \frac{k_E}{k_l} > \frac{1+\alpha}{1+2\alpha} \) are illustrated in Figure 6. Similar to the base model, the pooling equilibrium arises when \( \gamma \) is large and the separating equilibrium arises otherwise.

\(^\text{10}\) The expressions of \( \gamma^*, \tau^*, q_{j,sep}^*, p_{pool}^*, \pi_{j,pool}^* \) are given in the online Technical Appendix.
In contrast to the base model, in our current setting a costless-separating equilibrium occurs in two different scenarios. First, if the difference in efficiency between the firms is large enough (i.e., $\frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha}$), the $E$-type firm will find it more profitable to offer high quality to increase the consumer’s overall willingness-to-pay, rather than to mimic the $I$-type firm’s low first-best quality to take advantage of the consumer’s higher willingness-to-pay for quality to the $I$-type firm. This is consistent with the equilibrium outcome in the base model. Second, if the fraction of $F$-type consumers is small enough (i.e., $\tau \leq \frac{\alpha}{1+2\alpha}$), both types of firms will target only the $N$-type consumers, hence resulting in a natural, costless-separating outcome, where the existence of the small number of consumers with fairness concerns does not affect the firm’s strategies.

Now we examine the costly-separating equilibrium outcome. In Section 3.2.2, we find that when consumers have the same degree of inequity aversion, the $I$-type firm separates itself by lowering its quality from the first-best but setting its first-best price conditional on that quality. When consumers have different degrees of inequity aversion, this type of “quality-separating” equilibrium still exists as long as the fraction of $F$-type consumers is high enough (i.e., $\tau > \tau^*$). However, if $\frac{\alpha}{1+2\alpha} \leq \tau < \tau^*$, we will obtain a “price-separating” equilibrium, where the $I$-type firm will choose the first-best quality as in the case of symmetric information (i.e., $q_{I,sep}^* = q_I^*$) but a higher price to serve only $N$-type consumers whereas the $E$-type firm targets both types of consumers as it does under symmetric information. This contrasts with the finding in the base model, where the price-separating equilibrium cannot be obtained.

Figure 6 shows that when $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma < \gamma^*$, as the fraction of $F$-type consumers ($\tau$) decreases, the equilibrium switches from quality separating to price separating, then to costless separating. An $F$-type consumer’s willingness-to-pay is always lower than an $N$-type consumer. When the proportion of $F$-type
consumers is large (i.e., $\tau > \tau^*$), both types of firms should target all consumers. The $I$-type firm will lower its quality from its first-best and choose the first-best price conditional on its quality, which leads to quality-separating equilibrium. This is consistent with the base model, which is actually a special case of $\tau = 1$. As the proportion of $F$-type consumers decreases, the opportunity cost (profit loss) of not selling to $F$-type consumers decreases. But the $E$-type firm’s opportunity cost is higher than that of the $I$-type firm since the $E$-type firm’s profit margin is higher. Thus when $\tau$ decreases to the middle region (i.e., $\frac{a}{1+2a} < \tau < \tau^*$), setting a relatively high price and targeting only $N$-type consumers will not reduce the $I$-type firm’s profit as much as it will reduce the $E$-type’s profit, even though the $I$-type firm’s first-best strategy is also to target all consumers. Thus the $I$-type firm should target only $N$-type consumers to effectively prevent the $E$-type firm from mimicking profitably. In this case, the $I$-type firm’s corresponding optimal quality is the same as its first-best quality, which is independent of $\alpha$, while its optimal price is higher than its first-best price since it targets only $N$-type consumers rather than both types of consumers. Thus the price-separating equilibrium arises. When the proportion of $F$-type consumers is very low (i.e., $\tau < \frac{a}{1+2a}$), both types of firms prefer to set a relatively high price and not selling to $F$-type consumers. Since $N$-type consumers do not have fairness concerns and their utilities do not directly depend on the firm’s cost, both $I$-type and $E$-type firms will pursue their respective first-best strategies, which gives rise to the costless-separating equilibrium.

5. Discussion

In this section, we will discuss an extension of our model to the case of fixed costs rather than variable costs and other alternative models that can capture consumer’s fairness concerns.

In our base model, we focus on the product with significant variable costs. However, in some industries, such as software or digital goods, the firm’s marginal cost is negligible but its fixed cost (e.g., R&D) is very significant. We can extend our model to such markets. Let $f_j(q) = K_jq^2$, where $j \in \{I, E\}$, denote the firm’s fixed cost, where $K_j$ represents the firm’s cost efficiency and $K_E < K_I$. The consumer’s disutility from inequity aversion is based on her own monetary surplus relative to the firm’s net unit profit, i.e., $-\frac{f_j(q)}{m}$, where $m$ is the market demand. Thus, the consumer’s overall utility, conditional on knowing the firm’s cost type, is given by $U_c(q, p, K_j) = q\theta - p - \alpha \cdot \max\{(p - \frac{f_j(q)}{m}) - (\theta q - p), 0\}$. Our analysis shows that the main results and intuitions from the base model remain qualitatively the same.

Our base model assumes that the consumer’s fairness perception is based on an equal split of the economic surplus. Note that consumers’ fairness concerns can be context- or industry-dependent. The benchmark for fairness or what consumers consider as “most fair” or most reasonable can vary across different industries and competitive contexts. One may consider two alternative models. First, to
accommodate the possibility of more general split of surplus as the fairness benchmark, one can model the utility function of consumers with fairness concerns in the following way:

\[
U_t(q, p, k_j) = \theta_t q - p - \alpha \cdot \max\{\beta(p - k_j q^2) - (\theta_t q - p), 0\},
\]

where \( \beta \geq 0 \) is used to capture some industry or market context (note that \( \beta = 1 \) in our base model).

The second alternative fairness model one might consider is based on the firm’s percentage profit margin rather than some equitable split of the total economic surplus between the firm and the consumer:

\[
U_t(q, p, k_j) = \theta_t q - p - \alpha \cdot k_j q^2 \cdot \max\left\{\frac{p - k_j q^2}{k_j q^2} - r, 0\right\} = \theta_t q - p - \alpha \cdot \max\{p - k_j q^2 - rk_j q^2, 0\},
\]

where \( r \geq 0 \) represents the firm’s percentage profit margin that is considered fair in the industry.

From the above utility function, we see that for any given product quality, an inequity-averse consumer’s utility is higher when he knows that the product’s cost is higher. Thus, under asymmetric information, the \( E \)-type firm has an incentive to mimic the \( I \)-type firm’s strategy to take advantage of the consumer’s higher willingness to pay while the \( I \)-type firm may try to credibly reveal its type by choosing the price and quality strategy that the \( E \)-type firm finds unprofitable to mimic. The tradeoff that both types of firms face is still qualitatively the same in the above proposed frameworks as it is in the framework formally analyzed in our base model. Thus, we expect our results to hold qualitatively the same under different parameter regions, though formal analyses of such alternative or more general models for fairness may not lead to closed form solutions.

Both types of definitions—one based on equitable split of surplus versus one based on the firm’s percentage profit margin—have limitations and can be reasonable in some situations/industries but neither should be applied to all industries or all competitive contexts without reservation. The consumer’s fairness concerns in reality may be more nonlinear depending on different ranges of qualities, costs, and prices. However, we believe that the fundamental tradeoffs and incentives of the firm will persist.

6. Conclusion
As anecdotal evidence, existing literature and our survey all show, consumers may perceive the firm’s price as unfair when its profit from the transaction is too high relative to the consumer’s surplus. They may derive a disutility from buying the product at an unfair price. Consumers with strong inequity aversion may even decide to forgo their own benefits from using a product if they perceive the firm’s profit as excessive. In reality, consumers may not be certain of the fairness of a price since they may not know the firm’s true cost. When inequity-averse consumers do not observe the firm’s cost, a cost-efficient firm has an incentive to mimic an inefficient firm since for any given quality consumers are willing to pay more to an inefficient
firm. We have developed a game-theoretic model to investigate how consumer fairness concerns influence the firm’s pricing and quality decisions and what implications consumer inequity aversion has on firm profitability, the consumer’s surplus, and the social welfare.

We have shown several interesting findings. First, when consumers are uncertain about the firm’s cost, the firm’s equilibrium quality may be non-monotone with respect to the consumer’s inequity aversion. One may intuit that as the consumer’s inequity aversion becomes stronger, the consumer’s willingness-to-pay for quality decreases and the firm should therefore reduce its equilibrium quality. We show that this may not necessarily be the case. As the consumer becomes more inequity-averse, the cost-efficient firm has an increased incentive to mimic the inefficient firm, to which the consumer has a higher willingness-to-pay for any given quality level. Stronger inequity aversion may make it too costly for the inefficient firm to credibly signal its type with its quality and price, leading to a pooling outcome, in which the inefficient firm raises its quality and the efficient firm reduces it to the same level, resulting in an increase in the expected quality in the market.

Second, though stronger inequity aversion always makes an inefficient firm worse off, it can benefit an efficient firm in some situations. With stronger inequity aversion, the efficient firm may be able to pool with the inefficient firm at the new equilibrium, at which the consumer’s willingness-to-pay for quality (when not knowing the firm’s cost type) is higher than that for the efficient firm’s quality. Hence, the efficient firm may actually be better off when the consumer’s inequity aversion becomes stronger.

Third, stronger inequity aversion leads to a higher monetary payoff for the consumer when she knows the firm’s cost. In contrast, when the consumer is uncertain about the firm’s cost, stronger inequity aversion may actually lead to a lower monetary payoff for the consumer, because the firm may reduce quality to a greater extent than it reduces its price. This implies that consumers might not want to exaggerate their fairness concerns.

Fourth, one may intuit that, if the probability of the firm being inefficient increases or the expected efficiency in the market decreases, both the expected quality and the social welfare will decrease. On the contrary, we have shown that, because of the cost information asymmetry and the efficient firm’s incentives to mimic the inefficient firm, both the expected quality and the social welfare may increase when the expected cost-efficiency in the market decreases.

We have shown that these main results remain qualitatively the same even when consumers are heterogeneous in their inequity aversion. We have also shown that, if the fraction of inequity-averse consumers is high, the inefficient firm tends to signal its type by choosing less than first-best quality with the first-best price conditional on that quality. In contrast, if the fraction of inequity-averse consumers is the middle region, the inefficient firm tends to signal its type by choosing its first-best quality but a higher than first-best price to not serve the inequity-averse consumers.
In this paper, we have examined a monopoly, search goods market in which consumers have fairness concerns but do not directly observe the firm’s cost. Consumers’ fairness concerns may well depend on the competitive context and our results should be interpreted in the monopoly context of our model and our definition of fairness. Instead of defining fairness by some equitable split of the total economic surplus between the firm and the consumer, we may alternatively define fairness based on some percentage profit margin of the firm. Both types of definitions have limitations and can be reasonable in some situations but neither should be blindly applied to all industry or competitive contexts. Future research may study the effects of consumer fairness concerns in the presence of multiple competing firms and product offerings, although the complexity of an oligopoly model with asymmetric information may preclude analytical solutions. Lastly, our paper shows what strategic firms should do with respect to their product quality and pricing decisions when they face rational consumers with fairness concerns. We hope our paper motivates future empirical research to study the signaling phenomenon in this context to indirectly determine a firm’s quality or price deviations.

References


Signaling through Price and Quality to Consumers with Fairness Concerns

Technical Appendix

**PROPOSITION 1.** If the firm’s type $j \in \{I, E\}$ is common knowledge,
(a) its optimal quality $\tilde{q}_j^*$, price $\bar{p}_j^*(\tilde{q}_j^*)$ and the corresponding profit $\bar{\pi}_j^*$ are

$$
\tilde{q}_j^* = \begin{cases} 
\frac{\theta_I}{2k_j}, & \text{if } \lambda \geq \frac{\theta_I^2}{\theta_H^2}, \\
\frac{\theta_E}{2k_j}, & \text{otherwise}
\end{cases}, \quad 
\bar{p}_j^*(\tilde{q}_j^*) = \begin{cases} 
\frac{(1+\alpha)\theta_I\tilde{q}_j^*+\alpha k_j q_j^2}{1+2\alpha}, & \text{if } \lambda \geq \frac{\theta_I^2}{\theta_H^2}, \\
\frac{(1+\alpha)\theta_E\tilde{q}_j^*+\alpha k_j q_j^2}{1+2\alpha}, & \text{otherwise}
\end{cases}, \quad \text{and } \bar{\pi}_j^* = \begin{cases} 
\frac{\lambda(1+\alpha)q_j^2}{4k_j(1+2\alpha)}, & \text{if } \lambda \geq \frac{\theta_I^2}{\theta_H^2}, \\
\frac{\theta_I^2(1+\alpha)}{4k_j(1+2\alpha)}, & \text{otherwise}
\end{cases};
$$

(b) $\bar{p}_j^*(\tilde{q}_j^*)$ and $\bar{\pi}_j^*$ both decrease in $\alpha$, while $\tilde{q}_j^*$ is independent of $\alpha$;
(c) the consumer’s monetary payoff $M_t(\tilde{q}_j^*, \bar{p}_j^*)$ increases in $\alpha$, where $t \in \{H, L\}$.

**PROOF OF PROPOSITION 1.** (a) We first solve the firm’s optimal price and quality when its cost type is common knowledge. Let $\bar{p}_{j,t}(q)$ denote the $t$-type consumer’s maximum willingness-to-pay for the $j$-type firm’s product of quality $q$ under symmetric information; it is the maximum $p$ such that $U_t(q, p, k_j) \geq 0$.

We compute $\bar{p}_{j,t}(q)$ by finding $p$ such that $U_t(q, p, k_j) = 0$, i.e.,

$$
\theta_t q - \bar{p}_{j,t}(q) - \alpha \cdot \max\{\bar{p}_{j,t}(q) - k_j q_j^2\} - \lambda q - \bar{p}_{j,t}(q) = 0. \quad (A1)
$$

We need to evaluate the max function in the following two cases.

**Case 1:** $\bar{p}_{j,t}(q) - k_j q_j^2 < \theta_t q - \bar{p}_{j,t}(q)$, or equivalently $\bar{p}_{j,t}(q) < \frac{\theta_t q + k_j q_j^2}{2}$

Solving (A1), we easily find $\bar{p}_{j,t}(q) = \theta_t q$. This solution is valid if and only if $\bar{p}_{j,t}(q) = \theta_t q < \frac{\theta_t q + k_j q_j^2}{2}$, which leads to $q > \frac{\theta_t}{k_j}$.

**Case 2:** $\bar{p}_{j,t}(q) - k_j q_j^2 \geq \theta_t q - \bar{p}_{j,t}(q)$, or equivalently $\bar{p}_{j,t}(q) \geq \frac{\theta_t q + k_j q_j^2}{2}$

From (A1), we readily obtain $\bar{p}_{j,t}(q) = \frac{(1+\alpha)q\theta_t + \alpha k_j q_j^2}{1+2\alpha}$. This solution is valid if and only if it satisfies

$$
\bar{p}_{j,t}(q) = \frac{(1+\alpha)q\theta_t + \alpha k_j q_j^2}{1+2\alpha} \geq \frac{\theta_t q + k_j q_j^2}{2}, \text{ which leads to } q \leq \frac{\theta_t}{k_j}.
$$

Combining Case 1 and 2, we have shown a $t$-type consumer’s maximum willingness-to-pay for the $j$-type firm’s product of quality $q$ is

$$
\bar{p}_{j,t}(q) = \begin{cases} 
\theta_t q, & \text{if } q > \frac{\theta_t}{k_j}, \\
\frac{(1+\alpha)q\theta_t + \alpha k_j q_j^2}{1+2\alpha}, & \text{if } q \leq \frac{\theta_t}{k_j}
\end{cases} \quad (A2)
$$

where $j \in \{I, E\}$ and $t \in \{H, L\}$. 
If the $j$-type firm chooses quality $q$ and price $p$ such that $p > \bar{p}_{j,H}(q)$, it will make zero profit since even the $H$-type consumers will not purchase. If $\bar{p}_{j,L}(q) < p \leq \bar{p}_{j,H}(q)$, only $H$-type consumers will buy; if $p \leq \bar{p}_{j,L}(q)$, both types of consumers will buy. Thus, the $j$-type firm’s profit is

$$\pi_j(q, p) = \begin{cases} 
0, & \text{if } p > \bar{p}_{j,H}(q) \\
\lambda(p - k_jq^2), & \text{if } \bar{p}_{j,L}(q) < p \leq \bar{p}_{j,H}(q) \\
p - k_jq^2, & \text{if } p \leq \bar{p}_{j,L}(q)
\end{cases}$$

The firm’s problem is to choose some strategy $(q, p)$ with $q \geq 0$ and $p \geq 0$ to maximize its profit. Since any $(q, p)$ satisfying $p > \bar{p}_{j,H}(q)$ leads to zero profit and is obviously sub-optimal. And conditional on $q$, the firm’s profit $\pi_j(q, p)$ increases in $p$ when $\bar{p}_{j,L}(q) < p \leq \bar{p}_{j,H}(q)$ and $p \leq \bar{p}_{j,L}(q)$. Thus conditional on $q$, the $j$-type firm’s optimal price is either $\bar{p}_{j,H}(q)$ or $\bar{p}_{j,L}(q)$. We need to analyze these two scenarios. We first find the quality that maximizes the firm’s profit in each scenario and then compare the two scenarios to determine the global profit-maximizing strategy and the corresponding profit.

**Scenario 1.** $p = \bar{p}_{j,H}(q)$

When $p = \bar{p}_{j,H}(q)$, the firm’s profit is $\pi_j(q, \bar{p}_{j,H}(q)) = \lambda(\bar{p}_{j,H}(q) - k_jq^2)$. Substituting (A2) into this expression, we obtain

$$\pi_j(q, \bar{p}_{j,H}(q)) = \begin{cases} 
\lambda(\theta_H q - k_jq^2), & \text{if } q > \frac{\theta_H}{k_j} \\
\frac{\lambda(1+\alpha)(\theta_H q - k_jq^2)}{1+2\alpha}, & \text{if } q \leq \frac{\theta_H}{k_j}
\end{cases}$$

The firm’s optimal quality falls in the range $q \leq \frac{\theta_H}{k_j}$ since $\pi_j(q, \bar{p}_{j,H}(q)) < 0$ for any $q > \frac{\theta_H}{k_j}$. The first-order condition yields $q = \frac{\theta_H}{2k_j}$, which is easily shown to be the optimal quality for $q \leq \frac{\theta_H}{k_j}$ case. In summary, in Scenario 1, the optimal quality and price are $q = \frac{\theta_H}{2k_j}$ and $p = \bar{p}_{j,H}(\frac{\theta_H}{2k_j}) = \frac{(2+3\alpha)\theta_H}{4k_j(1+2\alpha)}$, respectively, leading to a profit of $\pi_j(\frac{\theta_H}{2k_j}, \bar{p}_{j,H}(\frac{\theta_H}{2k_j})) = \lambda \frac{(1+\alpha)\theta_H^2}{4(1+2\alpha)k_j}$.

**Scenario 2.** $p = \bar{p}_{j,L}(q)$

Similar to the analysis in Scenario 1, one can readily show that in Scenario 2, the firm’s optimal quality and price are $q = \frac{\theta_L}{2k_j}$ and $p = \bar{p}_{j,L}(\frac{\theta_L}{2k_j}) = \frac{(2+3\alpha)\theta_L^2}{4k_j(1+2\alpha)}$, respectively, leading to a profit of $\pi_j(\frac{\theta_L}{2k_j}, \bar{p}_{j,L}(\frac{\theta_L}{2k_j})) = \frac{(1+\alpha)\theta_L^2}{4(1+2\alpha)k_j}$.
Comparing the firm’s optimal profits in Scenario 1 and Scenario 2, one can readily verify that

\[ \bar{\pi}_j \left( \frac{\theta_H}{2k_j}, \bar{p}_{j,H} \left( \frac{\theta_H}{2k_j} \right) \right) \geq \bar{\pi}_j \left( \frac{\theta_L}{2k_j}, \bar{p}_{j,L} \left( \frac{\theta_L}{2k_j} \right) \right) \text{ if and only if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}. \]

That is, if \( \lambda \geq \frac{\theta_L^2}{\theta_H^2} \), it is most profitable to serve only \( H \)-type consumers with quality \( \frac{\theta_H}{2k_j} \) and price \( \bar{p}_{j,H} \left( \frac{\theta_H}{2k_j} \right) \); if \( \lambda < \frac{\theta_L^2}{\theta_H^2} \), it is most profitable to serve both types of consumers with quality \( \frac{\theta_L}{2k_j} \) and price \( \bar{p}_{j,L} \left( \frac{\theta_L}{2k_j} \right) \). Thus the \( j \)-type firm’s optimal quality and price are

\[
\bar{q}_j = \begin{cases} 
\frac{\theta_H}{2k_j}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2} \\
\frac{\theta_L}{2k_j}, & \text{otherwise}
\end{cases}
\]

and the corresponding optimal profit is

\[
\bar{\pi}_j = \begin{cases} 
\frac{\lambda \theta_H^2 (1+\alpha)}{4k_j (1+2\alpha)^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2} \\
\frac{\lambda \theta_L^2 (1+\alpha)}{4k_j (1+2\alpha)^2}, & \text{otherwise}
\end{cases}, \quad \text{where } j \in \{I,E\}
\]

(b) The comparative statics are easily computed from the above results.

\[
\frac{d\bar{q}_j}{d\alpha} = 0; \quad \frac{d\bar{p}_j}{d\alpha} = \begin{cases} 
-\frac{\theta_H^2}{4k_j (1+2\alpha)^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2} \geq 0; \\
-\frac{\theta_L^2}{4k_j (1+2\alpha)^2}, & \text{otherwise}
\end{cases}
\]

(c) A \( t \)-type consumer’s equilibrium monetary payoff is given by

\[ M_t(\bar{q}_j^*, \bar{p}_j^*) = \begin{cases} 
\theta_t \bar{q}_j^* - \bar{p}_j^*, & \text{if } U_t(\bar{q}_j^*, \bar{p}_j^*, k_j) \geq 0 \\
0, & \text{otherwise}
\end{cases}, \quad \text{where } t \in \{H,L\}.
\]

Substituting \( \bar{q}_j^* \) and \( \bar{p}_j^* \) (see Equation (A3)) into \( M_t(\bar{q}_j^*, \bar{p}_j^*) \), we obtain

\[ M_H(\bar{q}_j^*, \bar{p}_j^*) = \begin{cases} 
\frac{a \theta_H^2}{4k_j (1+2\alpha)^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2} \text{ and } M_L(\bar{q}_j^*, \bar{p}_j^*) = \begin{cases} 
\frac{a \theta_L^2}{4k_j (1+2\alpha)^2}, & \text{otherwise}
\end{cases}
\]

Thus, \( \frac{dM_H(\bar{q}_j^*, \bar{p}_j^*)}{d\alpha} \) is, the consumer’s monetary payoff (weakly) increases in \( \alpha \). \( \square \)

**Proposition 2.** There exists a unique \( \gamma^* \in [0,1] \) such that
(a) if \( \gamma < \gamma^* \), the separating equilibrium \((q_{j,sep}^*, p_{j,sep}^*, \pi_{j,sep}^*)\) is the unique undefeated equilibrium outcome, where \( j \in \{I, E\} \), \( q_{E,sep}^* = \bar{q}_E^* \), \( p_{E,sep}^* = \bar{p}_E^* \), \( q_{I,sep}^* = \begin{cases} \bar{q}_I^* & \text{if } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha}, \\ \hat{q} & \text{otherwise} \end{cases} \), \( p_{I,sep}^* = \bar{p}_I^*(q_{I,sep}^*) \) and \( \hat{q} = \begin{cases} \frac{k_I(1+\alpha) - \alpha(1+\alpha)k_E(k_I-k_E)}{2k_E(1+2\alpha)k_E-k_I)}, & \text{if } \frac{\theta_E^2}{\theta_H^2} \geq \frac{\theta_E^2}{\theta_H^2}, \\ \frac{k_I(1+\alpha) - \alpha(1+\alpha)k_E(k_I-k_E)}{2k_E(1+2\alpha)k_E-k_I)}, & \text{if } \frac{\theta_E^2}{\theta_H^2} < \frac{\theta_E^2}{\theta_H^2}. \end{cases} \)

(b) if \( \gamma > \gamma^* \), the pooling equilibrium \((q_{pool}^*, p_{pool}^*, \pi_{j,pool}^*)\) is the unique undefeated equilibrium outcome, where \( j \in \{I, E\} \),

\[
q_{pool}^* = \begin{cases} \frac{(1+\alpha)\theta_H}{2[(1+2\alpha-\gamma)k_I-\alpha(1-\gamma)k_E]}, & \text{if } \frac{\theta_E^2}{\theta_H^2} \geq \frac{\theta_E^2}{\theta_H^2} \text{ and } p_{pool}^* = \begin{cases} \frac{(1+\alpha)^2(2+4\alpha-\gamma)k_I-\alpha(1-\gamma)k_E\theta_H^2}{4(1+2\alpha)(1+2\alpha-\gamma)k_I-\alpha(1-\gamma)k_E^2}, & \text{if } \frac{\theta_E^2}{\theta_H^2} \geq \frac{\theta_E^2}{\theta_H^2} \text{ and } \lambda < \frac{\theta_E^2}{\theta_H^2}. \end{cases} \\
\frac{(1+\alpha)^2(2+4\alpha-\gamma)k_I-\alpha(1-\gamma)k_E\theta_H^2}{4(1+2\alpha)(1+2\alpha-\gamma)k_I-\alpha(1-\gamma)k_E^2}, & \text{if } \frac{\theta_E^2}{\theta_H^2} \geq \frac{\theta_E^2}{\theta_H^2} \text{ and } \lambda > \frac{\theta_E^2}{\theta_H^2}. \end{cases} 
\]

**Proof of Proposition 2.** We prove Proposition 2 by showing Lemmas A1 to A4. Lemma A1 identifies the unique least-cost separating equilibrium outcome (from the I-type’s perspective); Lemma A2 identifies the unique most-efficient pooling equilibrium outcome for the I-type. Lemma A3 shows that when \( \gamma < \gamma^* \) (the expression for \( \gamma^* \) is provided in the proof of Lemma A3) the least-cost separating equilibrium exists and is the unique undefeated equilibrium outcome. Lemma A4 shows that when \( \gamma > \gamma^* \) the most-efficient pooling equilibrium exists and is the unique undefeated equilibrium. This completes the proof of Proposition 2.

**Lemma A1.** The least-cost separating equilibrium outcome is (a) the E-type firm’s quality, price and profit are \( q_{E,sep}^* = \bar{q}_E^*, p_{E,sep}^* = \bar{p}_E^*, \) and \( \pi_{E,sep}^* = \bar{\pi}_E^* \); (b) the I-type firm’s quality, price and profit are

\[
q_{I,sep}^* = \begin{cases} \bar{q}_I^* & \text{if } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha}, \\ \hat{q} & \text{otherwise} \end{cases}, \quad p_{I,sep}^* = \bar{p}_I^*(q_{I,sep}^*), \quad \pi_{I,sep}^* = \begin{cases} \bar{\pi}_I^* & \text{if } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \text{ and } \lambda \geq \frac{\theta_E^2}{\theta_H^2}, \\ \frac{\lambda\theta_E^2(1+\alpha)[(1+\alpha)k_E-\gamma][k_I+s(k_E+s)]}{4k_E(1+2\alpha)(1+2\alpha-k_E-k_I)} & \text{if } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \text{ and } \lambda \geq \frac{\theta_E^2}{\theta_H^2}, \end{cases}
\]

where \( \hat{q} = \begin{cases} \frac{k_E(1+\alpha) - \alpha(1+\alpha)k_E(k_I-k_E)}{2k_E(1+2\alpha)k_E-k_I)}, & \text{if } \frac{\theta_E^2}{\theta_H^2} \geq \frac{\theta_E^2}{\theta_H^2}, \\ \frac{k_E(1+\alpha) - \alpha(1+\alpha)k_E(k_I-k_E)}{2k_E(1+2\alpha)k_E-k_I)}, & \text{if } \frac{\theta_E^2}{\theta_H^2} < \frac{\theta_E^2}{\theta_H^2}. \end{cases} \)

and \( s = \sqrt{\alpha(1+\alpha)k_E(k_I-k_E)}. \)
**Proof of Lemma A1.** The proof of Lemma A1 can be completed in five steps. Step 1, we show that at any separating equilibrium, the \( E \)-type firm chooses quality \( q_{E,sep}^* = \tilde{q}_E^* \) and price \( p_{E,sep}^* = \tilde{p}_E^* \). Step 2, we identify the strategy set \( \Phi \) to which the \( I \)-type firm’s separating-equilibrium strategy \((q_{I,sep}, p_{I,sep})\) must belong. Step 3, we find among all possible separating equilibria one that is most profitable (least-cost) to the \( I \)-type firm and label it \((q_{I,sep}^*, p_{I,sep}^*)\), and one of the supporting belief systems, \( \mu_{sep}^* \), is given as an example in Equation (A14).

**Step 1.** Note that under asymmetric information, the \( E \)-type firm has an incentive to hide its type. In any separating equilibrium, consumers will correctly infer the firm’s type. Thus at any separating equilibrium, the \( E \)-type firm should choose its first-best strategy \((q_{I,sep}, p_{I,sep})\) regardless of the consumer’s belief (even under the most favorable belief that the firm is \( E \)-type firm). In this case, the condition for no profitable deviation by the \( I \)-type firm is given by Equation (A2). One can show that \( \pi_{E,sep}^* \) (see Equation (A4)) as it does in under symmetric information.

**Step 2.** At any separating equilibrium, the \( E \)-type firm must not be able to profitably deviate to the \( I \)-type firm’s strategy \((q_{I,sep}, p_{I,sep})\); that is, 
\[
\pi_{E,sep}^* = \pi_E(q_{E,sep}, p_{E,sep}) = 0 \geq \pi_E(q_{I,sep}, p_{I,sep}) | \mu(l|q, p) = 1, \]
where \( \pi_E(q, p) \) represents the \( E \)-type firm’s profit when it adopts strategy \((q, p)\) and is believed to be \( I \)-type with probability \( \mu(l|q, p) \). Let \( \Phi \) be the set of strategies that are equilibrium-dominated for the \( E \)-type firm regardless of the consumer’s belief (even under the most favorable belief that the firm is \( I \)-type with probability 1). That is, \( \Phi \equiv \{(q, p) : \pi_{E,sep}^* \geq \pi_E(q, p) | \mu(l|q, p) = 1\} \). At any separating equilibrium, the condition for no profitable deviation by the \( E \)-type firm requires \((q_{I,sep}, p_{I,sep}) \in \Phi \).

We can compute \( \pi_E(q, p) | \mu(l|q, p) = 1 \) below,
\[
\pi_E(q, p) | \mu(l|q, p) = 1 = \begin{cases} 
0, & \text{if } p > \tilde{p}_{I,H}(q) \\
\lambda(p - k_E q^2), & \text{if } \tilde{p}_{I,L}(q) < p \leq \tilde{p}_{I,H}(q), \\
\lambda(p - 2k_E q^2), & \text{if } p \leq \tilde{p}_{I,L}(q)
\end{cases} \tag{A8}
\]
where \( \tilde{p}_{I,t}(q) \) is given by Equation (A2). One can show that \( \pi_{E,sep}^* \geq \pi_E(q, p) | \mu(l|q, p) = 1 \) if and only if \( p > \tilde{p}_{I,H}(q), \) or \( \tilde{p}_{I,L}(q) < p \leq \min\{\frac{\pi_{E,sep}^*}{\lambda} + k_E q^2, \tilde{p}_{I,H}(q)\}, \) or \( p \leq \min\{\pi_{E,sep}^* + k_E q^2, \tilde{p}_{I,L}(q)\} \). For notational convenience, let us define two expressions:
\[
\mathcal{P}_H(q) \equiv \min\{\frac{\pi_{E,sep}^*}{\lambda} + k_E q^2, \tilde{p}_{I,H}(q)\} = \begin{cases} 
\min\{\frac{\theta_{E}^2(1+\alpha)}{4k_E(1+2\alpha)} + k_E q^2, \tilde{p}_{I,H}(q)\}, & \lambda \geq \frac{\theta_{E}^2}{\theta_{H}^2}, \\
\min\{k_E q^2, \tilde{p}_{I,H}(q)\}, & \lambda < \frac{\theta_{E}^2}{\theta_{H}^2},
\end{cases} \tag{A9}
\]
\[
\mathcal{P}_L(q) \equiv \min\{\pi_{E,sep}^* + k_E q^2, \tilde{p}_{I,L}(q)\} = \begin{cases} 
\min\{\frac{\lambda \theta_{E}^2(1+\alpha)}{4k_E(1+2\alpha)} + k_E q^2, \tilde{p}_{I,L}(q)\}, & \lambda \geq \frac{\theta_{E}^2}{\theta_{H}^2}, \\
\min\{k_E q^2, \tilde{p}_{I,L}(q)\}, & \lambda < \frac{\theta_{E}^2}{\theta_{H}^2}
\end{cases} \tag{A10}
\]
To summarize, we have shown that $\Phi = \{(q,p): p \in [0, P_L(q)] \cup (\bar{p}_{IL}(q), P_H(q)] \cup (\bar{p}_{IH}(q), +\infty)\}$.

**Step 3.** Let $(q^*_\text{sep}, p^*_\text{sep}) \in \Phi$ be the $l$-type’s strategy from the most profitable (or least-cost) separating equilibrium, i.e., $(q^*_\text{sep}, p^*_\text{sep}) = \arg\max_{(q_{l,sep}, p_{l,sep}) \in \Phi} \pi_{l,sep}(q_{l,sep}, p_{l,sep})$. For any $(q_{l,sep}, p_{l,sep}) \in \Phi$, the $l$-type firm’s separating-equilibrium profit, $\pi_{l,sep}(q_{l,sep}, p_{l,sep}) \equiv \pi_{l}(q_{l,sep}, p_{l,sep} | \mu(l) | q_{l,sep}, p_{l,sep} = 1)$ is computed below:

$$\pi_{l,sep}(q_{l,sep}, p_{l,sep}) = \begin{cases} 0, & \text{if } p_{l,sep} > \bar{p}_{I,H}(q_{l,sep}) \\ \lambda(p_{l,sep} - k_l q^2_{l,sep}), & \text{if } P_L(q_{l,sep}) < p_{l,sep} \leq P_H(q_{l,sep}), \\ P_{l,sep} - k_l q^2_{l,sep}, & \text{if } p_{l,sep} \leq P_L(q_{l,sep}). \end{cases} \quad (A11)$$

Any $(q_{l,sep}, p_{l,sep})$ satisfying $p_{l,sep} > \bar{p}_{I,H}(q_{l,sep})$ is sub-optimal since it leads to zero profit. And conditional on $q_{l,sep}$, the firm’s profit $\pi_{l,sep}(q_{l,sep}, p_{l,sep})$ increases in $p$ when $P_L(q_{l,sep}) < p_{l,sep} \leq P_H(q_{l,sep})$ and $p_{l,sep} \leq P_L(q_{l,sep})$. Thus conditional on $q_{l,sep}$, the firm’s optimal price is either $P_H(q_{l,sep})$ or $P_L(q_{l,sep})$. We need to analyze these two cases. We first find the quality that maximizes the $l$-type firm’s profit in each case and then compare the two cases to determine the global profit-maximizing strategy and the corresponding profit.

**Case 1.** $p_{l,sep} = P_H(q_{l,sep})$: only $H$-type consumers purchase the product.

When $p_{l,sep} = P_H(q_{l,sep})$, the firm’s profit is $\pi_{l,sep}(q_{l,sep}, P_H(q_{l,sep})) = \lambda(P_H(q_{l,sep}) - k_l q^2_{l,sep})$. Using the definitions of $P_H(q)$ in Equation (A9) and $\bar{p}_{I,H}(q)$ in (A2), one can verify that, if $q_{l,sep} > \frac{\theta_H}{k_l}$, then $P_H(q_{l,sep}) \leq \bar{p}_{I,H}(q_{l,sep}) = \theta_H q_{l,sep}$ and $\pi_{l,sep}(q_{l,sep}, P_H(q_{l,sep})) < 0$. Thus, the $l$-type firm’s most profitable quality is in the range $0 \leq q_{l,sep} \leq \frac{\theta_H}{k_l}$. In this quality range, we need to examine two subcases for $\lambda$.

**Subcase 1.1:** $\lambda \geq \frac{\theta_L^2}{\theta_H^2}$

With $q_{l,sep} \leq \frac{\theta_H}{k_l}$ and $\lambda \geq \frac{\theta_L^2}{\theta_H^2}$, we can rewrite and simplify $P_H(q_{l,sep})$ as

$$P_H(q_{l,sep}) = \begin{cases} \frac{\theta_H^2(1+\alpha)}{4k_E(1+2\alpha)} + k_E q_{l,sep}^2, & \text{if Condition 1 holds}, \\ \bar{p}_{I,H}(q_{l,sep}), & \text{otherwise} \end{cases} \quad (A12)$$

where Condition 1 is $Q_{H}^{(1)} \leq q_{l,sep} \leq Q_{H}^{(2)}$ and $\frac{k_E}{k_l} \geq \frac{\alpha}{1+2\alpha}$, or $q_{l,sep} \geq Q_{H}^{(1)}$ and $\frac{k_E}{k_l} < \frac{\alpha}{1+2\alpha}$, in

which $Q_{H}^{(1)} = \frac{\theta_H(1+\alpha)k_E - \sqrt{(1+\alpha)k_E(k_l-k_E)}}{2k_E(k_E+2\alpha k_E-\alpha k_l)}$ and $Q_{H}^{(2)} = \frac{\theta_H(1+\alpha)k_E + \sqrt{(1+\alpha)k_E(k_l-k_E)}}{2k_E(k_E+2\alpha k_E-\alpha k_l)}$.

Substituting (A12) into $\pi_{l,sep}(q_{l,sep}, P_H(q_{l,sep}))$, under Subcase 1.1, we can simplify it to
\[ \pi_{I, \text{sep}}(q_{I, \text{sep}}, \mathcal{P}_H(q_{I, \text{sep}})) = \begin{cases} \lambda \left( \frac{\theta_H^2(1+\alpha)}{4\lambda k_E(1+2\alpha)} + k_E q_{I, \text{sep}}^2 - k_I q_{I, \text{sep}}^2 \right), & \text{if Condition 1 holds} \\ \frac{\lambda(1+\alpha)q_{I, \text{sep}} - k_I q_{I, \text{sep}}^2}{1+2\alpha}, & \text{otherwise} \end{cases} \]

If Condition 1 holds, \( \pi_{I, \text{sep}}(q_{I, \text{sep}}, \mathcal{P}_H(q_{I, \text{sep}})) \) decreases in \( q_{I, \text{sep}} \) and thus the most profitable quality is \( q_{I, \text{sep}} = Q_H^{(1)} \). If Condition 1 does not hold, \( \pi_{I, \text{sep}}(q_{I, \text{sep}}, \mathcal{P}_H(q_{I, \text{sep}})) \) is concave in \( q_{I, \text{sep}} \) for \( q_{I, \text{sep}} < Q_H^{(1)} \) and decreasing in \( q_{I, \text{sep}} \) for \( q_{I, \text{sep}} > Q_H^{(2)} \) when \( k_E \frac{k_I}{k_I} \geq \frac{\alpha}{1+2\alpha} \), and the first-order condition leads to \( q_{I, \text{sep}} = \frac{\theta_H}{2k_I} \). Thus under Subcase 1.1, \( \pi_{I, \text{sep}}(q_{I, \text{sep}}, \mathcal{P}_H(q_{I, \text{sep}})) \) is unimodal in \( q_{I, \text{sep}} \) and the global maximum is achieved at \( \min\{\frac{\theta_H}{2k_I}, Q_H^{(1)} \} \). The I-type firm’s most profitable separating strategy is

\[ q_{I, \text{sep}} = \min\left( \frac{\theta_H}{2k_I}, Q_H^{(1)} \right) = \begin{cases} \frac{\theta_H}{2k_I}, & \text{if } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \\ Q_H^{(1)}, & \text{if } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \end{cases} \]

and \( p_{I, \text{sep}} = \bar{p}_{H,I}(q_{I, \text{sep}}) \).

Subcase 1.2: \( \lambda < \frac{\theta_H^2}{\theta_H} \)

With \( q_{I, \text{sep}} \leq \frac{\theta_H}{k_I} \) and \( \frac{\theta_H^2}{\theta_H} \), we can rewrite and simplify \( \mathcal{P}_H(q_{I, \text{sep}}) \) as

\[ \mathcal{P}_H(q_{I, \text{sep}}) = \begin{cases} \frac{\theta_H^2(1+\alpha)}{4\lambda k_E(1+2\alpha)} + k_E q_{I, \text{sep}}^2, & \text{if Condition 2 holds} \\ \bar{p}_{I,H}(q_{I, \text{sep}}), & \text{otherwise} \end{cases} \] (A13)

where Condition 2 is \( Q_H^{(3)} < q_{I, \text{sep}} < Q_H^{(4)} \) and \( k_E \frac{k_I}{k_I} \geq \frac{\alpha}{1+2\alpha} \), or \( q_{I, \text{sep}} > Q_H^{(3)} \) and \( k_E \frac{k_I}{k_I} < \frac{\alpha}{1+2\alpha} \), in

\[ Q_H^{(3)} = \frac{(1+\alpha)k_E \theta_H \lambda - \sqrt{\lambda(1+\alpha)k_E [ak_H^2 + k_E (\lambda(1+\alpha) \theta_H (1+2\alpha) \theta_L^2)]}}{2ak_E (k_E + 2ak_E - k_I)} \]

and \( Q_H^{(4)} = \frac{(1+\alpha)k_E \theta_H \lambda + \sqrt{\lambda(1+\alpha)k_E [ak_H^2 + k_E (\lambda(1+\alpha) \theta_H (1+2\alpha) \theta_L^2)]}}{2ak_E (k_E + 2ak_E - k_I)} \).

Substituting (A13) into \( \pi_{I, \text{sep}}(q_{I, \text{sep}}, \mathcal{P}_H(q_{I, \text{sep}})) \), under Subcase 1.2, we can simplify it to

\[ \pi_{I, \text{sep}}(q_{I, \text{sep}}, \mathcal{P}_H(q_{I, \text{sep}})) = \begin{cases} \lambda \left( \frac{\theta_H^2(1+\alpha)}{4\lambda k_E(1+2\alpha)} + k_E q^2 - k_I q^2 \right), & \text{if Condition 2 holds} \\ \lambda \left( (1+\alpha)q \theta_H + ak_H^2 \right) \frac{1}{1+2\alpha} - k_I q^2, & \text{otherwise} \end{cases} \]

Similar to the analysis in Subcase 1.1, we can show that in Subcase 1.2, the I-type’s most profitable separating strategy is \( q_{I, \text{sep}} = \min\{\frac{\theta_H}{2k_I}, Q_H^{(3)} \} = \begin{cases} Q_H^{(3)}, & \text{if } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \text{ and } \frac{\theta_H}{\theta_H} < \min\{1, \sqrt{\frac{\lambda k_E (2k_I + 3ak_H^2 - 2ak_H^2)}{(1+\alpha)k_H^2}} \} \\ \frac{\theta_H}{2k_I}, & \text{otherwise} \end{cases} \)

and \( p_{I, \text{sep}} = \bar{p}_{H,I}(q_{I, \text{sep}}) \).

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In summary, combining Subcase 1.1 and 1.2, we conclude that the I-type firm’s most profitable separating strategy under Case 1 is \( q_{I,sep} = q_{I,H} \) and \( p_{I,sep} = \bar{p}_{H,I}(q_{H,I}) \), where

\[
q_{I,H} = \begin{cases}
\min\left\{\frac{\theta_H}{2k_I}, Q_H^{(1)}\right\}, & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2}\ \\
\min\left\{\frac{\theta_H}{2k_I}, Q_H^{(3)}\right\}, & \text{if } \lambda < \frac{\theta_H^2}{\theta_H^2}
\end{cases}
\]

and the corresponding profit is

\[
\pi_{I,sep}(q_{I,H}, \bar{p}_{H,I}(q_{H,I})) = \lambda \frac{(1+\alpha)(\theta_H - k_I q_{I,H}) q_{I,H}}{1+2\alpha}.
\]

**Case 2.** \( p_{I,sep} = \mathcal{P}_L(q_{I,sep}) \): both types of consumers buy the product.

Similar to the analysis in Case 1, we can show that the I-type firm’s most profitable separating strategy is

\[
q_{I,sep} = q_{I,L} = \begin{cases}
\min\left\{\frac{\theta_L}{2k_I}, Q_L^{(1)}\right\}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_L^2} \text{ and } \lambda < \frac{\theta_L^2}{\theta_L^2}
\end{cases}
\]

where \( Q_L^{(1)} = \frac{(1+\alpha)k_E \theta_L^2 - \sqrt{(1+\alpha)k_E \theta_L^2 + k_E(1+\alpha)\theta_L^2 - \lambda (1+2\alpha)\theta_L^2}}{2k_E(k_E+2ak_E-ak)} \) and \( Q_L^{(3)} = \frac{\theta_L[(1+\alpha)k_E \sqrt{\theta_L^2 + (1+\alpha)k_E \theta_L^2}] - \lambda (1+2\alpha)\theta_L^2}{2k_E(k_E+2ak_E-ak)} \).

The firm’s corresponding profit is

\[
\pi_{I,sep}(q_{I,L}, \bar{p}_{I,L}(q_{I,L})) = \frac{(1+\alpha)(\theta_L - k_I q_{I,L}) q_{I,L}}{1+2\alpha}.
\]

Now, by comparing the I-type firm’s maximum profits in Case 1 and Case 2, we readily obtain that

\[
\pi_{I,sep}(q_{I,L}, \bar{p}_{I,H}(q_{I,L})) > \pi_{I,sep}(q_{I,L}, \bar{p}_{I,L}(q_{I,L})) \text{ if and only if } \lambda > \frac{\theta_L^2}{\theta_L^2}.
\]

Thus, the I-type firm’s global optimal separating strategy is

\[
q_{I,sep}^* = \begin{cases}
q_{H,I} \text{ if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} = \begin{cases}
\min\left\{\frac{\theta_H}{2k_I}, Q_H^{(1)}\right\}, & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2}
\end{cases}
\text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}
\end{cases}
\]

and

\[
p_{I,sep}^* = \begin{cases}
\bar{p}_{H,I}(q_{I,sep}^*), & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} = \begin{cases}
\bar{p}_{I,H}(q_{I,H}^*), & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2}
\end{cases}
\text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}
\end{cases}
\]

and

\[
p_{I,sep}^* = \begin{cases}
\bar{p}_{I,L}(q_{I,sep}^*), & \text{if } \lambda < \frac{\theta_L^2}{\theta_L^2} = \begin{cases}
\bar{p}_{I,L}(q_{I,L}^*), & \text{if } \lambda < \frac{\theta_L^2}{\theta_L^2}
\end{cases}
\text{ and } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha}
\end{cases}
\]
Let us define $\hat{q} \equiv \begin{cases} Q_H^{(1)}, & \text{if } \lambda \geq \frac{\theta^2_l}{\theta^2_{H}} \\ Q_L^{(3)}, & \text{if } \lambda < \frac{\theta^2_l}{\theta^2_{H}} \end{cases}$. Using $q_I^*$ and $\hat{q}$, we can rewrite $q_{l,sep}^*$ as $q_{l,sep}^* = \min\{\hat{q}, q_I^*\}$ where $q_I^*$ is given as an example in Equation (A22). And the $I$-type firm’s profit $\pi_{l,sep}^* \equiv \pi_{l,sep}^*(q_{l,sep}, p_{l,sep})$ can be written as

$$
\pi_{l,sep}^* = \begin{cases} \lambda(p_{l,H}(q_{l,sep}) - k_Iq_{l,sep}^2), & \text{if } \lambda \geq \frac{\theta^2_l}{\theta^2_{H}} \\
\bar{p}_{l,L}(q_{l,sep}) - k_Iq_{l,sep}^2, & \text{if } \lambda < \frac{\theta^2_l}{\theta^2_{H}} \end{cases}
$$

where $s = \sqrt{\alpha(1 + \alpha)k_E(k_I - k_E)}$.

Many belief systems can support the above separating equilibrium and one of the examples is given below,

$$
\mu_{sep}(I|q,p) = \begin{cases} 1, & \text{if } (q,p) \in \Phi \\
0, & \text{otherwise} \end{cases}
$$

This completes the proof of Lemma A1. □

**Lemma A2.** At the most-efficient pooling equilibrium, both types of firms will choose

$$
q_{pool}^* = \begin{cases} \frac{(1+\alpha)\theta_H}{2((1+2a-\alpha)k_I-a(1-\gamma)k_E)}, & \text{if } \lambda \geq \frac{\theta^2_l}{\theta^2_{H}} \\
\frac{(1+\alpha)\theta_L}{2((1+2a-\alpha)k_I-a(1-\gamma)k_E)}, & \text{if } \lambda < \frac{\theta^2_l}{\theta^2_{H}} 
\end{cases}
$$

$$
p_{pool}^* = \begin{cases} \frac{(1+\alpha)^2[(2+4a-\alpha)k_I-a(1-\gamma)k_E]\theta_H^2}{4((1+2a)(1+2a-\alpha)k_I-a(1-\gamma)k_E)^2}, & \text{if } \lambda \geq \frac{\theta^2_l}{\theta^2_{H}} \\
\frac{(1+\alpha)^2[(2+4a-\alpha)k_I-a(1-\gamma)k_E]\theta_H^2}{4((1+2a)(1+2a-\alpha)k_I-a(1-\gamma)k_E)^2}, & \text{if } \lambda < \frac{\theta^2_l}{\theta^2_{H}} 
\end{cases}
$$

**Proof of Lemma A2.** We complete the proof of Lemma A2 in two steps. Step 1, we show that at any pooling equilibrium, a $t$-type consumer’s willingness-to-pay conditional on the pooling quality $q$ is at most $p_t(q)$ as given in Equation (A18). Step 2, we identify among all pooling outcome the most profitable pooling equilibrium for the $I$-type firm and label it ($q_{pool}^*$, $p_{pool}^*$), and one of the supporting belief systems, $\mu_{pool}^*$, is given as an example in Equation (A22).

**Step 1.** At any pooling equilibrium, the consumer’s posterior belief is the same as the prior. Let $p_t(q)$ denote a $t$-type consumer’s maximum willingness-to-pay for quality $q$ at a pooling equilibrium. We compute $p_t(q)$ by finding $p$ such that $EU_t(q, p)\mu(I|p, q) = \gamma$; that is, $p_t(q)$ is the solution of $p$ to

$$
EU_t(q, p)\mu(I|p, q) = \gamma = \gamma U_t(q, p, k_I) + (1 - \gamma)U_t(q, p, k_E) = 0.
$$
To solve Equation (A17), we need to consider three cases.

**Case 1.** \( p - k_E q^2 < \theta_t q - p \), or equivalently \( p < \frac{\theta_t q + k_E q^2}{2} \)

In this case, \( U_t(q, p, k_E) = U_t(q, p, k_I) = \theta_t q - p \) and Equation (A17) can be rewritten as
\[
EU_t(q, p | \mu(l | p, q) = \gamma) = \theta_t q - p = 0.
\]
Thus, the potential solution in this case is \( p_t(q) = \theta_t q \). This solution is consistent with the condition for Case 1 only if \( q > \frac{\theta_t}{k_E} \).

**Case 2.** \( p - k_I q^2 < \theta_t q - p \leq p - k_E q^2 \), or equivalently \( \frac{\theta_t q + k_I q^2}{2} \leq p < \frac{\theta_t q + k_E q^2}{2} \)

In this case, \( U_t(q, p, k_E) = U_t(q, p, k_l) = \theta_t q - p - \alpha[(p - k_E q^2) - (\theta_t q - p)] \) and \( U_t(q, p, k_I) = \theta_t q - p \). Thus, Equation (A17) can be rewritten as
\[
EU_t(q, p | \mu(l | p, q) = \gamma) = (\theta_t q - p) - (1 - \gamma)\alpha[(p - k_E q^2) - (\theta_t q - p)] = 0,
\]
which leads to \( p_t(q) = \frac{[1 + \alpha(1 - \gamma)]q\theta_t + \alpha(1 - \gamma)k_E q^2}{1 + 2\alpha(1 - \gamma)} \). This solution is consistent with the condition for

Case 2 only if \( \frac{[1 + \alpha(1 - \gamma)]q\theta_t + \alpha(1 - \gamma)k_E q^2}{1 + 2\alpha(1 - \gamma)} \in \left( \frac{\theta_t}{2}, \frac{\theta_t}{2} \right) \), i.e., \( \frac{(1 + 2\alpha - 2\gamma)k_I - (2\alpha - 2\gamma)k_E}{(1 + 2\alpha - 2\gamma)k_I - (2\alpha - 2\gamma)k_E} < q \leq \frac{(1 + 2\alpha - 2\gamma)k_I - (2\alpha - 2\gamma)k_E}{(1 + 2\alpha - 2\gamma)k_I - (2\alpha - 2\gamma)k_E} \).

**Case 3.** \( p - k_I q^2 \geq \theta_t q - p \), or equivalently \( p \geq \frac{\theta_t q + k_I q^2}{2} \)

In this case, \( U_t(q, p, k_E) = \theta_t q - p - \alpha[(p - k_E q^2) - (\theta_t q - p)] \) and \( U_t(q, p, k_I) = \theta_t q - p - \alpha[(p - k_I q^2) - (\theta_t q - p)] \). Thus Equation (A17) can be rewritten as
\[
EU_t(q, p | \mu(l | p, q) = \gamma) = \theta_t q - p - \alpha \cdot [\gamma k_I + (1 - \gamma)k_E)q^2] - (\theta_t q - p) = 0.
\]
The solution is \( p_t(q) = \frac{(1 + \alpha)\theta_t + \alpha(1 - \gamma)k_E q^2}{1 + 2\alpha} \). This solution is consistent with the condition for Case 3 only if \( p_t(q) \geq \frac{\theta_t}{k_E} \), i.e., \( q \leq \frac{\theta_t}{(1 + 2\alpha - 2\gamma)k_I - (2\alpha - 2\gamma)k_E} \).

Combining the results in all three cases, the \( t \)-type consumer’s maximum willingness-to-pay for the pooling quality \( q \) is \( p_t(q) \) as given below:

\[
p_t(q) = \begin{cases} 
\frac{\theta_t q}{2}, & \text{if } q > \frac{\theta_t}{k_E} \\
\frac{[1 + \alpha(1 - \gamma)]q\theta_t + \alpha(1 - \gamma)k_E q^2}{1 + 2\alpha(1 - \gamma)}, & \text{if } \frac{\theta_t}{(1 + 2\alpha - 2\gamma)k_I - 2\alpha(1 - \gamma)k_E} < q \leq \frac{\theta_t}{k_E} \text{ for } t \in \{H, L\} \\
\frac{(1 + \alpha)\theta_t + \alpha(1 - \gamma)k_E q^2}{1 + 2\alpha}, & \text{if } q \leq \frac{\theta_t}{(1 + 2\alpha - 2\gamma)k_I - 2\alpha(1 - \gamma)k_E}
\end{cases}
\]

**Step 2.** Let us first find the \( l \)-type firm’s profit expression \( \pi_{l, polo}(q, p) \). If \( (q, p) \) is such that \( p > p_H(q) \), no consumers will purchase. If \( p_L(q) < p \leq p_H(q) \), only \( H \)-type consumers will buy; if \( p \leq p
\( p_L(q) \), both types of consumers will buy. Thus, the \( I \)-type firm’s profit, if pooling at \((q,p)\) under some belief system, is given by

\[
\pi_{I,\text{pool}}(q,p) = \pi_I(q,p|\mu(I,q,p) = \gamma) = \begin{cases} 
\lambda p - k_I q^2, & \text{if } p > p_H(q) \\
0, & \text{if } p \leq p_H(q)
\end{cases} \tag{A19}
\]

Note that each strategy pair \((q,p)\) can correspond to a possible pooling equilibrium under some belief system. Next we find, among all possible pooling equilibria, the most-efficient pooling equilibrium outcome (most profitable to the \( I \)-type firm).

In Equation (A19), any \((q,p)\) satisfying \( p > p_H(q) \) leads to zero profit and is obviously not the most profitable pooling outcome. And conditional on \( q \), the firm’s profit \( \pi_{I,\text{pool}}(q,p) \) increases in \( p \) when \( p_L(q) < p \leq p_H(q) \) and \( p \leq p_L(q) \). Thus conditional on \( q \), the firm’s optimal price is either \( p_H(q) \) or \( p_L(q) \). We need to analyze these two scenarios. We first find the quality that maximizes the \( I \)-type firm’s profit in each scenario and then compare the two scenarios to determine the global profit-maximizing strategy and the corresponding profit.

**Scenario 1.** \( p = p_H(q) \)

When \( p = p_H(q) \), the \( I \)-type firm’s pooling profit is \( \pi_{I,\text{pool}}(q,p_H(q)) = \lambda (p_H(q) - k_I q^2) \).

Substituting \( p_H(q) \) (Equation (A18)) into this expression, we obtain

\[
\pi_{I,\text{pool}}(q,p_H(q)) = \begin{cases} 
\lambda \left( \frac{1+\alpha(1-\gamma)}{1+2\alpha} \right) \left( \frac{\theta_H}{k_I} \right) q - k_I q^2, & \text{if } q > \frac{\theta_H}{k_E} \\
\lambda \left( \frac{1+\alpha(1-\gamma)}{1+2\alpha} \right) \left( \frac{\theta_H}{k_I} \right) q - k_I q^2, & \text{if } \frac{\theta_H}{k_I - 2\alpha(1-\gamma)k_E} < q \leq \frac{\theta_H}{k_E} \\
\lambda \left( \frac{1+\alpha(1-\gamma)}{1+2\alpha} \right) \left( \frac{\theta_H}{k_I} \right) q - k_I q^2, & \text{if } q \leq \frac{\theta_H}{k_I - 2\alpha(1-\gamma)k_E}
\end{cases}
\]

Next we find which \( q \) corresponds to a pooling outcome with the highest profit for the \( I \)-type firm. Since \( \pi_{I,\text{pool}}(q,p_H(q)) < 0 \) for any \( q > \frac{\theta_H}{k_E} \), we need to analyze only the two remaining intervals of \( q \).

Note that on the interval \( \frac{\theta_H}{k_I - 2\alpha(1-\gamma)k_E} < q \leq \frac{\theta_H}{k_E} \), \( \pi_{I,\text{pool}}(q,p_H(q)) \) is continuous and concave in \( q \), but the first-order condition yields \( q = \frac{(1+\alpha(1-\gamma))\theta_H}{2[\alpha k_I - k_E(1-\gamma)]} \), which is easily verified to be to the left of the quality interval. Thus, on this quality interval, \( q = \frac{(1+\alpha(1-\gamma))\theta_H}{2[\alpha k_I - k_E(1-\gamma)]} \) gives the highest profit. On the interval \( q \leq \frac{\theta_H}{k_I - 2\alpha(1-\gamma)k_E} \), the first-order condition yields \( q = \frac{(1+\alpha)\theta_H}{2[\alpha + 2\alpha(1-\gamma)k_E]} \), which is easily verified to be within the current quality interval and leads to the highest profit on all intervals.

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To summarize, in Scenario 1, the I-type firm’s most profitable pooling equilibrium corresponds to
\[ q = q_{H, pool} \] and 
\[ p = p_H(q_{H, pool}) = \frac{(1+\alpha)^2[(2+4\alpha-\gamma)k_f - \alpha(1-\gamma)k_E] \theta_H^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}, \]
and its profit is thus
\[ \pi_{I, pool}(q_{H, pool}, p_H(q_{H, pool})) = \frac{\lambda(1+\alpha)^2 \theta_H^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}. \]

**Scenario 2.** \( p = p_L(q) \)

When \( p = p_L(q) \), the I-type firm’s pooling profit is \( \pi_{I, pool}(q, p_L(q)) = p_L(q) - k_I q^2 \). Substituting \( p_L(q) \) (Equation (A18)) into this expression, we obtain
\[
\pi_{I, pool}(q, p_L(q)) = \begin{cases} 
    \theta_L q - k_I q^2, & \text{if } q > \frac{\theta_L}{k_E}, \\
    \frac{\theta_L}{(1+2\alpha-\gamma)k_f - \alpha(1-\gamma)k_E} - k_I q^2, & \text{if } \frac{\theta_L}{(1+2\alpha-\gamma)k_f - \alpha(1-\gamma)k_E} < q \leq \frac{\theta_L}{k_E}.
\end{cases}
\]

Similar analysis to that in Scenario 1, we can show that \( q = q_{L, pool} \equiv \frac{(1+\alpha)^2 \theta_L}{2[(1+2\alpha-\gamma)k_f - \alpha(1-\gamma)k_E]} \) corresponds to the pooling outcome with the highest profit for the I-type firm. In summary, in Scenario 2, the I-type firm’s most profitable pooling outcome corresponds to \( q = q_{L, pool} \) and \( p = \)
\[
p_L(q_{L, pool}) = \frac{(1+\alpha)^2[(2+4\alpha-\gamma)k_f - \alpha(1-\gamma)k_E] \theta_L^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}, \]
and its profit is thus \( \pi_{I, pool}(q_{L, pool}, p_L(q_{L, pool})) = \frac{(1+\alpha)^2 \theta_L^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}. \)

Comparing the most profitable pooling outcomes in Scenario 1 and Scenario 2, one can readily show
\[ \pi_{I, pool}(q_{H, pool}, p_H(q_{H, pool})) \geq \pi_{I, pool}(q_{L, pool}, p_L(q_{L, pool})) \] if and only if \( \lambda \geq \frac{\theta_L^2}{\theta_H^2} \). Therefore, the most profitable pooling outcome for the I-type firm corresponds to

\[ q_{pool}^* = \begin{cases} 
    \frac{(1+\alpha)^2 \theta_L}{2[(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E]}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
    \frac{(1+\alpha)^2 \theta_L}{2[(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E]}, & \text{otherwise}
\end{cases} \]

\[ p_{pool}^* = \begin{cases} 
    p_H(q_{pool}^*), & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
    p_L(q_{pool}^*), & \text{otherwise}
\end{cases} = \begin{cases} 
    \frac{(1+\alpha)^2[(2+4\alpha-\gamma)k_f - \alpha(1-\gamma)k_E] \theta_H^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
    \frac{(1+\alpha)^2[(2+4\alpha-\gamma)k_f - \alpha(1-\gamma)k_E] \theta_H^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}, & \text{if } \lambda < \frac{\theta_L^2}{\theta_H^2}.
\end{cases} \]

And the I-type’s and E-type firm’s corresponding profits at this pooling outcome are
\[
\pi_{I, pool}^* = \pi_{I, pool}(q_{pool}^*, p_{pool}^*) = \begin{cases} 
    \frac{\lambda(1+\alpha)^2 \theta_H^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
    \frac{(1+\alpha)^2 \theta_L^2}{4(1+2\alpha)(1+2\alpha-\alpha)k_f - \alpha(1-\gamma)k_E^2}, & \text{otherwise}
\end{cases}, \quad \text{(A20)}
\]
\[ \pi_{E,\text{pool}} \equiv \pi_{E,\text{pool}}(q_{\text{pool}}^*, p_{\text{pool}}^*) = \begin{cases} \frac{(1+\alpha)^2(2+4\alpha-\gamma)\beta_k-(1+3\alpha-\gamma)\beta_k}{4(1+\alpha)[(1+2\alpha)\beta_k-(1+\gamma)\beta_k]^2}, & \text{if } \lambda \geq \frac{\theta}{\theta_H}, \\ \frac{(1+\alpha)^2(2+4\alpha-\gamma)\beta_k-(1+3\alpha-\gamma)\beta_k}{4(1+\alpha)[(1+2\alpha)\beta_k-(1+\gamma)\beta_k]^2}, & \text{otherwise} \end{cases} \quad (A21) \]

Many belief systems can support the above pooling equilibrium and one of the examples is given below,
\[ \mu_{\text{pool}}^*(l|q, p) = \begin{cases} \gamma, & \text{if } (q, p) = (q_{\text{pool}}^*, p_{\text{pool}}^*), \\ 0, & \text{otherwise} \end{cases} \quad (A22) \]

This completes the proof of Lemma A2. □

**Lemma A3.** There exists a unique \( \gamma^* \in [0, 1] \) such that if \( \gamma < \gamma^* \), the least-cost separating equilibrium characterized in Lemma A1 exists and is the unique undefeated equilibrium outcome.

**Proof of Lemma A3.** Let \( \epsilon_{\text{sep}}^* \equiv (q_{E,\text{sep}}^*, p_{E,\text{sep}}^*), (q_{I,\text{sep}}^*, p_{I,\text{sep}}^*), \mu_{\text{sep}}^* \) denote the least-cost separating equilibrium characterized in Lemma A1 with belief \( \mu_{\text{sep}}^* \) as given in Equation (A14). We complete the proof in four steps. Step 1, we determine \( \gamma^* \). Step 2, we show that when \( \gamma < \gamma^* \), \( \epsilon_{\text{sep}}^* \) exists. Step 3, we show that when \( \gamma < \gamma^* \), \( \epsilon_{\text{sep}}^* \) is undefeated. Step 4, we show that when \( \gamma < \gamma^* \), all other separating equilibria except \((q_{E,\text{sep}}^*, p_{E,\text{sep}}^*), (q_{I,\text{sep}}^*, p_{I,\text{sep}}^*)\) and all pooling equilibria are defeated.

**Step 1.** We determine \( \gamma^* \). \( \pi_{l,\text{sep}}^* \) and \( \pi_{E,\text{pool}}^* \) (see Equation (A7) and Equation (A20)) are the \( I \)-type firm’s profits in the least-cost separating equilibrium and the most-efficient pooling equilibrium, respectively. Note that \( \pi_{E,\text{pool}}^* \) is increasing in \( \gamma \) and \( \pi_{l,\text{sep}}^* \) is independent of \( \gamma \). And one can easily verify that \( \pi_{E,\text{pool}}^* \geq \pi_{l,\text{sep}}^* \) when \( \gamma = 1 \) and \( \pi_{E,\text{pool}}^* < \pi_{l,\text{sep}}^* \) when \( \gamma = 0 \). Thus there exists a unique \( \gamma^* \in [0, 1] \) such that \( \pi_{E,\text{pool}}^* < \pi_{l,\text{sep}}^* \) if and only if \( \gamma < \gamma^* \). \( \gamma^* \) is solved from \( \pi_{E,\text{pool}}^* = \pi_{l,\text{sep}}^* \) and given as below
\[ \gamma^* \equiv \begin{cases} \gamma, & \text{if } \gamma \geq \frac{1+\alpha}{1+2\alpha}, \\ \gamma^* & \text{otherwise} \end{cases} \quad (A23) \]
in which \( s = \sqrt{\kappa_k(\kappa_k \gamma - \kappa_k)}(1+\alpha) \). And one can also verify that when \( \gamma^* \) is defeated.

**Step 2.** To show that the separating equilibrium \( \epsilon_{\text{sep}}^* \equiv ((q_{E,\text{sep}}^*, p_{E,\text{sep}}^*), (q_{I,\text{sep}}^*, p_{I,\text{sep}}^*), \mu_{\text{sep}}^*) \) exists when \( \gamma < \gamma^* \), we just need to show that,

1. given the belief system \( \mu_{\text{sep}}^*(l|q, p) \) and the \( E \)-type’s strategy \((q_{E,\text{sep}}^*, p_{E,\text{sep}}^*) \), the \( I \)-type cannot be strictly better off by deviating from strategy \((q_{I,\text{sep}}^*, p_{I,\text{sep}}^*) \) to any other strategy \((q, p)\), i.e.,
   \[ \pi_{l,\text{sep}}^* \geq \pi_I(q, p|\mu_{\text{sep}}^*(l|q, p)); \]
(2) given the belief system \( \mu_{se}^*(l|q, p) \) and the I-type’s strategy \((q_{I, sep}^*, p_{I, sep}^*)\), the E-type cannot be better off by deviating from strategy \((q_{E, sep}^*, p_{E, sep}^*)\) to any other strategy \((q, p)\), i.e., \(\pi_{E, sep}^* \geq \pi_E(q, p|\mu_{se}^*(l|q, p))\).

By the definition of \(\epsilon_{se}^*\) (see Lemma A1), obviously \(\pi_{I, sep}^* \geq \pi_I(q, p|\mu_{se}^*(l|q, p))\) for any \((q, p) \in \Phi\). And when \(\gamma < \gamma^*\), by the definition of \(\gamma^*\) in Step 1, we have \(\pi_{I, sep}^* > \pi_{I, pool}^*\) for any \((q, p) \notin \Phi\). Therefore, we have shown that \(\pi_{I, sep}^* \geq \pi_I(q, p|\mu_{se}^*(l|q, p))\) for any \((q, p)\) when \(\gamma < \gamma^*\).

By the definition of \(\Phi\), \(\pi_{E, sep}^* \geq \pi_E(q, p|\mu_{se}^*(l|q, p))\) for any \((q, p) \in \Phi\). And also \(\pi_{E, sep}^* = \max_{q \geq 0, p \geq 0} \pi_E(q, p|\mu_{se}^*(l|q, p))\) for any \((q, p) \notin \Phi\). Thus \(\pi_{E, sep}^* \geq \pi_E(q, p|\mu_{se}^*(l|q, p))\) for any \((q, p)\).

**Step 3.** We show that \(\epsilon_{se}^*\) is undefeated when \(\gamma < \gamma^*\). Let \(\epsilon_{se} = ((q_{E, sep}^*, p_{E, sep}^*), (q_{I, sep}^*, p_{I, sep}^*), \mu_{se}^*)\) denote any separating equilibrium in which the I-type’s equilibrium strategy is \((q_{I, sep}^*, p_{I, sep}^*) \in \Phi \setminus (q_{E, sep}^*, p_{E, sep}^*)\) and the consumer’s belief is \(\mu_{se}^*\). From the definition of \(\epsilon_{se}^*\), we know that \(\pi_{I, sep}(\epsilon_{se}) \leq \pi_{I, sep}^*\) and \(\pi_{E, sep}(\epsilon_{se}) = \pi_{E, sep}^*\), in other words, the firm’s profit at \(\epsilon_{se}\) is less than or equal to that at \(\epsilon_{se}^*\). Thus, \(\epsilon_{se}\) cannot defeat \(\epsilon_{se}^*\) by the definition of one equilibrium defeats another (Definition 1).

Let \(\epsilon_{pool} = ((q_{pool}^*, p_{pool}^*), \mu_{pool}^*)\) denote any pooling equilibrium in which both types of the firm’s equilibrium strategy is \((q_{pool}^*, p_{pool}^*)\) and the consumer’s belief is \(\mu_{pool}^*\). From the definition of \(\gamma^*\) in Equation (A23), for any pooling equilibrium \(\epsilon_{pool}^*\), we know \(\pi_{I, pool}(\epsilon_{pool}) \leq \pi_{I, pool}^*\) if \(\gamma < \gamma^*\). If \(\pi_{E, pool} \leq \pi_{E, sep}^*\), then \(\epsilon_{pool}\) cannot defeat \(\epsilon_{se}^*\). If \(\pi_{E, pool} > \pi_{E, sep}^*\), \(\epsilon_{pool}\) defeats \(\epsilon_{se}^*\) if and only if \(\mu_{sep}^*(l|q_{pool}, p_{pool}) \neq 0\). Thus, for \(\epsilon_{se}^*\) to be undefeated by any \(\epsilon_{pool}\), we just need to show \(\mu_{sep}^*(l|q_{pool}, p_{pool}) = 0\). It is obvious that \((q_{pool}, p_{pool}) \notin \Phi\) from the definition of \(\Phi \equiv \{(q, p): \pi_{E, sep}^* \geq \pi_E(q, p|\mu(l|q, p) = 1)\}\). From the belief system \(\mu_{se}^*\) as given in Equation (A14), we know \(\mu_{se}^*(l|q_{pool}, p_{pool}) = 0\). Therefore, \(\epsilon_{se}^*\) is undefeated.

**Step 4.** We first show that all the other separating equilibria except \((q_{E, sep}^*, p_{E, sep}^*), (q_{I, sep}^*, p_{I, sep}^*)\) are defeated by \(\epsilon_{se}^*\), and then show that all pooling equilibria are defeated by \(\epsilon_{se}^*\) when \(\gamma < \gamma^*\).

First, let \(\epsilon_{se} = ((q_{E, sep}^*, p_{E, sep}^*), (q_{I, sep}^*, p_{I, sep}^*), \mu_{se}^*)\) denote any separating equilibrium in which the I-type’s equilibrium strategy is \((q_{I, sep}^*, p_{I, sep}^*) \in \Phi \setminus (q_{E, sep}^*, p_{E, sep}^*)\) and the consumer’s belief is \(\mu_{se}^*\). In the
proof of Lemma A1, we have shown that $\pi_{i,sep}(q_{sep}^i, p_{sep}^i) > \pi_{i,sep}(q_{sep}, p_{sep})$ for $(q_{sep}, p_{sep}) \neq (q_{sep}^i, p_{sep}^i)$. From the definition of one equilibrium defeats another (Definition 1), to show $\epsilon_{sep}$ defeats $\epsilon_{sep}$, we just need to show $\mu_{sep}(l | q_{sep}^i, p_{sep}^i) \neq 1$. Suppose $\mu_{sep}(l | q_{sep}^i, p_{sep}^i) = 1$, then the $I$-type firm will be strictly better off by deviating from $(q_{sep}, p_{sep})$ to $(q_{sep}^i, p_{sep}^i)$ under the belief system $\mu_{sep}$ since $\pi_i(q_{sep}^i, p_{sep}^i | \mu_{sep}(l | q_{sep}^i, p_{sep}^i) = 1) = \pi_{i,sep}(q_{sep}^i, p_{sep}^i) > \pi_{i,sep}(q_{sep}, p_{sep})$, which contradicts the fact that $\epsilon_{sep}$ is an equilibrium. Thus, $\mu_{sep}(l | q_{sep}^i, p_{sep}^i) \neq 1$. Therefore, all the other separating equilibria except $((q_{E,sep}^i, p_{E,sep}^i), (q_{sep}^i, p_{sep}^i))$ are defeated.

Second, let $\epsilon_{pool} = (q_{pool}, p_{pool}, \mu_{pool})$ denote any pooling equilibrium in which both types of the firm’s equilibrium strategy is $(q_{pool}, p_{pool})$ and the consumer’s belief is $\mu_{pool}$. We want to show that $\epsilon_{pool}$ is defeated by $\epsilon_{sep}^*$. From the definition of $\gamma^*$, when $\gamma < \gamma^*$, we have $\pi_{i,sep}^* > \pi_{i,pool}^* = \max_{q,p} \pi_{i,pool}(q_{pool}, p_{pool}) \geq \pi_{i,pool}(q_{pool}, p_{pool})$. To show $\epsilon_{sep}$ defeats $\epsilon_{pool}$, we just need to show $\mu_{pool}(l | q_{pool}, p_{pool}) \neq 1$. Suppose $\mu_{pool}(l | q_{pool}, p_{pool}) = 1$, then the $I$-type firm can profitably deviate from $(q_{pool}, p_{pool})$ to $(q_{pool}^*, p_{pool}^*)$ since $\pi_i(q_{pool}^*, p_{pool}^* | \mu_{pool}(l | q_{pool}^*, p_{pool}^*) = 1) = \pi_{i,sep}^* > \pi_{i,pool}(q_{pool}, p_{pool})$. This contradicts the fact that $((q_{pool}, p_{pool}, \mu_{pool})$ is an equilibrium. Therefore, $\epsilon_{sep}$ defeats $\epsilon_{pool}$.

Therefore, when $\gamma < \gamma^*$, all separating equilibria except $((q_{E,sep}^i, p_{E,sep}^i), (q_{sep}^i, p_{sep}^i))$ and all pooling equilibria are defeated by $\epsilon_{sep}^*$.

Altogether, we have shown that when $\gamma < \gamma^*$, the least-cost separating equilibrium is the unique undefeated equilibrium outcome. □

**Lemma A4.** If $\gamma > \gamma^*$, the most-efficient pooling equilibrium characterized in Lemma A2 exists and is the unique undefeated equilibrium outcome.

**Proof of Lemma A4.** Let $\epsilon_{pool}^* = (q_{pool}, p_{pool}^*, \mu_{pool}^*)$ denote the most-efficient pooling equilibrium characterized in Lemma A2 with belief $\mu_{pool}^*$ as given in Equation (A22). We complete the proof in three steps. Step 1, we show that when $\gamma > \gamma^*$, $\epsilon_{pool}^*$ exists. Step 2, we show that when $\gamma > \gamma^*$, $\epsilon_{pool}^*$ is undefeated. Step 3, we show that when $\gamma > \gamma^*$, all the other pooling equilibria except $(q_{pool}, p_{pool}^*)$ and all separating equilibrium are defeated.

**Step 1.** To show that the pooling equilibrium $\epsilon_{pool}^* = (q_{pool}, p_{pool}^*, \mu_{pool}^*)$ exists when $\gamma > \gamma^*$, we just need to show that,
(1) given the belief system \( \mu_{\text{pool}}(l|q,p) \) and the \( E \)-type’s strategy \((q_{\text{pool}}^*,p_{\text{pool}}^*)\), the \( I \)-type cannot be strictly better off by deviating from \((q_{\text{pool}}^*,p_{\text{pool}}^*)\) to any other strategy \((q,p)\), i.e., \( \pi_{I,pool} > \pi_I(q,p|\mu_{\text{pool}}(l|q,p)) \);

(2) given the belief system \( \mu_{\text{pool}}(l|q,p) \) and the \( I \)-type’s strategy \((q_{\text{pool}}^*,p_{\text{pool}}^*)\), the \( E \)-type cannot be strictly better off by deviating from \((q_{\text{pool}}^*,p_{\text{pool}}^*)\) to any other strategy \((q,p)\), i.e., \( \pi_{I,pool} \geq \pi_I(q,p|\mu_{\text{pool}}(l|q,p)) \).

By the definition of \( \epsilon_{\text{pool}}^* \) (see Lemma A2), \( \pi_{I,pool} = \max_{q \geq 0, p \geq 0} \pi_I(q,p|\mu(l|q,p) = \gamma) \geq \max_{q \geq 0, p \geq 0} \pi_I(q,p|\mu_{\text{pool}}(l|q,p)) \), thus \( \pi_{I,pool} \geq \pi_I(q,p|\mu_{\text{pool}}(l|q,p)) \) for any \((q,p)\).

And when \( \gamma > \gamma^* \), one can show \( \pi_{E,\text{pool}} > \pi_{E,\text{sep}} = \max_{q \geq 0, p \geq 0} \pi_E(q,p|\mu(l|q,p) = 0) \). Thus, \( \pi_{E,\text{pool}} \geq \pi_E(q,p|\mu_{\text{pool}}(l|q,p)) \) for any \((q,p)\). Therefore both the above requirements are satisfied.

**Step 2.** We show that \( \epsilon_{\text{pool}}^* \) is undefeated when \( \gamma > \gamma^* \). Let \( \epsilon_{\text{pool}} \equiv (q_{\text{pool}}, p_{\text{pool}}), \mu_{\text{pool}} \) denote any other pooling equilibrium in which \((q_{\text{pool}}, p_{\text{pool}}) \neq (q_{\text{pool}}^*, p_{\text{pool}}^*) \) with belief system \( \mu_{\text{pool}} \). We know from Lemma A2 that \( \pi_{I,pool}(q_{\text{pool}}, p_{\text{pool}}) \leq \pi_{I,pool}^* \). If \( \pi_{E,\text{pool}}(q_{\text{pool}}, p_{\text{pool}}) \leq \pi_{E,\text{pool}}^* \), then \( \epsilon_{\text{pool}} \) cannot defeat \( \epsilon_{\text{pool}}^* \). If \( \pi_{E,\text{pool}}(q_{\text{pool}}, p_{\text{pool}}) > \pi_{E,\text{pool}}^* \), \( \epsilon_{\text{pool}} \) defeats \( \epsilon_{\text{pool}}^* \) if and only if \( \mu_{\text{pool}}(l|q_{\text{pool}}, p_{\text{pool}}) \geq \gamma \). Thus, for \( \epsilon_{\text{pool}}^* \) to be not defeated by \( \epsilon_{\text{pool}} \), it is sufficient to show that \( \mu_{\text{pool}}(l|q_{\text{pool}}, p_{\text{pool}}) = 0 \), which is obvious true since \((q_{\text{pool}}, p_{\text{pool}}) \neq (q_{\text{pool}}^*, p_{\text{pool}}^*) \). Thus, \( \epsilon_{\text{pool}}^* \) cannot be defeated by \( \epsilon_{\text{pool}} \).

Let \( \epsilon_{\text{sep}} \equiv (q_{E,\text{sep}}, p_{E,\text{sep}}), (q_{I,\text{sep}}, p_{I,\text{sep}}), \mu_{\text{sep}} \) denote any separating equilibrium with consumer’s belief \( \mu_{\text{sep}} \). When \( \gamma > \gamma^* \), both types of firms prefer \( \epsilon_{\text{pool}} \) to \( \epsilon_{\text{sep}}^* \) (i.e., \( \pi_{I,\text{pool}} > \pi_{I,\text{sep}}^* \) and \( \pi_{E,\text{pool}} > \pi_{E,\text{pool}}^* \)). Hence, at any separating equilibrium \( \epsilon_{\text{sep}} \), we have \( \pi_{I,\text{sep}}(\epsilon_{\text{sep}}) \leq \pi_{I,\text{pool}} < \pi_{I,\text{pool}}^* \) and \( \pi_{E,\text{sep}}(\epsilon_{\text{sep}}) = \pi_{E,\text{sep}}^* \). Thus, \( \epsilon_{\text{sep}} \) cannot defeat \( \epsilon_{\text{pool}}^* \) by definition.

Therefore, \( \epsilon_{\text{pool}}^* \) is undefeated when \( \gamma > \gamma^* \).

**Step 3.** Similar to the analysis in the proof of Lemma A3 (Step 4), we first show that all the other pooling equilibria except \((q_{\text{pool}}^*, p_{\text{pool}}^*)\) are defeated by \( \epsilon_{\text{pool}}^* \), and then show that all separating equilibria are defeated by \( \epsilon_{\text{pool}}^* \) when \( \gamma > \gamma^* \).

First, let \( \epsilon_{\text{pool}} \equiv (q_{\text{pool}}, p_{\text{pool}}), \mu_{\text{pool}} \) denote any pooling equilibrium in which both types of the firm’s equilibrium strategy is \((q_{\text{pool}}, p_{\text{pool}}) \neq (q_{\text{pool}}^*, p_{\text{pool}}^*) \) and the consumer’s belief is \( \mu_{\text{pool}} \). Since \((q_{\text{pool}}, p_{\text{pool}}^*) \) is the \( I \)-type firm’s (unique) most-efficient pooling equilibrium strategy, we have
We want to show that $\pi^*_{pool}(q^*_{pool}, p^*_{pool}) > \pi^*_{pool}(q^*_{pool}, p^*_{pool})$. We need to consider two cases. First, when $\pi^*_E(q^*_{pool}, p^*_{pool}) \geq \pi^*_E(q^*_{pool}, p^*_{pool})$, we just need to show that $\mu^*_{pool}(l|q^*_{pool}, p^*_{pool}) < \gamma$. Second, when $\pi^*_E(q^*_{pool}, p^*_{pool}) < \pi^*_E(q^*_{pool}, p^*_{pool})$, we just need to show that $\mu^*_{pool}(l|q^*_{pool}, p^*_{pool}) \neq \gamma$. In the equilibrium $\epsilon_{pool}$, no profitable deviation by the $I$-type firm implies that $\mu^*_{pool}(l|q^*_{pool}, p^*_{pool}) < \gamma$, which completes, in both cases above, the proof that $\epsilon_{pool}^*$ defeats $\epsilon_{pool}$.

Second, let $\epsilon_{sep} = ((q^*_{sep}, p^*_{sep}), (q_{l,sep}, p_{l,sep}), \mu_{sep})$ denote any separating equilibrium with the consumer’s belief $\mu_{sep}$. We want to show that $\epsilon_{sep}$ is defeated by $\epsilon_{pool}^*$ when $\gamma > \gamma^*$. Note that when $\gamma > \gamma^*$, we have $\pi^*_{l,sep} > \pi^*_l = \max_{(q,p) \in \Phi} \pi_l(q,p) \geq \pi_{l,sep}(q_{l,sep}, p_{l,sep})$ and $\pi^*_{E,sep} > \pi^*_E = \pi_{E,sep}(q^*_{E,sep}, p^*_E)$. To show that $\epsilon_{pool}^*$ defeats $\epsilon_{sep}$, we just need to show $\mu_{sep}(l|q^*_{pool}, p^*_{pool}) \neq \gamma$. Suppose $\mu_{sep}(l|q^*_{pool}, p^*_{pool}) = \gamma$, then the $I$-type firm can profitably deviate from $(q_{l,sep}, p_{l,sep})$ to $(q^*_{pool}, p^*_{pool})$ since $\pi_I(q^*_{pool}, p^*_{pool}, \mu_{sep}(l|q^*_{pool}, p^*_{pool}) = \gamma) = \pi^*_l > \pi_{l,sep}(q_{l,sep}, p_{l,sep})$. This contradicts the fact that $\epsilon_{sep}$ is an equilibrium. Thus, $\mu_{sep}(l|q^*_{pool}, p^*_{pool}) \neq \gamma$, which completes the proof that $\epsilon_{pool}^*$ defeats $\epsilon_{sep}$.

Therefore, when $\gamma > \gamma^*$, all pooling equilibria except $(q^*_{pool}, p^*_{pool})$ and all separating equilibria are defeated by $\epsilon_{pool}^*$.

Altogether, we have shown that when $\gamma > \gamma^*$, the most-efficient pooling equilibrium is the unique undefeated equilibrium outcome. □

**Corollary 1.** In the costless-separating parameter region (i.e., $\frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha}$), $q^*_{l,sep} = \bar{q}_I^*$, $p^*_{l,sep} = \bar{p}_I^*$ and $\pi^*_{l,sep} = \bar{\pi}_I^*$; in the costly-separating parameter region (i.e., $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma < \gamma^*$), $q^*_{l,sep} < \bar{q}_I^*$, $p^*_{l,sep} < \bar{p}_I^*$ and $\pi^*_{l,sep} < \bar{\pi}_I^*$; in the pooling parameter region (i.e., $\frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}$ and $\gamma > \gamma^*$), $q^*_{pool} < \bar{q}_E^*$, $q^*_{pool} < \bar{q}_E^*$, $p^*_{pool} < \bar{p}_E^*$, $\pi^*_{pool} < \bar{\pi}_E^*$ and $\pi^*_{E,pool} > \bar{\pi}_E^*$.

**Proof of Corollary 1.** $(\bar{q}_j^*, \bar{p}_j^*$, $\bar{\pi}_j^*)$ is given in Proposition 1 (see Equations (A3) and (A4)), and $(q^*_{l,sep}, p^*_{l,sep}, \pi^*_{l,sep})$ and $(q^*_{pool}, p^*_{pool}, \pi^*_{pool})$ for $j \in \{I, E\}$ are given in the proof of Proposition 2 (see Equations (A5), (A6), (A7), (A15),(A16), (A20),(A21)). From Equations (A5)-(A7), we can see that when $\frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha}$, $q^*_{l,sep} < \bar{q}_I^*$, $p^*_{l,sep} = \bar{p}_I^*$ and $\pi^*_{l,sep} = \bar{\pi}_I^*$.
In the costly-separating region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma < \gamma^* \)), we have
\[
\bar{q}_I^* - q_{I,sep}^* = \left\{ \begin{array}{ll}
\left( \sqrt{k_E(k_I-k_E)\alpha(1+\alpha)} - \alpha(k_I-k_E) \right) \theta_H \frac{\lambda}{2k_E(1+2\alpha)k_E-k_I(k_E-k_I)} & , \text{if } \lambda \geq \frac{\theta^2}{\theta_H^2} \\
\left( \sqrt{k_E(k_I-k_E)\alpha(1+\alpha)} - \alpha(k_I-k_E) \right) \theta_L & , \text{otherwise}
\end{array} \right.
\]

The part of the numerator in the above \( \bar{q}_I^* - q_{I,sep}^* \) expression \( \sqrt{k_E(k_I-k_E)\alpha(1+\alpha)} - \alpha(k_I-k_E) \) is equivalent to \( \frac{k_E}{k_I} > \frac{\alpha}{1+2\alpha} \), so \( \bar{q}_I^* - q_{I,sep}^* > 0 \) in the costly-separating region.

Now we show \( p_I^* - p_{I,sep}^* = \bar{p}_I^* - \bar{p}_{I,sep}^* > 0 \). Note that \( \bar{p}_I(q) \) (see Equation (A3)) is increasing in \( q \) since \( \frac{dp_I(q)}{dq} = \frac{2k_Ek_I(1+2\alpha)\alpha(1+\alpha)\theta_H^2 \lambda}{(k_I-k_E)^2(\alpha(1+\alpha) - \theta_H^2 \lambda)} > 0 \). Thus, \( \bar{q}_I^* > q_{I,sep}^* \) leads to \( \bar{p}_I(q_I^*) > \bar{p}_I(q_{I,sep}^*) \).

The profit difference between the symmetric-information and asymmetric-information cases is calculated below:
\[
\bar{\pi}_I^* - \pi_{I,sep}^* = \left\{ \begin{array}{ll}
\frac{(k_I-k_E)(1+\alpha)^2k_E(k_I-k_E)+k_I(k_I(1+\alpha)-2\alpha(1+2\alpha))}{4k_Ek_I(1+2\alpha)(k_E(1+2\alpha)-k_I(1+2\alpha)^2)} & , \text{if } \lambda \geq \frac{\theta^2}{\theta_H^2} \\
\frac{(k_I-k_E)(1+\alpha)^2k_E(k_I-k_E)+k_I(k_I(1+\alpha)-2\alpha(1+2\alpha))}{4k_Ek_I(1+2\alpha)(k_E(1+2\alpha)-k_I(1+2\alpha)^2)} & , \text{otherwise}
\end{array} \right.
\]

where \( s = \sqrt{k_E(k_I-k_E)\alpha(1+\alpha)} \). Let us define the part of the numerator in the above \( \bar{\pi}_I^* - \pi_{I,sep}^* \) expression as a function \( f(k_E) \equiv (1+2\alpha)^2k_E(k_I-k_E)+k_I(k_I(1+\alpha)-2\alpha(1+2\alpha)) \). One can easily verify that the \( f(k_E) \) increases in \( k_E \) when \( k_I \leq k_E < k_I \). Thus for any \( k_I < k_E < k_If(k_E) > f\left(\frac{1+\alpha}{1+2\alpha}k_I\right) = 0 \). Therefore we have \( \bar{\pi}_I^* - \pi_{I,sep}^* > 0 \) in the costly-separating region.

In the pooling region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma > \gamma^* \)), similarly, we can show that \( q_{pool}^* < \bar{q}_I^* < \bar{q}_E^* \), \( p_{pool}^* < \bar{p}_I^* < \bar{p}_E^* \), \( \pi_{I,pool}^* < \bar{\pi}_I^* \) and \( \pi_{E,pool}^* > \bar{\pi}_E^* \). \( \Box \)

**PROPOSITION 3.** (a) The \( j \)-type firm’s profit is always decreasing in \( k_j \), i.e. \( \frac{d\pi_{j,sep}^*}{dk_j} < 0 \) and \( \frac{d\pi_{j,pool}^*}{dk_j} < 0 \), for \( j \in \{I,E\} \); (b) the \( E \)-type firm’s profit is always higher than the \( I \)-type firm, i.e., \( \pi_{E,sep}^* > \pi_{I,sep}^* \) when \( \gamma < \gamma^* \) and \( \pi_{E,pool}^* > \pi_{I,pool}^* \) when \( \gamma > \gamma^* \).

**PROOF OF PROPOSITION 3.** (a) \( \pi_{E,sep}^* = \bar{\pi}_E^* \), \( \pi_{I,sep}^* \), \( \pi_{E,pool}^* \) and \( \pi_{I,pool}^* \) are given in Equations (A4), (A7), (A21) and (A20), respectively.

In the costless-separating region (i.e., \( \frac{k_E}{k_I} < \frac{1+\alpha}{1+2\alpha} \)), differentiating \( \pi_{j,sep}^* \) with respect to \( k_j \), we have...
It is easy to verify that in the above expression, the part of the denominator $k_j(1 + \alpha - \gamma) > 0$, and the part of the numerator $k_j(1 + \alpha + 2\gamma) - \alpha^2(2 - 5\gamma + \gamma^2) > 0$. Thus we can directly see that $\frac{d\pi^*_{E, pool}}{dk_E} < 0$. At last $\frac{d\pi^*_{I, pool}}{dk_I} < 0$ is obvious.

(b) In the costless-separating region (i.e., $\frac{k_E}{k_I} < \frac{1 + \alpha}{1 + 2\alpha}$), we have
\[
\pi_{E,\text{sep}}^* - \pi_{I,\text{sep}}^* = \begin{cases} 
\frac{(k_I - k_E)(1 + \alpha)\theta_H^2}{4k_Ek_I(1 + 2\alpha)}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
\frac{(k_I - k_E)(1 + \alpha)\theta_E^2}{4k_Ek_I(1 + 2\alpha)}, & \text{otherwise}
\end{cases}
\]

In the costly-separating region (i.e., \( \frac{k_E}{k_I} > \frac{1 + \alpha}{1 + 2\alpha} \) and \( \gamma < \gamma^* \)), we have

\[
\pi_{E,\text{sep}}^* - \pi_{I,\text{sep}}^* = \begin{cases} 
\frac{(k_I - k_E)(1 + \alpha)(k_E + k_I\alpha - 2s)\theta_H^2}{4k_E(k_E + 2k_I\alpha - k_E)^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
\frac{(k_I - k_E)(1 + \alpha)(k_E + k_I\alpha - 2s)\theta_E^2}{4k_E(k_E + 2k_I\alpha - k_E)^2}, & \text{otherwise}
\end{cases}
\]

where \( s = \sqrt{k_E(k_I - k_E)\alpha(1 + \alpha)} \). Let us define the part of the numerator in the above \( \pi_{E,\text{sep}}^* - \pi_{I,\text{sep}}^* \) expression as a function \( g(k_E) = k_E + k_I\alpha - 2s \). One can easily show that \( g(k_E) \) is increasing in \( k_E \) when \( k_I \frac{1 + \alpha}{1 + 2\alpha} \leq k_E < k_I \). Thus for any \( k_I \frac{1 + \alpha}{1 + 2\alpha} < k_E < k_I \), we have \( g(k_E) \geq g(k_I \frac{1 + \alpha}{1 + 2\alpha}) = \frac{k_I}{1 + 2\alpha} > 0 \). Therefore \( \pi_{E,\text{sep}}^* > \pi_{I,\text{sep}}^* \).

In the pooling region (i.e., \( \frac{k_E}{k_I} > \frac{1 + \alpha}{1 + 2\alpha} \) and \( \gamma > \gamma^* \)), we have

\[
\pi_{E,\text{pool}}^* - \pi_{I,\text{pool}}^* = \begin{cases} 
\frac{(k_I - k_E)(1 + \alpha)\theta_H^2}{4\left(k_I(1 + \alpha(2 - \gamma)) - k_E\alpha(1 - \gamma)\right)^2}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
\frac{(k_I - k_E)(1 + \alpha)\theta_E^2}{4\left(k_I(1 + \alpha(2 - \gamma)) - k_E\alpha(1 - \gamma)\right)^2}, & \text{otherwise}
\end{cases}
\]

which is obviously greater than 0. □

**Lemma 1.** (a) In the costly-separating parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1 + \alpha}{1 + 2\alpha} \) and \( \gamma < \gamma^* \)), the I-type firm’s quality \( q_{l,\text{sep}}^* \) and price \( p_{l,\text{sep}}^* \) decrease in \( \alpha \) and are independent of \( \gamma \); (b) in the pooling parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1 + \alpha}{1 + 2\alpha} \) and \( \gamma > \gamma^* \)), the equilibrium quality \( q_{p,\text{pool}}^* \) and price \( p_{p,\text{pool}}^* \) decrease in \( \alpha \) and increase in \( \gamma \); (c) when \( \gamma = \gamma^* \) or \( \alpha = \alpha^* \), \( q_{l,\text{sep}}^* < q_{p,\text{pool}}^* < q_{E,\text{sep}}^* \) and \( q_{p,\text{pool}}^* > \gamma q_{l,\text{sep}}^* + (1 - \gamma)q_{E,\text{sep}}^* \).

**Proof of Lemma 1.** (a) \( q_{l,\text{sep}}^* \) and \( p_{l,\text{sep}}^* \) are given in Lemma A1 (see Equations (A5) and (A6)). They are obviously independent of \( \gamma \). When \( \frac{k_E}{k_I} > \frac{1 + \alpha}{1 + 2\alpha} \) and \( \gamma < \gamma^* \), differentiating \( q_{l,\text{sep}}^* \) with respect to \( \alpha \), we obtain

\[
\frac{dq_{l,\text{sep}}^*}{d\alpha} = \begin{cases} 
\frac{-\theta_H(k_I - k_E)(k_E + k_I\alpha - 2s)\sqrt{k_E(k_I - k_E)\alpha(1 + \alpha)}}{4(k_E + 2k_I\alpha - k_E)^2\sqrt{k_E(k_I - k_E)\alpha(1 + \alpha)}}, & \text{if } \lambda \geq \frac{\theta_L^2}{\theta_H^2}, \\
\frac{-\theta_E(k_I - k_E)(k_E + k_I\alpha - 2s)\sqrt{k_E(k_I - k_E)\alpha(1 + \alpha)}}{4(k_E + 2k_I\alpha - k_E)^2\sqrt{k_E(k_I - k_E)\alpha(1 + \alpha)}}, & \text{otherwise}
\end{cases}
\]
In the above expression, the part of the numerator \( k_E + k_I - 2\sqrt{k_E(k_I - k_E)\alpha(1 + \alpha)} > 0 \) when \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \). Thus \( \frac{dq_{i,\text{sep}}}{d\alpha} < 0 \).

Differentiating \( p_{I,\text{sep}}^{*} \) (see Equation (A21) with respect to \( \alpha \), we obtain

\[
\frac{dp_{I,\text{sep}}^{*}}{d\alpha} = \begin{cases} 
\frac{\partial \tilde{p}_{I,H}(q)}{\partial \alpha} \bigg|_{q=q_{I,\text{sep}}} + \frac{\partial \tilde{p}_{I,I}(q_{I,\text{sep}})}{\partial \alpha} \frac{dq_{I,\text{sep}}}{d\alpha}, & \text{if } \lambda \geq \frac{\theta_{L}}{\theta_{H}} \\
\frac{\partial \tilde{p}_{I,I}(q)}{\partial \alpha} \bigg|_{q=q_{I,\text{sep}}} + \frac{\partial \tilde{p}_{I,L}(q_{I,\text{sep}})}{\partial \alpha} \frac{dq_{I,\text{sep}}}{d\alpha}, & \text{if } \lambda < \frac{\theta_{L}}{\theta_{H}}
\end{cases}
\]

Obviously, \( \frac{\theta H q_{I,\text{sep}}^{*} - k_I q_{I,\text{sep}}^{*}}{(1+2\alpha)^2} < 0 \) and \( \frac{(1+\alpha)\theta H + 2ak_I q_{I,\text{sep}}^{*}}{1+2\alpha} > 0 \), for \( t \in \{H,L\} \). Note also that \( \frac{dq_{I,\text{sep}}}{d\alpha} \leq 0 \). Thus, \( \frac{dp_{I,\text{sep}}}{d\alpha} < 0 \).

(b) In the pool equilibrium parameter region (i.e., \( \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma > \gamma^{*} \)), differentiating \( q_{\text{pool}}^{*} \) and \( p_{\text{pool}}^{*} \) with respect to \( \gamma \) and \( \alpha \), we obtain,

\[
\frac{dq_{\text{pool}}^{*}}{d\gamma} = \begin{cases} 
\frac{\alpha(1+\alpha)(k_I - k_E)\theta_H}{2((1+2\alpha-\alpha)k_I - \alpha(1-\gamma)k_E)^2}, & \text{if } \lambda \geq \frac{\theta_{L}}{\theta_{H}} \\
\frac{\alpha(1+\alpha)(k_I - k_E)\theta_L}{2((1+2\alpha-\alpha)k_I - \alpha(1-\gamma)k_E)^2}, & \text{otherwise}
\end{cases}
\]

\[
\frac{dq_{\text{pool}}^{*}}{d\alpha} = \begin{cases} 
-\frac{\theta_H(k_I - k_E)(1-\gamma)}{2((1+2\alpha-\alpha)k_I - \alpha(1-\gamma)k_E)^2}, & \text{if } \lambda \geq \frac{\theta_{L}}{\theta_{H}} \\
-\frac{\theta_L(k_I - k_E)(1-\gamma)}{2((1+2\alpha-\alpha)k_I - \alpha(1-\gamma)k_E)^2}, & \text{otherwise}
\end{cases}
\]

\[
\frac{dp_{\text{pool}}^{*}}{d\gamma} = \begin{cases} 
\frac{\alpha(1+\alpha)^2(k_I - k_E)(3+6\alpha-\gamma)k_I - \alpha(1-\gamma)k_E\theta_H}{4(1+2\alpha)((1+2\alpha-\gamma)k_I - \alpha(1-\gamma)k_E)^3}, & \text{if } \lambda \geq \frac{\theta_{L}}{\theta_{H}} \\
\frac{\alpha(1+\alpha)^2(k_I - k_E)(3+6\alpha-\gamma)k_I - \alpha(1-\gamma)k_E\theta_L}{4(1+2\alpha)((1+2\alpha-\gamma)k_I - \alpha(1-\gamma)k_E)^3}, & \text{otherwise}
\end{cases}
\]

\[
\frac{dp_{\text{pool}}^{*}}{d\alpha} = \begin{cases} 
\frac{((1+\alpha)(k_E k_I k_I - k_E)(3+2\alpha(3-\gamma))(-1+\gamma) + k_E^2(1+3\alpha)(1-\gamma)^2 + k_E^2(4+3\alpha(16+15\gamma+\gamma^2)+\alpha^2(16-18\gamma+3\gamma^2))(1-\gamma)^3)}{4(1+2\alpha)^2((1+\alpha(2-\gamma)) - k_E(1-\gamma)^3)}, & \text{if } \lambda \geq \frac{\theta_{L}}{\theta_{H}} \\
\frac{((1+\alpha)(k_E k_I (1+\alpha(3+2\alpha(3-\gamma))(-1+\gamma) + k_E^2(1+3\alpha)(1-\gamma)^2 + k_E^2(4+3\alpha(16+15\gamma+\gamma^2)+\alpha^2(16-18\gamma+3\gamma^2))(1-\gamma)^3))\theta_L}{4(1+2\alpha)^2((1+\alpha(2-\gamma)) - k_E(1-\gamma)^3)}, & \text{otherwise}
\end{cases}
\]

With tedious but algebraically straightforward analysis, one can show \( \frac{dq_{\text{pool}}^{*}}{d\alpha} < 0 \), \( \frac{dq_{\text{pool}}^{*}}{d\gamma} > 0 \), \( \frac{dp_{\text{pool}}^{*}}{d\alpha} < 0 \) and \( \frac{dp_{\text{pool}}^{*}}{d\gamma} > 0 \).

(c) Plugging \( \gamma = \gamma^{*} \) (see Equation (A23) into \( q_{\text{pool}}^{*} \)), then we can obtain
\[ q_{\text{pool}} - q_{l, \text{sep}}^* = \begin{cases} \frac{(k_E(1+\alpha) - s)(k_E k_f (1+2\alpha) - k_E (k_f + 2k_f \alpha)) \theta_H}{2k_E(k_f + 2k_f \alpha - k_f \alpha)^2}, & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H} \\ \frac{(k_E(1+\alpha) - s)(k_E^2 k_f + 2k_f \alpha - k_f \alpha)^2}{2k_E(k_f + 2k_f \alpha - k_f \alpha)^2}, & \text{otherwise} \end{cases}, \]

\[ q_{\text{pool}}^* - q_{E, \text{sep}}^* = \begin{cases} \frac{-(k_f k_E)(1+2\alpha)(k_E k_f (1+2\alpha) - 2s(3+4\alpha) + k_E^2(2k_f \alpha (2+3\alpha) + s(4\alpha^2 - 1)) - k_f^2 \alpha^2 s - k_E^2 \alpha (3+4\alpha(2+\alpha))) \theta_H}{2\alpha(k_f + 2k_f \alpha - k_f \alpha)^2(k_E k_f + k_f \alpha - s)(k_f^2 + k_E^2 (2+4\alpha) - k_f (k_f + 3k_f \alpha))}, & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H} \\ \frac{-(k_f k_E)(1+2\alpha)(k_E k_f (1+2\alpha) - 2s(3+4\alpha) + k_E^2(2k_f \alpha (2+3\alpha) + s(4\alpha^2 - 1)) - k_f^2 \alpha^2 s - k_E^2 \alpha (3+4\alpha(2+\alpha))) \theta_l}{2\alpha(k_f + 2k_f \alpha - k_f \alpha)^2(k_E k_f + k_f \alpha - s)(k_f^2 + k_E^2 (2+4\alpha) - k_f (k_f + 3k_f \alpha))}, & \text{otherwise} \end{cases}, \]

where \( s = \sqrt{k_f(k_f - k_E)\alpha(1+\alpha)} \). With tedious but algebraically straightforward analysis, one can show that when \( \gamma = \gamma^* \), \( q_{\text{pool}} - q_{l, \text{sep}}^* > 0, q_{\text{pool}} - q_{E, \text{sep}}^* < 0 \) and \( q_{\text{pool}}^* - (\gamma q_{l, \text{sep}}^* + (1-\gamma)q_{E, \text{sep}}^*) > 0 \) holds.

From the definition of \( \gamma^* \) in Equation (A23), one can easily show that \( \gamma^* \) is strictly decreasing in \( \alpha \) when \( \frac{k_E}{k_f} > \frac{1+\alpha}{1+2\alpha} \). And \( \frac{k_E}{k_f} > \frac{1+\alpha}{1+2\alpha} \) is equivalent to \( \alpha > \frac{k_f - k_E}{2k_f k_f - k_f} \text{ and } k_f > \frac{k_f}{2} \). Thus there exists a unique \( \alpha^*(\gamma) \), which can be solved from \( \gamma = \gamma^*(\alpha) \), such that the unique undefeated equilibrium is the least-cost separating when \( \alpha < \alpha^* \) and the most-efficient pooling when \( \alpha > \alpha^* \). And since \( \gamma = \gamma^* \) is equivalent to \( \alpha = \alpha^* \), obviously when \( \alpha = \alpha^* \), \( q_{\text{pool}}^* - q_{l, \text{sep}}^* > 0, q_{\text{pool}}^* - q_{E, \text{sep}}^* < 0 \) and \( q_{\text{pool}}^* - (\gamma q_{l, \text{sep}}^* + (1-\gamma)q_{E, \text{sep}}^*) > 0 \) holds. □

**PROPOSITION 4.** (a) The ex-ante expected quality may be higher in a less cost-efficient market (i.e., a larger \( \gamma \)). (b) The ex-ante expected quality may be higher when consumers have stronger inequity aversion.

**PROOF OF PROPOSITION 4.** (a) Recall from Lemma 1(b), we have \( \frac{dq_{\text{pool}}^*}{d\gamma} > 0 \), that is, the equilibrium quality is increasing in \( \gamma \) in the pooling region.

(b) Lemma 1(c) shows that the expected quality increases as \( \alpha \) increases from below \( \alpha^* \) to above \( \alpha^* \). So, there exists \( \alpha_1 > \alpha^* \) and \( \alpha_2 < \alpha^* \) such that the expected quality is higher when \( \alpha \in (\alpha^*, \alpha_1) \) than that when \( \alpha \in (\alpha_2, \alpha^*) \) (as illustrated in Figure A1). This completes the proof. □
LEMMA 2. (a) In the costly-separating parameter region (i.e., \( \frac{k_E}{k_l} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma < \gamma^* \)), the I-type firm’s profit \( \pi_{I,sep}^* \) decreases in \( \alpha \) and is independent of \( \gamma \); (b) in the pool parameter region (i.e., \( \frac{k_E}{k_l} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma > \gamma^* \)), the j-type firm’s profit \( \pi_{j,pool}^* \) decreases in \( \alpha \) and increases in \( \gamma \), for \( j \in \{I, E\} \); (c) when \( \gamma = \gamma^* \) or \( \alpha = \alpha^* \), \( \pi_{I,sep}^* = \pi_{I,pool}^* \) and \( \pi_{E,sep}^* < \pi_{E,pool}^* \).

PROOF OF LEMMA 2. (a) In the costly-separating parameter region (i.e., \( \frac{k_E}{k_l} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma < \gamma^* \)), \( \pi_{I,sep}^* \) is given in Equation (A7). It is obvious that \( \frac{d\pi_{I,sep}^*}{d\gamma} = 0 \), i.e., \( \pi_{I,sep}^* \) is independent of \( \gamma \). Differentiating \( \pi_{I,sep}^* \) with respect to \( \alpha \), we obtain,

\[
\frac{d\pi_{I,sep}^*}{d\alpha} = \begin{cases} 
\frac{\gamma q_{sep}^*}{\lambda \theta_H^2(1+\alpha)} - \frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} - \frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} , & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} \\
\frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} , & \text{otherwise} 
\end{cases}
\]

It is also algebraically straightforward (albeit tedious) to show \( \frac{d\pi_{I,sep}^*}{d\alpha} < 0 \).

(b) In the pool parameter region (i.e., \( \frac{k_E}{k_l} > \frac{1+\alpha}{1+2\alpha} \) and \( \gamma > \gamma^* \)), Differentiating \( \pi_{I,pool}^* \) and \( \pi_{E,pool}^* \) with respect to \( \gamma \) and \( \alpha \), we obtain

\[
\frac{d\pi_{I,pool}^*}{d\gamma} = \begin{cases} 
-\frac{\alpha \lambda \theta_H^2(1+\alpha)^2(k_j-k_E)}{4(1+2\alpha)(1+2\alpha-\gamma)k_j-\alpha(1-\gamma)k_E} , & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} \\
\frac{\alpha \lambda \theta_H^2(1+\alpha)^2(k_j-k_E)}{4(1+2\alpha)(1+2\alpha-\gamma)k_j-\alpha(1-\gamma)k_E} , & \text{otherwise} 
\end{cases}
\]

\[
\frac{d\pi_{I,pool}^*}{d\gamma} = \begin{cases} 
\frac{\alpha \lambda \theta_H^2(1+\alpha)^2(k_j-k_E)}{4(1+2\alpha)(1+2\alpha-\gamma)k_j-\alpha(1-\gamma)k_E} , & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} \\
\frac{\alpha \lambda \theta_H^2(1+\alpha)^2(k_j-k_E)}{4(1+2\alpha)(1+2\alpha-\gamma)k_j-\alpha(1-\gamma)k_E} , & \text{otherwise} 
\end{cases}
\]

\[
\frac{d\pi_{I,sep}^*}{d\gamma} = \begin{cases} 
\frac{\gamma q_{sep}^*}{\lambda \theta_H^2(1+\alpha)} - \frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} - \frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} , & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} \\
\frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} , & \text{otherwise} 
\end{cases}
\]

\[
\frac{d\pi_{I,sep}^*}{d\alpha} = \begin{cases} 
\frac{\gamma q_{sep}^*}{\lambda \theta_H^2(1+\alpha)} - \frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} - \frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} , & \text{if } \lambda \geq \frac{\theta_H^2}{\theta_H^2} \\
\frac{\gamma q_{sep}^*}{\lambda (1+2\alpha)} , & \text{otherwise} 
\end{cases}
\]
Stronger inequity aversion (i.e., a larger \( \alpha \)), we can obtain the general condition for \( \frac{d\pi^*_E_{pool}}{d\alpha} < 0 \) when \( \lambda < \frac{\theta^*_T}{\theta^*_H} \) (i.e., both types of consumers are targeted), a very cumbersome expression omitted here. We complete the proof by providing a sufficient parameter region in which an \( H \)-type consumer’s monetary payoff decreases in \( \alpha \).

Given \( k_I = 1, k_E = 0.9, \theta_H = 1, \theta_L = 0.03 \) and \( \gamma = 0.9 \), one can verify \( \frac{dM_H}{d\alpha} < 0 \) if \( \lambda \in (0, \frac{\theta^*_T}{\theta^*_H}) \) and \( \alpha \in (0.36, 1) \), e.g., \( M_H|_{\alpha=0.5} = 0.014558 > M_H|_{\alpha=0.9} = 0.014554 \) (see Figure 4).

**PROPOSITION 7.** (a) A less cost-efficient market (i.e., large \( \gamma \)) may have higher expected social welfare; (b) Stronger inequity aversion (i.e., a larger \( \alpha \)) may lead to higher expected social welfare.

\( \mathcal{P} \)
PROOF OF PROPOSITION 7. The ex-ante expected social welfare is defined as \( E[S] = \gamma \pi_I + (1 - \gamma)\pi_E + \lambda M_H + (1 - \lambda)M_L \). Let \( E[S_{\text{sep}}] \) and \( E[S_{\text{pool}}] \) denote the ex-ante social welfare under the separating and pooling outcome, respectively. Substituting the separating outcomes into \( E[S] \), one can verify that \( \frac{dE[S_{\text{sep}}]}{d\gamma} < 0 \) and \( \frac{dE[S_{\text{pool}}]}{d\alpha} \leq 0 \) when \( \gamma > \gamma^* \), i.e., the expected social welfare is also (weakly) decreasing in \( \gamma \) and \( \alpha \) in the pooling region. One can also verify that \( \lim_{\gamma \to \gamma^* -} E[S_{\text{sep}}] < \lim_{\gamma \to \gamma^* +} E[S_{\text{pool}}] \), i.e., the expected social welfare increases as \( \gamma \) increases from below \( \gamma^* \). With similar argument to the proof of Proposition 4(b), we can show that the expected social welfare may be higher for a larger \( \gamma \) (i.e., in a less cost-efficient market). Similarly, we can easily show that the expected social welfare may be higher with stronger inequity aversion. □

PROPOSITION 8. There exists a unique \( \gamma^* \in [0,1] \) such that if \( \gamma < \gamma^* \), the separating equilibrium \((q_{j,\text{sep}}^*, p_{j,\text{sep}}^*, \pi_{j,\text{sep}}^*)\) is the unique undefeated equilibrium outcome, where \( q_{E,\text{sep}}^* = \bar{q}_E^* \), \( p_{E,\text{sep}}^* = \bar{p}_E^* \), \( \pi_{E,\text{sep}}^* = \pi_{\text{E,sep}}^* \); (b) the I-type firm’s quality, price and profit are

\[
\begin{align*}
\text{ALPHA}_1, & \quad \text{if } \tau \leq \tau^* \quad \text{or} \quad \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \\
\bar{q}_I, & \quad \text{if } \tau > \tau^* \quad \text{and} \quad \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}
\end{align*}
\]

and \( p_{I,\text{sep}}^* = \left\{ \begin{array}{ll}
\bar{p}_I, & \text{if } \tau \leq \frac{\alpha}{1+2\alpha} \quad \text{or} \quad \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \\
\theta \bar{q}_I, & \text{if } \frac{\alpha}{1+2\alpha} < \tau \leq \tau^* \quad \text{and} \quad \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \\
(1+\alpha)^2 p_{\text{pool}} - \bar{p}_E, & \text{if } \tau > \tau^* \quad \text{and} \quad \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}
\end{array} \right. \)

where \( \bar{q} = \frac{k_E(1+\alpha)\theta - \sqrt{\alpha(1+\alpha)k_E(k_I-k_E)}}{2k_E[(1+2\alpha)k_E-ak_I]} \); if \( \gamma > \gamma^* \), the pooling equilibrium \((q_{\text{pool}}^*, p_{\text{pool}}^*, \pi_{\text{pool}}^*)\) is the unique undefeated equilibrium outcome.

PROOF OF PROPOSITION 8. The proof of Proposition 8 can be completed by showing Lemmas A5 to A8. Lemma A5 identifies the unique least-cost separating equilibrium outcome (from the I-type’s perspective); Lemma A6 identifies the unique most-efficient pooling equilibrium outcome for the I-type. Lemma A7 shows that when \( \gamma < \gamma^* \) (the expression of \( \gamma^* \) is provided in Lemma A7) the least-cost separating equilibrium exists and is the unique undefeated equilibrium outcome. Lemma A8 shows that when \( \gamma > \gamma^* \) the most-efficient pooling equilibrium exists and is the unique undefeated equilibrium.

LEMMA A5. The least-cost separating equilibrium outcome is (a) the E-type firm’s quality, price and profit are: \( q_{E,\text{sep}}^* = \bar{q}_E^* \), \( p_{E,\text{sep}}^* = \bar{p}_E^* \), and \( \pi_{E,\text{sep}}^* = \bar{\pi}_E^* \); (b) the I-type firm’s quality, price and profit are
\[ q^*_{I, sep} = \begin{cases} \tilde{q}_I^*, & \text{if } \tau \leq \tau^* \text{ or } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \\ \tilde{q}, & \text{if } \tau > \tau^* \text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \end{cases} \]

\[ p^*_{I, sep} = \begin{cases} \tilde{p}_I^*, & \text{if } \tau \leq \frac{\alpha}{1+2\alpha} \text{ or } \frac{k_E}{k_I} \leq \frac{1+\alpha}{1+2\alpha} \\ \theta\tilde{q}_I^*, & \text{if } \frac{\alpha}{1+2\alpha} < \tau \leq \tau^* \text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}, \\ \frac{(1+\alpha)\theta\tilde{q}+ak_E\tilde{q}^2}{1+2\alpha}, & \text{if } \tau > \tau^* \text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha} \end{cases} \]

where \( \tilde{q} \equiv \frac{k_E(1+\alpha)\theta-\theta\sqrt{\alpha(1+\alpha)k_E(k_I-k_E)}}{2k_E[(1+2\alpha)k_E-ak_I]} \) and

\[ \tau^* = 1 - \frac{(1+\alpha)k_I k_E - \sqrt{\alpha(1+\alpha)k_E k_I(k_I-k_E)}}{(1+2\alpha)((1+2\alpha)k_E-ak_I)^2}, \]

**Lemma A6.** At the most-efficient pooling equilibrium, both types of firms will choose \( q^*_{pool} = \frac{(1+\alpha)\theta}{2[(1+2\alpha-\gamma)k_I-\gamma(1-\gamma)k_E]} \) and \( p^*_{pool} = \frac{(1+\alpha)\theta q^*_pool + ak_E q^*_pool^2}{1+2\alpha} \).

**Lemma A7.** There exists a unique \( \gamma^* \in [0,1] \) such that if \( \gamma < \gamma^* \), the least-cost separating equilibrium characterized in Lemma A5 exists and is the unique undefeated equilibrium outcome, in which

\[ \gamma^* = \begin{cases} \frac{(\alpha(2+3\alpha)-\tau(1+2\alpha)^2)k_I(k_I-1-\tau)(1+2\alpha)k_E}{(1-\tau)(1+2\alpha)(k_I-1)k_E}, & \text{if } \frac{\alpha}{1+2\alpha} < \tau \leq \tau^* \text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}, \\ \frac{-(1+2\alpha)(k_E^2(1+5\alpha+4\alpha^2)-k_E(k_E(1+4\alpha+3\alpha^2)+2\alpha s-k_I(k_E(1+\alpha)-2s(1+\alpha)))}{(\alpha(-2k_E^2(1+3\alpha+2\alpha^2)+k_E(k_E(k_E(1+\alpha)+2\alpha)+k_E(k_E(1+\alpha)-2s(1+\alpha)))}}}, & \text{if } \tau > \tau^* \text{ and } \frac{k_E}{k_I} > \frac{1+\alpha}{1+2\alpha}, \end{cases} \]

where \( s = \sqrt{k_E(k_I-1)k_E(1+\alpha)} \).

**Lemma A8.** If \( \gamma > \gamma^* \), the most-efficient pooling equilibrium characterized in Lemma A2 exists and is the unique undefeated equilibrium outcome.

The proofs of Lemma A5-A8 are similar to those of Lemma A1-A4 and are available from the authors upon request. □