Platform Competition with Endogenous Homing*

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Abstract

We consider two-sided markets in which consumers and firms endogenously determine whether they single-home (patronize only one platform), or multi-home (join competing platforms). We find that while there are many zero-profit tipping equilibrium configurations, there exists a unique symmetric price-constellation in which equilibrium participation occurs on both platforms. Even though this price-constellation is unique, three types of equilibrium allocations among consumers and firms are associated with these prices. An allocation in which all consumers single-home while firms multi-home always exists (mirroring smartphones). The second allocation has a mix of multi-homing and single-homing on both sides of the market (akin to game consoles). In the third allocation, firms single-home and some consumers multi-home (e.g., rideshare services). Competition leads to lower prices and greater market access, but results in cost duplication and disaggregates network effects compared to a monopoly platform. Moreover, cross-subsidization that increases total welfare only takes place in monopoly. Thus, which market structure results in more welfare depends on the interplay of these factors.

Keywords: two-sided markets, platforms, platform competition, multi-homing, single-homing, endogenous homing decisions, network effects, smartphones, video games and game consoles, rideshare services.
JEL Classifications: L14, L22, D40, L13.

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1 Introduction

Video game consoles and smartphones are two platforms that have grown in importance in many people's lives: Two thirds of U.S. households own a video game console, and the average time spent by gamers on their consoles is eight hours a week—the equivalent of a full work day. While this is a considerable amount of time, it pales in comparison to U.S. college students who reportedly spend as much as eight to ten hours on their phones *every day*—amounting to half their waking hours. In the U.K. 69 percent of smartphone users checked their phone within 30 minutes of waking up while 77 percent use their smartphone within an hour of going to bed, and people check their phones as often as 150 times per day.

Nearly 85 percent of 18–29 year-olds in the U.S. have smartphones, and Nielson data show a 65 percent increase in time spent using apps by Android and iPhone users over the last two years, with 18–44 year-olds using close to thirty different apps each month. The smartphone penetration rate for 25–34 year-olds was 78 percent in the U.K. and 72 percent in Italy; the numbers are slightly higher in Asia with penetration rates as high as 86 percent for South Korea, 82 percent for urban China, and 76 percent for Australia.[1]

The market structures in which platforms operate vary considerably in terms of the participation decisions that people make. When several platforms offer competing services, participants may join only one platform (called single-homing), or they may patronize several platforms (called multi-homing). For the smartphone industry, the majority of consumers single-home and only own one platform while most content (apps, videos, and music) is available on all platforms (Bresnahan et al., 2015). With video game consoles, many consumers own more than one console while others own only one; similarly, there are many games that are available on a single console while many others are available across consoles.[2]

[1] The data on gaming come from ESRP (2010), those on college students' phone usage from Roberts et al. (2014), and the data on prevalence of smartphones come from Smith (2012) in the U.S., Deloitte (2015a) in Asia Deloitte (2015b) in Australia, Deloitte (2015c) in Italy, and Deloitte (2015d) in the U.K. The Nielson data is at Nielsen (2014a,b).

[2] For example, Bungie, Sega, and Valve Corporation have all developed some video games that are exclusive to a particular console; whereas Blizzard, Electronic Arts, and Ubisoft primarily develop games that are available for all consoles.

Despite the prevalence of mixed homing configurations, the literature on platform competition generally abstracts from the issue of endogenous homing by exogenously fixing agents' homing decisions on each side of the market.[3] This leads to allocation-specific pricing decisions: exogenously fixed multi-homers face high prices as they cannot exit a platform without exiting the entire market, and exogenously fixed single-homers must be dislodged from a rival before they can be acquired as new customers. Moreover, postulating universal single- or multi-homing on one side precludes observed allocations in which a side of the market has a mix of single- and multi-homing agents. To understand how prices on each side relate to which homing decisions should be expected in equilibrium requires a model with endogenous homing decisions by agents.

We consider platform competition in which consumers and firms endogenously choose how many and which platforms to join. We show that in equilibrium different allocations of consumers and firms emerge, mirroring the configurations found on many platforms, including those for smartphones, game consoles, and ridesharing services. In comparing the multiple equilibrium allocations with allocations seen in platform industries we identify the parameters that are significant in determining which allocations are more likely to occur for a particular platform industry. We further identify under which circumstances a monopoly platform generates higher welfare than competing platforms across the potential equilibrium allocations.

One of the prominent concerns in studying platforms in the previous literature was in determining equilibrium participation: as agents choose which platforms to join, the availability of agents on the other side of the platform is critical in determining the equilibrium and potential coordination issues across the two sides of the platform can arise. Caillaud and Jullien (2003) assume that with platform competition coordination favors the incumbent platform; otherwise platforms may fail to gain a critical mass, i.e. "fail to launch." They argue this solves the "chicken and egg" problem of each side's action depending on the other

[3] See, e.g., Caillaud and Jullien (2003), Armstrong (2006), Hagiu (2006), Rochet and Tirole (2006) and subsequent work.

side's action. Hagiu (2006) shows the chicken and egg problem does not occur when sides join platforms sequentially; and Jullien (2011) investigates this further over a broader class of multi-sided markets. Ambrus and Argenziano (2009) show how prices can endogenize heterogeneity and steer agents to asymmetric allocation configurations; and Lee (2013) investigates the video game market, and shows that Xbox was able to enter the video game market because exclusive contracts with game developers allowed Microsoft to overcome the coordination issue. In addition to the primary participation decision, the role of beliefs and information play an important role in determining the equilibria as examined by Hagiu and Hałaburda (2014), who consider 'passive' price expectations on one side in contrast to complete information about prices on the second side, and Gabszewicz and Wauthy (2014) who also consider active and passive beliefs in determining platform allocations, or Hałaburda and Yehezkel (2013), who show how multi-homing alleviates coordination issues tied to asymmetric information.[4] These models of platform competition generally exogenously fix whether agents single-home or multi-home, whereas in our model the critical participation decision resolves not only whether and which platform to join, but also how many platforms to join.

In extending Armstrong (2006), Armstrong and Wright (2007) show how the common "competitive bottleneck" equilibrium, in which all consumers single-home and all firms multi-home, can endogenously arise. While this allocation is frequently seen in platform industries, a common feature of many of these markets is that there are a mix of single-homers and multi-homers. In order to understand how this affects competition, agents' allocation decisions must be derived endogenously as part of the equilibrium.

Another paper that allows for endogenous homing is Rochet and Tirole (2003), where buyers and sellers engage in a matched transaction that takes place on a platform. The model is best illustrated by the credit card market: it is assumed that card-issuers only charge per transaction and do not charge card-users or merchants any membership fees, so all agents

[4] In normative analyses Weyl (2010) and White and Weyl (2016) fully mitigate coordination issues through insulating tariffs in which prices are contingent upon participation, thus resolving failure-to-launch and multiple equilibrium concerns.

can costlessly multi-home. However, because card-users choose which card to use when they make a purchase, merchants may wish to single-home in order to limit the customers' options of which card to use. In contrast to per-transaction fees, many platform markets—including smartphones and gaming systems—are characterized by access or membership fees. Although we allow for both usage and stand-alone membership benefits, we consider platforms that compete by setting membership/access fees, as this relates more closely to the markets that we are concerned with.

There is also a nascent literature on endogenous multi-homing in media markets. There the focus is on determining the pricing structure on the advertising-side of the platform. Thus, in Anderson et al. (2015) consumers are not charged to join a platform, so the only cost is a nuisance cost of advertising. However, as consumers do not observe the prices platforms charge to advertisers, platforms cannot affect consumer participation. In Athey et al. (2016) the focus is on endogenous homing on the ad-side, while assuming that consumer allocations are exogenously fixed; and in a related model Ambrus et al. (2016) allow platform pricing to affect consumer participation; however, participation on any given platform does not affect demand on other platforms, so there is no competition between platforms for consumers. In contrast to these studies of media markets, we explicitly model the competition that takes place between platforms to attract agents on both sides of the market and we allow agents from both sides to endogenously make their homing decisions.

In allowing for endogenous homing decisions, we find two classes of equilibrium outcomes: Tipping outcomes, in which all active agents are found on one platform; and non-tipping outcomes, in which agents split across platforms. In the case of tipping, there is often a continuum of possible platform prices to support the equilibrium outcome, although profits are the same across all of these price-configurations. And while all market participants are found on one platform, due to the ability to endogenously multi-home, some agents may choose to multi-home and therefore also participate on the rival platform so the rival platform need not be entirely foreclosed.

In contrast to the continuum of possible prices that support the tipping outcomes, there is a unique price constellation that emerges for the case of a non-tipped market in which participation goes across platforms with each platform having some patrons that do not join the rival platform. In this case however—and again in some contrast to the tipping outcome—concurrent with the unique price constellation, several participation allocations can emerge. Specifically, there are as many as three possible types of equilibrium participation configurations, each reflecting a commonly found market structure.[5]

The first allocation equilibrium—which can always arise in equilibrium—has all consumers single-homing, whereas all firms multi-home. This is the allocation generally observed in the market for smartphones. Indeed, Bresnahan et al. (2015) find that the practice of multi-homing by app producers—that is their simultaneous presence on competing platforms—insures against a tipping in the market that would concentrate all economic activity on a single platform. The second equilibrium allocation is one in which there is a mix of single-homing and multi-homing on both sides. This is the division found in the market for game consoles (Lee, 2013): Many consumers have only one system, but others will buy competing platforms; and while some games are available only on one system, others can be purchased for multiple machines. In the third equilibrium allocation all firms single-home, whereas some or all consumers multi-home. An example of this is found in ride-sharing services such as Uber and Lyft or the nascent restaurant reservations market with OpenTable and the more recent Yelp Reservations. Here firms (drivers or restaurants) are members of one system, whereas consumers either have a preferred platform and single-home or they make use of both platforms and multi-home.

When comparing welfare between competing platforms and a monopoly platform, we find that lower prices and stronger platform differentiation favors competition, whereas more concentrated network effects and potential cost savings from not having to make apps compatible

[5]Interestingly, though, the case where agents on both sides of the market choose to single-home and the case where all agents multi-home do not actually arise in equilibrium, even though the former is a constellation that is frequently postulated in the literature.

across multiple platforms tend to favor a monopoly platform. In addition to these factors, we find that when firms' presence on the platform greatly benefit consumers, then a monopoly platform may increase welfare by subsidizing firm entry. Due to non-appropriability if firms multi-home in competition, competing platforms forgo such welfare enhancing investments, which tends to also favor the monopoly platform market in terms of total welfare.

2 The Model

2.1 Platforms

Two groups of agents can benefit from interaction, but require an intermediary in order to do so. The benefits from the interaction to an agent in one group depends on the number of agents of the other group that are made available through the intermediary. This intermediary—the platform—brings these two groups together by charging agents in each group a price to participate on the platform. We consider two platforms, indexed by $X \in \{A, B\}$.

The sequence of play is as follows: first the platforms simultaneously (and non-cooperatively) set prices to each of the two groups, and then the two groups of agents simultaneously choose whether and which platforms to join.

Agents on each side of the platform are described by continuous variables. Agents on Side 1 are consumers or buyers, and agents on Side 2 are firms or sellers. The number of consumers that join Platform X is $n_1^X \in [0, \overline{N}_1]$, and the number of firms on Platform X is $n_2^X \in [0, \overline{N}_2]$.

The cost to the platform of accommodating an agent on side $i \in \{1, 2\}$ who joins the platform is $f_i \geq 0$, and there are no fixed costs. Platform X has profits of

$$\Pi^X = n_1^X (p_1^X - f_1) + n_2^X (p_2^X - f_2), \tag{1}$$

where p_i^X is the (uniform) price that platform X charges to the agents on side i.

2.2 Side 1: Consumers

Consumers on Side 1 are indexed by $\tau \in [0, \overline{N}_1]$. All consumers' outside option is valued at 0, whereas the utility for a consumer of type τ from joining Platform X is

$$u_1^X(\tau) = v + \alpha_1(\tau) \cdot n_2^X - p_1^X. \tag{2}$$

Here $v \geq 0$ is the membership value every consumer receives from joining the platform. This is the stand-alone utility of being a member of the platform that one gets even if no firms join the platform. Note that it is possible for $v = 0$, but for smartphones and video game consoles $v > 0$. For smartphones v is the utility from using a smartphone as a phone, including the preloaded features, and for video game consoles v is the utility from using the console to watch Blu-ray discs. Consumers are homogeneous in their membership benefit to the platform; so v does not depend on consumer type τ; and the stand-alone value of joining a platform is the same regardless of which platform is joined.

Consumers are heterogeneous in their marginal benefit from firms. The network effect or the marginal benefit to a consumer of type τ for an additional firm on the platform is constant and given by $\alpha_1(\tau)$; and the number of firms that join the platform is n_2^X. We focus on the case when network effects are positive so $\alpha_1(\tau) \geq 0$ for all τ, with $\alpha_1(\cdot)$ decreasing and twice continuously differentiable. Since $\alpha_1(\tau)$ is decreasing, it orders consumers by their marginal benefits. Consumers whose type τ is close to zero have marginal benefits that are high relative to those consumers whose type is located far from zero.

The platform knows v and $\alpha_1(\cdot)$ but cannot distinguish the individual values for each consumer τ. Thus, it cannot price discriminate between consumers, so the price or membership fee that consumers pay the platform is a uniform price given by p_1^X.

With there being two platforms in the market, consumers can either join a single platform (single-home) or join both platforms (multi-home). A consumer who multi-homes has utility

$$u_1^M(\tau) = (1+\delta)v + \alpha_1(\tau) \cdot N_2 - p_1^A - p_1^B. \tag{3}$$

Notice that if a consumer participates on two platforms the intrinsic benefit from membership to the second platform diminishes by $\delta \in [0, 1]$ so that the total stand-alone membership benefit from the two platforms is $(1+\delta)v$. If $\delta = 0$, then there is no additional membership benefit from joining the second platform, and when $\delta = 1$ the membership benefit is unaffected by being a member of another platform.[6]

Apart from the positive membership value of being on a second platform, the main gain to joining a second platform is access to additional firms. Letting n_2^M denote the number of multi-homing firms, a consumer that multi-homes has access to $N_2 := n_2^A + n_2^B - n_2^M$ distinct firms: these are all the firms that join at least one platform. The above utility function implies a multi-homing firm provides only a one-time gain to a consumer that multi-homes. Having a firm available on both platforms to which the consumer has access provides no added benefit.

2.3 Side 2: Firms

On the other side of the platform, Side 2, are firms that are indexed by $\theta \in [0, \overline{N}_2]$. All firms' outside option is 0, while a firm's payoff from joining Platform X is

$$u_2^X(\theta) = \alpha_2 \cdot n_1^X - c\theta - p_2^X. \tag{4}$$

The marginal benefit firms receive from an additional consumer on the platform is α_2—which is the same for all firms, so firms' marginal benefits for an additional consumer are homogeneous across firm type. The logic here is that an additional consumer will (in expec-

[6]One can also consider the possibility that owning a second platforms is tedious so that $\delta < 0$, but this doesn't impact the main results. Also, depreciation in network benefits, α_1, is possible, as studied in Ambrus et al. (2016).

tation) shift the demand curve for a firm's app upward in the same way for all firms. The assumption entails that each consumer sees firm products—their app, or game—as homogeneous, but consumers differ in their preferences, resulting in different willingness to purchase apps and games.[7]

Firms incur a cost of $c > 0$ to join the platform. This cost reflects development and synchronization costs associated with programming and formatting their product to fit the platform. Firms are heterogeneous with respect to their development and synchronization costs. Firms with type θ close to zero have lower costs compared to those with higher θ.[8]

The platform knows the firm's profit structure but cannot identify firms individually; hence, it cannot price discriminate between firms and the price or membership fee the firms pay to the platform is given by p_2^X for all firms.

A firm that multi-homes has payoff

$$u_2^M(\theta) = \alpha_2 \cdot N_1 - (1+\sigma)c\theta - p_2^A - p_2^B, \tag{5}$$

where $N_1 := n_1^A + n_1^B - n_1^M$ is the number of distinct consumers to which the firm gains access; these are all the consumers that join at least one platform. As noted above, when a firm's product is available to the multi-homing consumer on both platforms, a consumer will only purchase the product at most once. Therefore a firm only cares about the number of distinct consumers that are available to it through the platforms.

When a firm participates on two platforms its development and synchronization cost for joining the second platform diminishes to $\sigma \in [0, 1]$. Thus, σ represents the amount of 'duplication economies' that exist when synchronizing an app or game to a second platform.

[7]Note that we abstract from the transactions costs that may occur between consumers and firms. Thus, the benefits that accrue to consumers from interacting with firms—apps and games—can be viewed as being net of prices paid to firms. Deltas and Jeitschko (2007) consider auctioneers setting optimal reserves on an auction hosting platform, and Reisinger (2014) generalizes Armstrong (2006) and considers tariff-pricing with heterogeneous trading. See Tremblay (2016b) for a more detailed analysis of pricing across the platform in a framework that is more similar to our current setting.

[8]The model is isomorphic to assuming there is a limited number of app developers with increasing costs of developing apps so the equilibrium number of apps brought to market is endogenous.

If $\sigma = 1$ then there are no economies of duplication and as σ decreases, there exists economies of duplication.[9]

2.4 Strategies and Equilibrium Notion

The sequence of events is that first platforms simultaneously set and publish prices p_i^X, which we refer to as the pricing game. Thereafter, upon observing all platform prices, consumers and firms simultaneously make participation decisions, which we refer to as the allocation subgame, which follows the platform pricing game.

A consumer of type τ has strategy $s^\tau : (p_1^A, p_2^A, p_1^B, p_2^B) \to \{\varnothing, A, B, M\}$ that maps the observed prices set by the platforms into the consumer's participation decision, where the empty set $\varnothing$ denotes not joining either platform, A or B denotes joining Platform A or B, and M denotes that the consumer multi-homes and joins both platforms. Similarly, a firm of type θ has strategy $s^\theta : (p_1^A, p_2^A, p_1^B, p_2^B) \to \{\varnothing, A, B, M\}$ that maps the observed prices set by the platforms into the firm's participation decision. Lastly, platforms' strategies are their prices, $s^X : (p_1^X, p_2^X)$, $X = A, B$.

We focus exclusively on pure strategies and solve for subgame-perfect Nash equilibrium configurations of the model using backward induction. In the allocation subgame, consumers and firms simultaneously make participation decisions given the platform prices. The equilibrium of the allocation subgame requires that for all consumer types τ, s^τ maximizes the consumer's utility $u_1^{s^\tau}$, given the price constellation and given the consumers' and firms' strategies. In addition, the allocation equilibrium requires that for all firm types θ, s^θ maximizes the firm's utility $u_2^{s^\theta}$ for the given price constellation and given the consumers' and firms' strategies.

Platform X chooses s^X to maximize profits Π^X, given the other platform's strategy and the anticipated consumer and firm allocations in light of these prices. Thus, platforms are

[9]The relative lack of duplication economies played a role in providing an app for Facebook in the tablet market. For some time the app 'Friendly for Facebook' was used by Facebook users because Facebook itself had not developed an app for the tablet. It was rumored that Apple later 'assisted' Facebook in developing the official Facebook app.

forward looking and so the pricing equilibrium requires that the anticipated allocation for given prices obtains in the subsequent allocation equilibrium.

Applying backward induction requires that all equilibrium allocations for all possible sets of platform price constellations are derived. To reduce the expositional burden—while leaving intact the set of equilibrium outcomes of the entire game—we focus on platform prices in which (1) neither platform makes loses if the market tips in its favor and (2) there exists at least one allocation equilibrium with positive participation for the platform for some price constellation of the rival (i.e., no platform sets prices under which it necessarily prices itself out of the market).[10]

In addition, we restrict the set of allocation equilibrium constellations by ruling out two strong forms of coordination failure in the allocation subgame. First, the no-trade subgame equilibrium where no consumers and no firms join any platform in the allocation subgame is ruled out on and off path.[11] Second, we preclude dis-coordinated allocation configurations in which despite having better (i.e., lower) prices a platform fails to attract any firms. Formally, if Platform X has prices that are strictly better on one side of the market and no worse on the other ($p_i^X < p_i^Y$ and $p_j^X \leq p_j^Y$ for $j \neq i$), then Platform X will have some firms join its platform ($n_2^X \neq 0$). Note that this assumption says nothing about the equilibrium allocation of consumers. And, importantly, it says nothing about consumers or firms for the case that one platform has a lower price on one side, but the rival platform has the lower price on the other side.[12]

[10]Very loosely speaking, the considerations eliminate price constellations that are 'too low' to be profitable or 'too high' to have anyone join. However, the notions of too low or too high are not well-defined in that the platform chooses price-pairs and allocations are made in light of the constellations of the price-pairs.

[11]If we allow these no-trade equilibria to exist in the allocation subgames, then there exists a continuum of trivial equilibria of the entire game where an arbitrary price constellation is an equilibrium because no-trade occurs off path for any other price constellation.

[12]The rationale for why the assumption pertains only to a minimum participation of the firms—rather than guaranteeing minimum participation by consumers—is twofold. First, a platform can always attract some consumers when it prices sufficiently low, because the platform offers a stand-alone value to consumers; and second, in the contexts we have in mind it is reasonable to assume that firms are aware of pricing on both sides of the platform, whereas consumers are likely to only observe prices on Side 1. Hence, firms are able to observe any price-advantages regardless of the side on which they are offered. Hagiu and Hałaburda (2014) use this fact to differentiate between information that firms have in contrast to beliefs that consumers have about prices.

3 Equilibrium

We now investigate the allocation subgame of consumers and firms in joining platforms for given (arbitrary) prices charged by the platforms; and then we determine the equilibria for the entire game by considering price competition between the two platforms, in light of the continuation equilibrium from the allocation subgame.

3.1 The Allocation Equilibrium for Arbitrary Prices

First consider the case where platforms have chosen symmetric pricing strategies.

Proposition 1 (Allocations under Symmetric Pricing). *If $p_i^X = p_i^Y = p_i$ there exist two types of equilibria:*

1. *A tipping equilibrium in which all participation takes place on one platform.*

2. *A symmetric participation equilibrium with $n_i^X = n_i^Y = n_i$, $i = 1, 2$.*

In the symmetric participation equilibrium there exist multiple equilibrium allocations in which the distribution of multi-homers and single-homers on each side depends on the distribution of multi-homers and single-homers on the other side.

In particular, the set of multi-homing consumers is given by $\tau \in [0, n_1^M]$, and the set of single-homing consumers is given by $\tau \in [n_1^M, N_1]$, with

$$n_1^M = \alpha_1^{-1}\left(\frac{p_1 - \delta v}{n_2 - n_2^M}\right) \quad \textit{and} \quad N_1 = \alpha_1^{-1}\left(\frac{p_1 - v}{n_2}\right). \tag{6}$$

The set of multi-homing firms is given by $\theta \in [0, n_2^M]$, and the set of single-homing firms is given by $\theta \in (n_2^M, N_2]$, with

$$n_2^M = \min\left\{\frac{\alpha_2 \cdot N_1 - 2p_2}{(1+\sigma)c}, \frac{\alpha_2 \cdot (n_1 - n_1^M) - p_2}{\sigma c}\right\} \quad \textit{and} \quad N_2 = \min\left\{\frac{\alpha_2 n_1 - p_2}{c}, n_2^M\right\}. \tag{7}$$

Proposition 1 says that when platforms set equal prices, then either the tipping allocation occurs or the platforms split both sides of the market equally. However, this equal division does not determine the extent to which consumers and firms multi-home in equilibrium. In fact, the allocation of one side of the market depends on the allocation on the other side; and this results in the possibility of multiple equilibrium allocations—depending on parameter values as well as the platform prices chosen.

Consider first consumers. Consumers always obtain an added benefit from joining a second platform, namely δv. Hence, if prices to consumers are low enough, $p_1 < \delta v$, then all consumers join both platforms: $n_1^M = N_1 = \overline{N}_1$.[13] For consumer prices above this threshold, but still below the stand-alone utility from a single platform membership, $\delta v < p_1 < v$, all consumers will join one platform, $N_1 = \overline{N}_1$; but whether any consumers join a second platform (multi-home) depends on whether firms multi-home. In particular, if the number of multi-homing firms is large ($n_2 - n_2^M$ is small), then consumers have access to many firms when joining the first platform and so the number of multi-homing consumers is small, or even zero. For even higher consumer prices, consumers with large values of τ even refrain from joining a single platform, $N_1 < \overline{N}_1$.

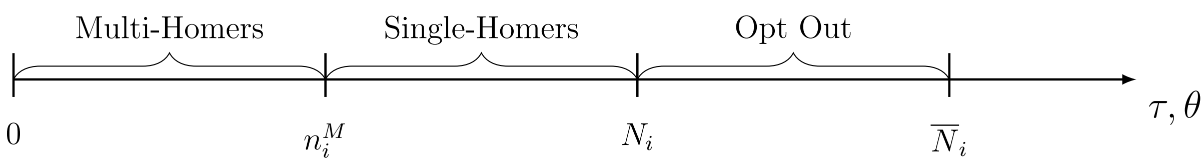

Figure 1: Homing Decisions by Consumers and Firms According to Type τ, θ

Unlike consumers, firms do not obtain a stand-alone benefit from joining a platform. However, they experience duplication economies in production when joining a second platform. This implies that a firm will multi-home only when the marginal gain from joining a second platform and the total payoff from being on two platforms are both positive. And hence, the set of multi-homing firms depends on the number of consumers that multi-home.

[13]Note that since $\alpha(\cdot)$ is positive and decreasing, so is $\alpha^{-1}(\cdot)$ and therefore when $p_1 - \delta v < 0$ the corner solution obtains in which $n_1^M = N_1 = \overline{N}_1$.

If all consumers multi-home then $n_1^M = n_1$ and no firm will multi-home, unless they are paid to do so (which requires $p_2 < 0$). Second, the firms that choose to multi-home instead of single-home are the firms with sufficiently low synchronization costs (θ close to zero). As the synchronization cost gets larger the marginal cost for joining another platform becomes larger than the marginal gain from having access to additional consumers. Hence, for firms with higher synchronization costs, θ farther from zero, it becomes too costly to join more than one platform. Thus, a firm is more likely to multi-home if it faces a lower synchronization cost to join a platform.

Note finally that if few consumers multi-home (n_2^M is small) and there are strong duplication economies (small σ), then it is possible that no firms single-home and all firms multi-home.

Proposition 2 (Allocations with Price-Undercutting)**.** *If $p_i^Y \leq p_i^X$ with at least one strict inequality then there exists a unique allocation equilibrium in which all active agents join Platform Y so that $n_i^Y = N_i$, $i = 1, 2$; and they multi-home onto Platform X iff $p_1^X \leq \delta v$ so that $n_1^X = n_1^M > 0$, or $p_2^X < 0$ so that $n_2^X = n_2^M > 0$.*

Proposition 2 shows that when one platform has better prices (at least one better price, and the other price no worse), then all agents—consumers and firms alike—join the platform with the price advantage. Whether agents also join the second platform (and, thus, multi-home) depends on the prices on the second platform. Consumers join the second platform only if the price is below their marginal stand-alone benefit from joining a second platform, $p_1^X < \delta v$, because they already have access to all active firms through the first platform so that any firm presence on the second platform is of no value to consumers. Similarly, because firms already have complete market access to all consumers on one platform, they only join the second platform if they are paid to do so, $p_2^X < 0$.

Proposition 3 (Allocations under Orthogonal Pricing)**.** *When $p_1^X > p_1^Y$ and $p_2^X < p_2^Y$ for $X \neq Y \in \{A, B\}$ then there are potentially two types of equilibria*

1. *All active agents join the same platform, with multi-homing occurring only when agents are subsidized to do so.*

2. *For consumer prices sufficiently close together, there also exists an equilibrium in which Platform Y only attracts multi-homing firms and Platform X also attracts single-homing firms, $n_2^M = n_2^Y < n_2^X$; and high-valued consumers single-home on Platform X, i.e., $\tau \in \left[0, n_1^X\right]$, and low-valued consumers single-home on Platform Y, $\tau \in \left[n_1^X, N_1\right]$, so $n_1^Y = N_1 - n_1^X$.*

The first case is essentially the tipping outcome which can occur for either platform. That is, all participation occurs on one of the platforms exclusively, unless there is an inducement to join the second platform as a multi-homer.

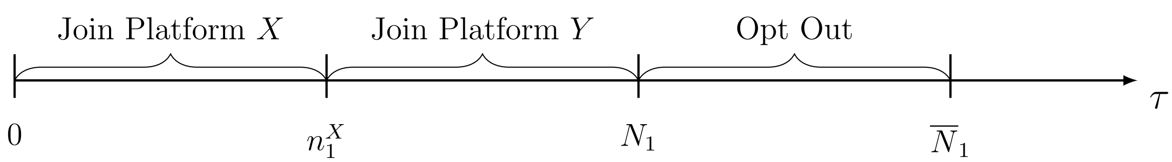

Figure 2: Homing Decisions by Consumers in Second Type of Equilibrium

In the second case, all active firms join Platform X and some also join Platform Y—so Platform Y has no firms that are exclusive to it. Consumers single-home across the two platforms, with high-valued consumers being willing to pay the higher price at Platform X in order to have access to all firms, whereas low-valued consumers are willing to forgo access to some firms in return for a lower price at Platform Y (see Figure 2). The price difference on the consumer side is limited by the degree of differentiation that is possible to attain from differential firm participation. Hence, if the difference in consumer prices is too large, only a tipping-type outcome is supported.

3.2 Equilibria of the Pricing Game

A recurring theme in the allocation configurations was whether a platform sets prices low enough to attract consumers merely for the marginal stand-alone value, δv. This pricing decision often plays a special role in determining whether consumers multi-home. In particular, if a platform sets $p_1^X < \delta v$, then it is sure to capture all consumers—regardless of all other prices and homing decisions. In light of this, when determining the platforms' pricing decisions it is important to consider the relationship between the cost of providing service to a consumer and the consumer's marginal stand-alone value for the second platform, i.e., $f_1 \gtreqless \delta v$.

We first suppose that $f_1 < \delta v$. In this case a platform can charge a consumer price of $p_1^X = \delta v > f_1$ and guarantee itself positive profits since consumers will either single-home on platform X or if a consumer is already on platform $Y \neq X$ then they will be willing to multi-home even absent any firms on platform X. Hence, both platforms are guaranteed profits and, in equilibrium, all consumers $\tau \in [0, \overline{N}_1]$ join at least one platform.

Theorem 1 (Weak Competition: $f_1 \leq \delta v$)**.** *There exists a unique symmetric equilibrium with $p_1^A = p_1^B = \delta v$ and $p_2^A = p_2^B = f_2$. All consumers multi-home, $n_1^M = \overline{N}_1$, and firms that join a platform single-home, $n_2^M = 0$. Platform profits are $\Pi^A = \Pi^B = \overline{N}_1(\delta v - f_1) > 0$.*

The case of 'weak competition' implies that failure to launch issues are not encountered, since both platforms are able to establish themselves on the consumer side of the market. This occurs because platforms are differentiated from the consumers' perspectives—there is a positive marginal value from joining a second platform—and so consumers are willing to join a platform even if there are no apps available on that platform. Because all consumers join both platforms and because the firm prices are the same across platforms firms that join a platform are indifferent between which platform they join—leading to a continuum of configurations, ranging from all joining platform A to all joining platform B and combination in between.

For many products the membership benefit depreciates almost to zero when a consumer multi-homes, $\delta \approx 0$. This implies that even for small f_1 the marginal cost of accommodating an additional consumer on the platform is greater than the additional membership benefit from joining another platform. In the smartphone case, for example, the membership benefit is the ability to make calls and use the phone's preloaded features. Since most phones have similar pre-loaded features, δ is close to zero and any additional benefit from a second phone would not overcome the production cost of an additional phone.

We now consider the case $f_1 \geq \delta v$, so the cost of attracting a consumer who has already joined the rival platform exceeds the platform's stand-alone value to the consumer. As a result, platforms compete head-to-head for single-homers, rather than trying to attract multi-homers. We refer to this as 'strong competition,' which leads to fierce price-competition resulting in a form of Bertrand Paradox.

Theorem 2 (Strong Competition; $f_1 > \delta v$)**.** *There exists a continuum of tipping equilibrium configurations in which all participation goes to one platform, with equilibrium prices so that* $\Pi^A = \Pi^B = 0$.

Furthermore, there exist unique equilibrium prices when both platforms have participation: $p_1^A = p_1^B = f_1$ *and* $p_2^A = p_2^B = f_2$. *For this case there exist at least one and possibly as many as three types of symmetric participation allocations* ($n_i^X = n_i^Y$ *for* $i = 1, 2$) *in equilibrium:*

I. *All active consumers single-home and all active firms multi-home:* $n_1^M = 0$, $n_2^M = N_2$. *This is always an equilibrium.*

II. *A mix of multi-homing and single-homing consumers with multi-homing and single-homing firms:* $n_i^M \in (0, N_i)$. *Existence requires that network effects are sufficiently strong.*[14]

III. *All active firms single-home and many, potentially all, active consumers multi-home:* $n_2^M = 0$, $n_1^M \in (0, N_1]$. *A sufficient condition for existence of this allocation is* $v = 0$.

Theorem 2 states that there exist continua of tipping equilibria, but in all of these the prices are such that profits are zero. In addition to the tipping configurations, there exists a unique set of prices under which participation takes place on both platforms and platforms make zero profits. In these equilibrium configurations prices are set at marginal costs, and

[14] This follows from Equations (14) and (15) and since x in the proof of Theorem 2 must be in $[0, 1]$.

there are up to three distinct equilibrium allocations of consumers and firms across the platforms.

The three allocations that occur when there is participation on both platforms resemble several two-sided market industries. Allocation I mirrors the two-sided market for smartphones. Almost all consumers single-home, they own only one phone; and almost all firms multi-home, the vast majority of apps are available across all types of smartphones. This is also the 'competitive bottleneck' allocation in Armstrong (2006). There, however, the homing decisions are exogenously assumed, rather than endogenously derived. As a result, firms face high prices, whereas in our model—in which platforms compete to attract firms—firms face marginal cost pricing.

Allocation II resembles current allocations seen in many two-sided markets, including those for game consoles: For video game platforms, there exist consumers who multi-home—buying several game consoles—and others who single-home; and there exists game designers whose games are available across platforms, i.e., they multi-home, while others are available on only one system, i.e., they single-home.

Allocation III is seen with the ride sharing companies Uber and Lyft. These are platforms that connect drivers (i.e., firms) with passengers seeking transportation (consumers). Drivers generally offer their services through one ride sharing company (i.e., they single-home); whereas many customers seeking rides use both companies and compare availability and prices (i.e., they multi-home). Since there is no benefit from linking to a ride sharing company that has no drivers $v = 0$, consumers can join for free $p_1 = 0$, and an additional consumer joining Uber is costless for the platform and this provides a example of an industry where Allocation III occurs.[15]

It is worth noting that the equilibrium allocation configurations in Theorem 2 are exhaus-

[15] Another example are antique malls with many individual stalls each rented out to individual antiques dealers (i.e., firms), and consumers who visit the mall to browse the individual stalls. Vendors sell their antiques in only one mall (single-home), yet consumers browse at different malls (multi-home). There is no benefit from going to a vacant antique mall so $v = 0$. Also, Yelp Reservations restaurant reservation system aims to compete with OpenTable. In this case, restaurants would work with one or the other system, but patrons could search either.

tive; that is, there are no other equilibrium allocations. In particular, while many papers on platform competition assume exclusive single-homing, there does not exist an equilibrium in which all active consumers and all active firms single-home. When all consumers single-home, then firms optimally multi-home in order to reach all consumers. Also, there is no equilibrium allocation in which all active firms and consumers multi-home. If all consumers are multi-homing, firms optimally respond by single-homing.

4 Monopoly versus Duopoly

Does strong competition between two platforms result in higher welfare when compared to a monopoly platform? The answer is not readily apparent. On the one hand competition results in lower prices and additional stand-alone membership benefits to consumers who multi-home. On the other hand, however, competition can increase synchronization costs, as well as destroy network surplus by fragmenting the market. Moreover—as we show in this section—competition may also undermine welfare-increasing cross-subsidization that takes place in the monopoly setting.

Since weak competition leads to all consumers multi-homing due to the positive incremental value of joining another platform, we consider the case of strong competition, and show that even in this case the monopoly equilibrium may welfare-dominate.

To obtain closed form solutions and welfare we assume that τ is distributed uniformly on $[0, a/b]$, which implies that $\alpha_1(\cdot)$ is linear: $\alpha_1(\tau) = a - b\tau$. The number of potential consumers is then $\overline{N}_1 = \frac{a}{b}$. We assume that θ is distributed uniformly, with $\overline{N}_2$ sufficiently large so that the platform can always attract more app producers. That is, there exists many potential app producers, many of which end up not developing an app because their synchronization costs are too high. To simplify calculations, we further let $v = f_1$ and $f_2 = 0$ (which implies the case of strong competition since $f_1 = v > \delta v$).[16]

[16]These assumptions are not that critical and they make computations straightforward: In the market for smartphones and video game consoles it would be the case that both the marginal cost to produce the

4.1 Monopoly

For given prices, the agents' participation decisions are implied by the marginal agents on both sides being indifferent between participation and opting out, in light of their expectations about the participation decisions on the opposite side of the platform. Thus, on the consumer side $u_1(\tau = N_1) \equiv 0$ implies $p_1 = v + (a - bN_1) \cdot N_2$ (see Equation 2); and on the firm side $u_2(\theta = N_2) \equiv 0$ yields $p_2 = \alpha_2 \cdot N_1 - cN_2$ (see Equation 4). We maintain the assumption of no coordination failure.

Using these relations between participation and prices in conjunction with the platform's profit function (1), the monopolist's objective is to chose the implied participation levels, N_1 and N_2 to maximize

$$\Pi^m = N_1(v + (a - bN_1) \cdot N_2 - f_1) + N_2(\alpha_2 \cdot N_1 - cN_2 - f_2). \tag{8}$$

With $\alpha_1(\tau) = a - b\tau$, the highest marginal benefit any consumer has (namely a consumer of type $\tau = 0$) for firm participation is a. If the firms' constant marginal valuation of consumer participation exceeds that of consumers, $\alpha_2 > a \geq \alpha_1(\tau)$, then firms' gross willingness to pay (gross of the synchronization costs $c\theta$) exceeds that of all consumers. The optimal platform strategy is to attract all consumers as this allows the platform to extract a larger surplus from firms than was the cost of attracting consumers. The highest price to consumers that still attracts all consumers is $p_1 = v$. Given this price and the implied consumer participation of $N_1 = a/b = \overline{N}_1$, the platform maximizes profits with respect to N_2 with $p_2 = p_2(N_1 = a/b, N_2) = \alpha_2 \frac{a}{b} - cN_2$. This yields

$$p_1^{m_C} = v, \quad N_1^{m_C} = \frac{a}{b} = \overline{N}_1, \text{ and} \tag{9}$$

$$p_2^{m_C} = \frac{a\alpha_2}{2b}, \quad N_2^{m_C} = \frac{a\alpha_2}{2bc}; \tag{10}$$

platform and the membership gains consumers receive are positive and the cost to platforms of adding an additional app or game is nearly costless.

where m_C is a mnemonic that denotes the monopoly corner solution (with respect to consumers).

In contrast, when $\alpha_2 \leq a$, the consumers with the highest marginal benefit from firm participation have a higher willingness to pay than any firm has for consumers. As a result, consumers are charged higher prices and an interior equilibrium emerges, in which some consumers do not join the platform. The second order conditions hold for this problem and it is straightforward to show that for the interior equilibrium the prices and allocations are

$$p_1^{m_I} = v + \frac{1}{16bc}(a+\alpha_2)^2(a-\alpha_2), \quad N_1^{m_I} = \frac{1}{2b}(a+\alpha_2), \text{ and} \tag{11}$$

$$p_2^{m_I} = \frac{1}{8b}(a+\alpha_2)(3\alpha_2 - a), \quad N_2^{m_I} = \frac{1}{8bc}(a+\alpha_2)^2; \tag{12}$$

where m_I denotes the interior monopoly solution.

There are two things worth noting in this equilibrium. First, recall the usual monopoly problem with linear inverse demand $P = a - bQ$ and marginal cost equal to zero, yielding the monopoly output of $Q^m = \frac{a}{2b}$. Now notice that $N_1^{m_I} > Q^m$, so in equilibrium, a monopoly platform will price to have more consumers than a traditional (one-sided) monopolist—even when there is no corner solution on the consumer side. This is because the added consumers generate additional surplus on the platform which makes the platform more attractive to firms.

More important is the second observation, namely that prices charged to firms can be negative, $p_2^{m_I} < 0$. That is, firms may be subsidized to join the platform. This occurs when consumers with a high willingness to pay for apps (small τ) value firm presence on the platform especially high compared to the value of a consumer to the firm, i.e., $a > 3\alpha_2$. The monopolist then invests in the attractiveness of the platform to consumers by paying firms to join. This investment is then recouped through higher prices to consumers.

4.2 Welfare Comparison

Consider now competing platforms. Note first that when under competition tipping occurs, then all activity takes place on one platform and so there is no breaking up of network benefits in that case, but prices are such that profits are zero.

Theorem 3 (Tipping v. Monopoly)**.** *When a tipping equilibrium occurs when platforms compete then the elimination of deadweight loss implies that potential competition generates more welfare compared to a monopoly market.*

Because we are dealing with the case of strong competition, Theorem 2 holds and when there is no tipping, $p_1^A = p_1^B = f_1 = v \geq 0$ and $p_2^A = p_2^B = f_2 = 0$. Moreover, from Theorem 2 we know that there can be up to three distinct allocations of consumers and firms in equilibrium.

Allocation I is an equilibrium allocation that exists for all parameter values. All consumers single-home and all firms multi-home: $n_1^M = 0$ and $n_2^A = n_2^B = n_2^M = N_2$. Given the prices from Theorem 2 in conjunction with our functional form assumptions, $n_1 := n_1^A = n_1^B = \frac{1}{2}\overline{N}_1 = \frac{1}{2}\frac{a}{b}$ and $n_2 := n_2^A = n_2^B = n_2^M = \frac{\alpha_2}{(1+\sigma)c}\frac{a}{b}$. From this follows:

Theorem 4 (Allocation I v. Monopoly)**.** *Whenever $\alpha_2 \geq a$ all consumers join a platform regardless of the market structure (corner monopoly solution); and there exists $\sigma^C := \frac{a+2\alpha_2}{3a+2\alpha_2} \in (3/5, 1)$ such that monopoly generates more welfare than competition* iff $\sigma \geq \sigma^C$.

When $\alpha_2 < a$ competition serves all consumer types, whereas the monopoly limits consumer participation (interior monopoly solution); and yet there exists $\sigma^I := \frac{64\alpha_2 a^2}{5(a+\alpha_2)^3} - 1 \in (-1, 1)$ such that monopoly generates more welfare than competition whenever $\sigma \geq \sigma^I$.

Notice that competition with Allocation I always leads to all consumers joining a platform, $N_1^{AI} = \overline{N}_1$, whereas a monopoly excludes some when the willingness to pay is sufficiently large for the consumers with the highest marginal benefits from firm participation (the interior solution), $N_1^{m_I} < \overline{N}_1$.

Despite the (weakly) greater market coverage on the consumer side when platforms compete, whenever $\sigma > \max\left\{\sigma^I, \sigma^C\right\}$ the monopoly generates higher surplus than competition. The intuition for a superior outcome under monopoly when duplication costs are high is quite straightforward: in the competitive equilibrium of Allocation I all firms that join a platform end up multi-homing, and therefore incur the duplication cost of $\sigma c\theta$. Hence, the larger are the duplication costs, the more costly is the competitive solution—so much so that for sufficiently high duplication costs the monopoly platform welfare-dominates; even when it reduces participation on the consumer side.

Notice, however, that it is possible that a monopoly generates greater welfare even when there are no duplication costs ($\sigma = 0$) so there are no additional costs associated with firms joining a second platform. This can happen when the monopoly interior solution obtains ($\alpha_2 < a$) and $\sigma^I \leq 0$. The reason for this is that whenever $\sigma^I \leq 0$ then it must be that $3\alpha_2 < a$;[17] and for this case the monopolist subsidizes firm entry (i.e., $p_2^{m_I} < 0$, see Equation 11). The reason for the firm subsidy is to increase the total welfare that the platform generates so that the monopolist can appropriate this through higher prices to consumers.[18]

In contrast, in the competitive equilibrium the welfare-enhancing investment in firm-entry does not take place: a platform that subsidizes firms must make up for the cost of doing so by charging higher consumer prices. However, when $\sigma = 0$, all firms that obtain the investment subsidy will join both platforms; and so the platform that doesn't make the investment in firm participation reaps the same reward as does the rival platform that does subsidize firm entry—thus placing the non-investing platform at a competitive advantage. Therefore welfare-increasing firm subsidies do not occur when platforms compete.[19]

Turning to the comparison between monopoly and Allocation II under competition, note from Theorem 2 that a type-II Allocation may not exist.

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[17]For $\alpha_2 < a$ (the interior solution) σ^I is strictly increasing in α_2. However, when $\alpha_2 = \frac{1}{3}a$, $\sigma^I > 0$, so whenever $\sigma^I \leq 0$, it follows that $\alpha_2 < \frac{1}{3}a$.

[18]See Hagiu (2007) for a similar comparison between "closed" platforms with open platforms.

[19]An obvious implication of this is that having firms enter into exclusive deals with a platform can be welfare enhancing as it solves the non-appropriability problem associated with firm subsidies when $\sigma = 0$.

Theorem 5 (Allocation II v. Monopoly)**.** *Regardless of whether the monopoly has an interior or a corner solution, there exists mixed allocations such that the welfare from the competitive mixed allocation equilibrium is greater than the welfare with the monopoly platform; even when there are no savings in duplication* ($\sigma = 1$)*.*

If under competition the mixed allocation emerges in equilibrium, then whether or not the monopoly outcome generates greater welfare depends on the type of monopoly solution (corner or interior), the amount of multi-homing compared to single-homing on each side of the market, and the strength of the firm duplication costs. Theorem 5 shows that even in the extreme case when it is costly for firms to multi-home in terms of production and synchronization costs ($\sigma = 1$), competition can generate greater welfare in the mixed allocation.

In light of Theorems 4 and 5 it is clear that when duplication economies are small (large σ) monopoly is more likely to be preferred to competition, although this need not be the case if under competition a mixed allocation emerges in which not all firms multi-home. Notice also that for the case of an interior monopoly solution and parameters such that $\sigma^I < 0$, then for sufficiently large duplication effects (σ small) competition generates less welfare than monopoly if with competition all consumers single-home, but competition generates more welfare if a mixed-homing equilibrium emerges, because multi-homing is not expensive and the lower prices increase network effects.

Consider now Allocation III. To assure existence of this equilibrium, let $v = f_1 = 0$. Prices are $p_1 = 0$ and $p_2 = 0$, all firms single-home, and all consumers multi-home.

Theorem 6 (Allocation III v. Monopoly)**.** *Whenever* $\alpha_2 \geq \frac{1}{4}a$ *the competitive equilibrium in which all firms single-home and all consumers multi-home generates greater welfare than the monopoly outcome. Otherwise, the monopoly outcome generates greater welfare.*

Notice that the welfare comparison between monopoly and competition does not involve duplication economies σ, since in either case firms only single-home. Also, when $\alpha_2 \in$

$\left(\frac{1}{4}a, \frac{1}{3}a\right)$, then the monopoly platform subsidizes firms to increase the value of the platform and increase overall welfare, but the lower prices on the consumer side that occur under competition more than offset this so that competition leads to greater welfare.

Lastly, because Allocation III does not involve any multi-homing by firms it is possible that it generates greater welfare than Allocation I.[20] This then implies that welfare can be non-monotone in duplication economies across the degree of competitiveness in the market: In particular, there are parameters under which for small enough σ competition with Allocation I generates the highest welfare, whereas increases in σ lead to the monopoly generating the greatest welfare, only to be dominated by the competitive Allocation III upon further increases in σ.

5 Extensions

Before concluding we briefly address additional issues that the model can account for. First, we consider entry beyond two platforms, decreasing marginal network effects, exclusive deals, and then we consider other issues addressed in the literature, in particular the case of one platform having an inherent advantage by being "focal."

5.1 Entry

If additional platforms enter the market, then multi-homing becomes more complex, as agents may join any number of platforms. However, welfare implications are unambiguously worse for entry beyond two platforms for the case of strong competition, where the cost of accommodating another consumer is greater than the marginal value of the second platform, $f_1 \geq \delta v$. If tipping occurs nothing is gained, and otherwise with even just two platforms prices are competed down to marginal costs, and so there are no additional beneficial price effects from entry. Moreover, since prices are above the marginal value of additional platforms to

[20]This occurs when both equilibrium configurations exist and $\sigma > \frac{\alpha_2}{3\alpha_2+4a}$.

consumers, there are no gains through platform differentiation. Hence there is no increased participation in the market on the consumer side. However, if consumers fragment across platforms, then firms' additional replication costs destroy welfare, and network effects that accrue to firms are also diminished.

The case of weak competition ($f_1 < \delta v$) is less clear. In this case competition beyond the duopoly level will also drive prices on the consumer side down, as now there is strong competition for multi-homing consumers. However, it requires a great deal of platform differentiation (large δ that doesn't decrease (too much) upon consumer participation on additional platforms), in order to add welfare in the market.

5.2 Decreasing marginal network effects

Marginal network effects are assumed to be constant in the base model, i.e., $\partial^2 u_i / \partial n_j^2 = 0$. However, in many settings it is the case that the value of additional interactions with agents from the other side of the platform decrease in the number of available agents, i.e., $\partial^2 u_i / \partial n_j^2 < 0$. For instance, consumers have decreasing marginal benefits from the number of apps available on their smartphone, and similarly the value of additional games declines as enjoying each game requires an investment in time.

Decreasing marginal valuations reduce the total surplus generated in both monopoly and competition. However, in the competitive market there are no price effects, as prices are already competed to the lowest levels. In contrast, the monopoly will raise prices, because the value of the marginal firm is decreased, and so the change in prices further destroys surplus. Indeed, if consumers' marginal network effects from firms' participation are strongly declining, then it is unlikely that subsidizing firm entry is welfare improving, and so one of the possible reasons for a monopoly to welfare-dominate becomes weaker.

In sum, decreasing marginal network effects reduce welfare in the monopoly market more than in competition, so the stronger are deceasing marginal network effects, the more likely it is that competition welfare-dominates the monopoly platform.

5.3 Exclusive deals

In some instances firms enter into exclusivity contracts with platforms postulating that they cannot offer their service on competing platforms. In our model this has two implications. First, there exist equilibrium configurations in which the market tips and only one platform survives. That is, one platform can use exclusive deals to lock in enough firms to have the market tip. To see this, recall from the discussion of Theorem 5 that when $\alpha_2 < \frac{1}{3}a$ subsidizing firm entry increases welfare in the market. In competition, in this case, this welfare-increasing subsidy does not take place because of the non-appropriability of the investment when firms subsequently multi-home. However, under exclusive deals, the investment in welfare can be appropriated allowing a single platform to corner the market. Of course, in this case overall welfare is actually greater than under competition, because the investment in welfare overcomes adverse price effects.

Exclusive deals can also be used to successfully enter into the platform market, as was studied by Lee (2013) for the case of the successful entry of the Xbox in the video game market. In our model there also exists exclusive contract equilibrium configurations without market tipping. These are distortions of the Allocation I equilibrium where in Theorem 2 all firms multi-home, all consumers single-home, and prices are $p_i^A = p_i^B = f_i$. To see this, take these prices as given on Platform X and suppose that Platform Y sets $p_2^Y = p_2^X = f_2$, but also offers some of the firms an exclusive contract with $p_{2,\text{excl}}^Y < f_2$. The firms who are offered the exclusive deal will join Platform Y and the remaining firms will multi-home so long as there are the same number of consumers on each platform. This can be the case, although with $p_{2,\text{excl}}^Y < f_2$, Platform Y must charge $p_1^Y > f_1 = p_1^X$. Now, with $p_1^Y > f_1 = p_1^X$ and $n_2^Y = N_2 > n_2^X$ consumers are indifferent when $u_1^Y(\tau) = v + \alpha_1(\tau) \cdot N_2 - p_1^Y = v + \alpha_1(\tau) \cdot n_2^X - p_1^X = u_1^X(\tau)$, so $n_1^Y = n_1^X$, and both platforms have zero profits, $\Pi^Y = \Pi^X = 0$; and so this characterizes an equilibrium.[21]

[21]It is worth pointing out that a similar asymmetric equilibrium can also be derived for the case that platforms differ in their stand-alone values, $v^A \lesseqgtr v^B$.

5.4 Favorable Beliefs/Focal Platform

In many settings one platform may have an inherent advantage over another that is not tied to superior technology or pricing. This may be due to incumbency, greater name recognition, successful marketing campaigns, or the like. We call Platform X *focal* when all agents believe that $n_1^X = N_1$, that is, all agents believe that all consumers that join at least one platform will join Platform X.[22]

In the case of weak competition ($f_1 \leq \delta v$), having Platform X be focal does not affect the previous results. The non-focal Platform Y can fully establish itself in the market and Theorem 1 holds: prices are $p_1^A = p_1^B = \delta v$ and $p_2^A = p_2^B = f_2$, all consumers multi-home, the firms that join a platform single-home, and platform profits are $\Pi^A = \Pi^B = \overline{N}_1(\delta v - f_1) > 0$.

However, under strong competition ($f_1 > \delta v$), having Platform X be focal makes it harder for Platform Y to compete. Nevertheless, Platform Y may be able to compete by using a muted divide-and-conquer strategy. Caillaud and Jullien (2003) note that by subsidizing one side of the market a platform "divides" that side, and by subsequently "conquering" the other side of the market it can recover the loss it made through subsidization. Jullien (2011) demonstrates this in a model where a second mover platform uses divide-and-conquer to compete against a first mover platform. However, all agents are assumed to single-home. With endogenous homing decisions, the divide-and-conquer strategy is muted: the non-focal platform's ability to attract consumers by pricing low is enhanced through the possibility of these choosing to multi-home. However, this need not lead a 'divide' (because the multi-homers remain on the focal platform as well), and hence the ability to recover the costs of the subsidy may be harder to achieve as the 'conquering' is less effective (firms that access multi-homing consumers on the focal platform have no need to join the 'conquering' platform).

In order for Platform Y to compete with the focal Platform X, Platform Y sets $p_1^Y =$

[22] Alternatively, we could have beliefs be that $n_1^X > n_1^Y$ instead of $n_1^X = N_1$. Either framework is largely consistent with the models developed by Caillaud and Jullien (2003), Hagiu (2006), Jullien (2011), and Tremblay (2016a) on favorable beliefs, focal platforms, and incumbent platforms.

$\delta v < f_1$. This guarantees $n_1^Y = N_1 = \overline{N}_1$ and, so long as f_1 is not too large relative to δv and p_2^X is not too low, Platform Y can charge a mark up on the firm side of the market that makes up for its losses on the consumer side. Thus, if Platform X attempts to use its advantage as a focal platform and charge very high prices then Platform Y can use this divide and conquer strategy to compete.[23] The limits of this strategy are captured in the following theorem.

Theorem 7 (Focal Platform Equilibrium)**.** *Let* $f_1^L := \delta v + \frac{\alpha_2^2 \overline{N}_1}{8c} < \delta v + \frac{\alpha_2^2 \overline{N}_1}{4c} =: f_1^H$*. Then,*

- *if* $f_1 < f_1^L$*, there is a competitive equilibrium in which* $p_1^X = p_1^Y = \delta v < f_1$*,* $p_2^X = p_2^Y > f_2$*,* $n_1^X = n_1^Y = n_1^M = N_1$*,* $n_2^M = 0$*, and* $\Pi^X = \Pi^Y = 0$*;*
- *if* $f_1 \in \left[f_1^L, f_1^H\right]$*, Platform* X *is a monopoly constrained to* $p_1^X = \delta v < f_1$ *and* $p_2^X > f_2$*;*
- *if* $f_1 > f_1^H$*, Platform* X *is an unconstrained monopoly.*

Theorem 7 shows that there exists two critical levels of marginal cost to the platform for an additional consumer, f_1. In particular, if f_1 is relatively close to δv competition is strong and profits are zero. For a moderately large f_1 relative to δv the presence of the non-focal platform will prevent monopoly prices by the focal platform, but participation will only occur on the focal platform (akin to a contestable market). Lastly, if f_1 is large relative to δv then the focal platform will charge monopoly prices.

Thus, when agents make endogenous homing decisions a focal platform cannot necessarily tip the market and become a monopoly platform; in fact, competition can be strong between the focal and non-focal platforms. Agents' abilities to either single-home or multi-home allows the non-focal platform to attract one side of the market to multi-home, which enables it to compete with the focal platform.

[23] Platform Y is unable to divide and conquer by pricing below marginal cost on the firm side, $p_2^Y < 0 = f_2$, since this only guarantees a small mass of firms will find this sufficient to multi-home. If $p_2^Y < f_2$ it must charge a consumer price such that $p_1^Y > f_1$ in order to have non-negative profits. However, $p_1^Y > f_1 \geq \delta v$ implies $n_1^Y = 0$ since Platform X is focal; this will result in negative profits for Platform Y.

6 Conclusion

In many markets in which platforms compete against each other agents choose to join either one platform (single-home) or they join several platforms (multi-home). While most of the previous literature on platform competition has assumed this decision to be given exogenously, we allow participants on both sides of the platform to endogenously make an optimal homing decision.

When platforms are sufficiently differentiated in the sense that membership at a second platform bestows an additional stand-lone benefit apart from access to more agents on the other side, then platforms set prices to attract multi-homers, blunting head-to-head price competition between platforms (weak competition). In contrast, when added stand-along values are small relative to the cost of providing membership benefits, platforms engage in fierce competition for single-homers (strong competition), which leads to a zero-profit equilibrium with marginal-cost pricing, akin to the Bertrand Paradox. When this happens, the market may tip so that all activity takes place on one platform and the other is foreclosed from the market, or prices are at marginal costs and both platforms have patrons. When prices are set at marginal cost, there are potentially multiple equilibrium allocations concerning how consumers and firms divide themselves onto the competing platforms.

One equilibrium allocation that always exists entails all consumers single-homing and all firms multi-homing. This mirrors the allocation in the market for smartphones, where virtually all consumers own only a single phone, but virtually all apps are available across competing smartphones.

When network effects are strong enough, another type of equilibrium allocation emerges in which there is a mix of multi-homing and single-homing on both sides of the platforms. This constellation is found in the market for video game consoles: while many consumers have only one console, serious gamers often have more than one system; and while some games are available across providing platforms, others are exclusive to one system.

The third possible equilibrium constellation has all firms single-homing, whereas some or all consumers multi-home. This occurs when there is no stand-alone benefit to consumers of accessing the platform, that is, consumers are exclusively interested in the service provided by firms. This market structure is found with the rideshare services Uber and Lyft, where drivers generally work off one or the other platform (single-home), but some consumers multi-home to compare prices and availability across the services.

Compared to the lower prices and possibly greater access to services provided by competition between platforms, a monopoly platform may (but need not) generate higher welfare compared to any of the three possible allocations under competition. This is because the monopoly may better concentrate network effects and prevents cost-redundancies when firms multi-home. Moreover, the monopolist may invest in the value of the platform by subsidizing firm entry and thereby increase total welfare—whereas such investments are not undertaken in competition due to non-appropriability of the investments.

We find that decreasing marginal network effects adversely affect welfare under monopoly more than with competition, but that increased entry beyond two platforms is unlikely to increase welfare. Finally we show that due to endogenous homing focal platforms may skew market shares, but a complete foreclosure of the market is often prevented as disadvantaged platforms can set prices so as to attract some multi-homers, which proves to be a sufficiently strong foothold to preserve some benefits of competition.

Appendix of Proofs

Proof of Proposition 1: First consider the case where $p_1^X = p_1^Y > \delta v$. To prove that symmetric participation and tipping are the equilibrium allocations, we consider the alternative. If $n_2^X > n_2^Y > 0$ then $u_1^X(\tau) > u_1^Y(\tau)$ for all τ which implies all consumers join X and none join Y; all consumers joining X implies that $u_2^X(\theta) > u_2^Y(\theta)$ so that all firms also join X. Similarly, if $n_1^X > n_1^Y > 0$ then all firms join X and none join Y. Thus, only the symmetric participation and the tipping allocations result in allocations where deviation will not occur. If $p_1^X = p_1^Y \leq \delta v$ then tipping is ruled out and all consumer multi-home since $u_1^M(\tau) > u_1^X(\tau), u_1^Y(\tau)$ for all τ.

(6) is generated using (2) and (3), by solving for τ when $u_1^X(\tau) = 0$ and $u_1^M(\tau) = 0$. Similarly, (7) stems from (4) and (5). Lastly, consider the case where $p_1^X = p_1^Y \leq \delta v$. In this case all consumer multi-home and so the firms single-home to either platform; unless $p_2 < 0$, in which case the firms also multi-home. □

Proof of Proposition 2: By assumption, no dis-coordination implies that $n_2^Y > 0$. Consider the case when $p_1^X > \delta v$ and $p_2^X > 0$, and suppose that X has participation. Note that either $u_2^X(\theta) > u_2^Y(\theta)$ and each agent on Y deviates to X, this contradicts $n_2^Y > 0$, or $u_2^X(\theta) < u_2^Y(\theta)$ and each agent on X deviates to Y. Thus, the only equilibrium allocation is when all participation occurs on Y when $p_1^X > \delta v$ and $p_2^X > 0$. This implies that $n_1^Y = N_1$ and $n_1^X = 0$ when $p_1^X > \delta v$ and if $p_1^X \leq \delta v$ then $n_1^X = n_1^M > 0$. On the firm side of the market, we have $n_2^Y = N_2$ and $n_2^X = 0$ when $p_2^X \geq 0$ and if $p_1^X < 0$ then $n_1^X = n_1^M > 0$. □

Proof of Proposition 3: The tipping configurations follow readily from having assumed that no platform prices itself out of the market. The other allocation is characterized as follows:

Using (2), the marginal consumer who is indifferent between X and Y is given by

$$u_1^X(\tau = n_1^X) = v + \alpha_1(n_1^X)n_2^X - p_1^X = v + \alpha_1(n_1^X)n_2^Y - p_1^Y = u_1^Y(\tau = n_1^X), \tag{13}$$

which implies that $n_2^X > n_2^Y$. The last consumer to join Y, $\tau \in (n_1^X, N_1]$ so that $n_1^Y = N_1 - n_1^X$ is characterized by $u_1^Y(\tau = N_1) = v + \alpha_1(N_1)n_2^Y - p_1^Y = 0$. Setting (4) and (5) equal to zero yields $n_2^X = \frac{\alpha_2 n_1^X - p_2^X}{c}$, $n_2^M = n_2^Y = \frac{\alpha_2 n_1^Y - p_2^Y}{(1+\phi)c}$, where multi-homing on Y follows from $n_2^X > n_2^Y$ being implied by (13).

The conditions jointly characterize the equilibrium provided that (13) yields a $\tau \in [0, \overline{N}_1]$. □

Proof of Theorem 1: Given these prices, platform profits are $\Pi^X = \Pi^Y = \overline{N}_1(\delta v - f_1) > 0$.

Furthermore, the allocation follows from Proposition 1 and all consumers multi-homing implies that all firms that participate will single-home. For existence of the equilibrium prices, note that any deviation below leads to forgone profit with stable participation, and any increase leads to forgone profit due to loss of participation.

To prove uniqueness, consider other pricing strategies. First, consider other symmetric pricing strategies. In this case, for any $p_1 < \delta v$ profits increase when increasing price. If $p_1 > \delta v$ a platform an incentive to undercut the other platform as $N_1(p_1 - \epsilon - f_1) > {}^{N_1}/_2(p_1 - f_1)$. Similarly, for any $p_2 \neq f_2$.

Second, consider price constellations where $p_i^X \geq p_i^Y$, $i = 1, 2$, with at least one inequality being strict. If $\Pi^Y > 0$ then X will undercut Y's prices and make positive profits, and if $\Pi^Y = 0$ then Y will increase its prices but still undercut X's prices.

Third, consider orthogonal pricing with $p_1^X > p_1^Y$, $p_2^X < p_2^Y$, and $p_1^X, p_1^Y > \delta v$. Given the multiple subgame equilibrium allocations found in Proposition 3, first consider the tipping allocations. These are ruled out as equilibria because the platform that is tipped out of the market would deviate to $p_1 = \delta v$ and $p_2 = f_2$ and make profit $\overline{N}_1(\delta v - f_1)$. If the allocation from the second bullet in Proposition 3 occurs, then profits for each platform must be greater than $\overline{N}_1(\delta v - f_1)$ in this allocation, otherwise a platform deviates to $p_1 = \delta v$ and $p_2 = f_2$. If both $\Pi^X, \Pi^Y > \overline{N}_1(\delta v - f_1) > 0$ then either platform can undercut the rival platform's prices and tip the market with profits $\Pi^X + \Pi^Y \pm \epsilon$ for small $\epsilon > 0$. When at least one price has $p_1^X, p_1^Y \leq \delta v$ with orthogonal prices, X can raise p_2^X and increase profits since all consumers are multi-homing. Thus, orthogonal prices cannot occur in equilibrium and so the equilibrium in the theorem is unique. □

Proof of Theorem 2: Consider first tipping: By Propositions 1–3 any price-configuration can lead to tipping. If the active platform makes zero profits, then there is no price response by the rival that makes the rival better off. A sufficient condition to support a continuum of tipping equilibria is to have the rival match the prices of the platform that has all the activity, leading to the tipping allocation in Proposition 1.

Now consider non-tipping. (1) Existence: when $p_1^X = p_1^Y = f_1$, $p_2^X = p_2^Y = f_2$, a platform charging a higher price looses all customers and a platform charging a lower price looses money. Orthogonal price-deviations result in zero profit provided that the market tips to the platform charging marginal cost prices. (2) Uniqueness: To show uniqueness of the non-tipping equilibrium outcome, we must rule out (i) symmetric prices not equal to marginal cost, (ii) weakly dominated prices, and (iii) orthogonal prices.

(i) With symmetric prices both above marginal costs or both below marginal costs lead to price-undercutting responses or negative profits (and hence higher prices). For $p_i < f_i$ and $p_j \geq f_j$ consider first $p_1 \leq \delta v < f_1$. By Proposition 1 all consumers multi-home and each platform can increase profit by reducing p_2 and tipping the market on the firm side. If $p_1 > \delta v$, then each platform can keep participation on the subsidized side in place while decreasing the price on the profitable side to undercut the rival and increase profit. Thus, $p_i = f_i$ with $i = 1, 2$ is the only symmetric price constellation that can occur in equilibrium.

(ii) By Proposition 2 weakly dominated prices lead to tipping.

(iii) If prices are orthogonal with one platform having both prices above marginal costs, then the rival platform increases profits by tipping the market by just undercutting these prices. If both platforms have one price above and the other price below marginal cost, then a platform can increase profit by undercutting the rival's price on the profitable side and matching the rival's price on the subsidized side while fixing participation.

Consider now the allocations.

Allocation I: Allocation (7) implies all firms multi-home when all consumers single-home, since $n_2^M > N_2$ i.e. $[n_2^M, N_2]$ is empty when $n_1^M = 0$. Furthermore, when $n_2^M = n_2^A = n_2^B$, allocation (6) implies no consumer multi-homes. Hence, all consumers single-home if and only if all firms multi-home. Thus, the allocation where all firms multi-home and all consumers single-home is an equilibrium allocation.

Allocation II: Since $p_1 > \delta v$, allocation (6) implies the set of multi-homing consumers is non-empty when the number of multi-homing firms is not to large. Let $x \in [0, 1]$ be the percent of consumers who multi-home of those n_1^X who join platform X so that $n_1^M = xn_1^X = xn_1^Y$. This implies $N_1 = (2-x)n_1^X$ since $N_1 = n_1^X + n_1^Y - n_1^M$ and $n_1^X = n_1^Y$. From the Allocation I $x > 0$ occurs when not all of the firms are multi-homing. From Equation 7, this occurs when $\min\left\{\frac{\alpha_2\cdot(2-x)n_1^X - 2p_2}{(1+\sigma)c}, \frac{\alpha_2\cdot(1-x)n_1^Y - p_2}{\sigma c}\right\} < \frac{\alpha_2 n_1^X - p_2}{c}$.

In the remainder of this proof we assume $\sigma = 1$, no economies of duplication. Using allocation (7) there exists x^M such that for $x > x^M$ no firm will multi-home. Allocation (7) implies $0 = \alpha_2(1-x^M)n_1^Y - p_2$. Thus, $x^M = 1 - \frac{p_2}{\alpha_2 n_1^Y}$. And for all $x > x^M$ no firm multi-homes. Note, $p_2 < \alpha_2 n_1^Y$ since otherwise the market collapses, hence $x^M \in (0,1)$.

If $0 < x < x^M$ then some firms will single-home and some firms will multi-home. Allocation (7) implies $n_2^M = \frac{\alpha_2(1-x)n_1^Y - p_2}{c}$ and allocation (7) implies $n_2^Y = (1/2)(N_2 + n_2^M) = (1/2c)[\alpha_2(2-x)n_1^X - 2p_2]$. Similarly, allocation (6) defines the number of multi-homing consumers: $0 = \delta v + \alpha_1(n_1^M)(n_2^Y - n_2^M) - p_1$; using this equation and the equations for n_2^M, n_2^Y, and $n_1^M = xn_1^X = xn_1^Y$ we can characterize x by:

$$\begin{aligned} 0 &= \delta v + \alpha_1(xn_1^X)(1/2c)[\alpha_2(2-x)n_1^X - 2p_2 - 2\alpha_2(1-x)n_1^Y + 2p_2)] - p_1 \\ &= \delta v + \alpha_1(xn_1^X)(1/2c)[\alpha_2 \cdot xn_1^X] - p_1, \end{aligned} \tag{14}$$

Furthermore, allocation (6) defines N_1, the number of consumers on Platform X: $0 = v + \alpha_1(N_1)n_2^X - p_1$. Thus we have:

$$0 = v + \alpha_1(N_1)n_2^X - p_1 = v + \alpha_1((2-x)n_1^X)(1/2c)(\alpha_2\cdot(2-x)n_1^X - 2p_2) - p_1. \tag{15}$$

Thus, we have two equations (14) and (15) and two unknowns, x and n_1^X. If the solution is $x \in (0, x^M)$ then we have an equilibrium allocation. Note, this equilibrium does not exist when $x \notin (0, x^M)$.

Allocation III**:** Allocation (7) implies all firms single-home when the number of multi-homing consumers is $n_1^Y \leq n_1^M + p_2/\alpha_2$. If $p_2 = 0$, then this holds when all consumers multi-home. By allocation (6), this will only be an equilibrium when $\delta v = p_1$. If $p_2 > 0$, then allocation (6) implies there exists an equilibrium where all firms single-home and a large portion of consumers multi-home given prices such that $N_1 - n_1^M = \alpha_1^{-1}(\frac{p_1 - v}{n_2^X}) - \alpha_1^{-1}(\frac{p_1 - \delta v}{n_2^X}) \leq \frac{2p_2}{\alpha_2}$, i.e., $\delta v + \epsilon = p_1$ for small $\epsilon > 0$.

Thus, there exists at least one and potentially three allocations that occur in equilibrium with unique non-tipping equilibrium prices $p_1^X = p_1^Y = f_1$ and $p_2^X = p_2^Y = f_2$. $\square$

Proof of Theorem 3: This follows since the platform with competition has more participation than the pure monopoly platform resulting in greater welfare. $\square$

Proof of Theorem 4: Given the monopoly corner solution, we calculate the standard welfare measures.

$$\Pi^{m_C} = N_1(p_1^{m_C} - f_1) + N_2(p_2^{m_C} - f_2) = N_1 \times 0 + N_2 p_2^{m_C} = \frac{a^2 \alpha_2^2}{4b^2 c}, \tag{16}$$

$$CS^{m_C} = \int_0^{N_1^{m_C}} (v + \alpha_1(\tau) N_2^{m_C} - p_1^{m_C})\, d\tau = \frac{a^3 \alpha_2}{4b^2 c}, \tag{17}$$

$$FS^{m_C} = \int_0^{N_2^{m_C}} (\alpha_2 N_1^{m_C} - c\theta - p_2^{m_C})\, d\theta = \frac{a^2 \alpha_2^2}{8b^2 c}, \tag{18}$$

$$W^{m_C} = \frac{a^2 \alpha_2}{8b^2 c}(3\alpha_2 + 2a). \tag{19}$$

For the monopoly interior solution:

$$\Pi^{m_I} = N_1(p_1^{m_I} - f_1) + N_2(p_2^{m_I} - f_2) = \frac{\left(a + \alpha_2\right)^4}{64b^2 c}, \tag{20}$$

$$CS^{m_I} = \int_0^{N_1^{m_I}} (v + \alpha_1(\tau) N_2^{m_I} - p_1^{m_I})\, d\tau = \frac{(a + \alpha_2)^4}{64b^2 c}, \tag{21}$$

$$FS^{m_I} = \int_0^{N_2^{m_I}} (\alpha_2 N_1^{m_I} - c\theta - p_2^{m_I})\, d\theta = \frac{(a + \alpha_2)^4}{128b^2 c}, \tag{22}$$

$$W^{m_I} = \frac{5(a + \alpha_2)^4}{128b^2 c}. \tag{23}$$

Lastly, the welfare results for Allocation I with competing platforms.

$$\Pi^A = \Pi^B = 0, \tag{24}$$

$$CS^{AI} = \int_0^{a/b} (a - b\tau)\frac{2\alpha_2 \cdot n_1}{(1+\sigma)c}d\tau = \frac{a^3\alpha_2}{2(1+\sigma)cb^2}, \tag{25}$$

$$FS^{AI} = \int_0^{n_2} (\alpha_2\overline{N}_1 - 2c\theta - 2p_2)d\theta = \frac{a^2\alpha_2^2}{2(1+\sigma)cb^2}, \tag{26}$$

$$W^{AI} = \frac{a^2\alpha_2}{2(1+\sigma)cb^2}(\alpha_2 + a); \tag{27}$$

where the superscript AI denotes Allocation I.

A monopoly corner solution occurs when $\alpha_2 \geq a$. Using the welfare equations (19) and (27), $W^{AI} < W^{m_C}$ occurs when $\sigma > \frac{a+2\alpha_2}{3a+2\alpha_2}$. A monopoly interior solution occurs when $\alpha_2 < a$. Using welfare equations (23) and (27), $W^{AI} < W^{m_I}$ occurs when $\sigma > \frac{64\alpha_2 a^2}{5(a+\alpha_2)^3} - 1$.

□

Proof of Theorem 5: Note first that Allocation II in Theorem 2 exists when $\frac{b(1-\delta)vc}{a^2\alpha_2} \in (0, \frac{1}{8})$: Equations (14) and (15) imply we have two equations and two unknowns, x and n_1^A. Solving these equations implies x is implicitly defined by: $t \equiv \frac{b(1-\delta)vc}{a^2\alpha_2} = \frac{(1-x)x}{(2-x)^2}$. This implies $0 = (1+t)x^2 - (1+4t)x + 4t$. Solving for x as a function of t and using the quadratic formula such that $x \in (0,1)$ implies we must have $t \in (0, \frac{1}{8})$. Thus, Allocation II exists if and only if $(1-\delta)v < \frac{a\alpha_2\overline{N}_1}{8c}$.

Consider the case where $\sigma = 1$. We show two cases where Allocation II dominates either the monopoly corner solution or the monopoly interior solution in terms of welfare. When $x = 1/2$, equations (14) and (15) imply half of firms and a third of consumers multi-home. The welfare from this allocation is greater than the welfare from the monopoly interior solution if and only if $0 > 135a^4 - 484a^3\alpha_2 - 150a^2\alpha_2^2 + 540a\alpha_2^3 + 135\alpha_2^4$. This occurs when $\alpha_2 \in [r_1 \cdot a, r_2 \cdot a]$ where $r_1 \cdot a$ and $r_2 \cdot a$ are the roots of the preceding polynomial with $1 > r_2 > r_1 > 0$ so that $a > \alpha_2$ so that we have an interior solution. This provides an example where Allocation II generates greater welfare than the monopoly interior solution when the interior solution exists. However, the welfare for $x = 1/2$ is never greater than the monopoly corner solution.

When $x = .9$, equations (14) and (15) imply a tenth of firms and (8/11)s of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly corner solution for all $\alpha_2 \geq a$ since $3.3388\alpha_2 + 3.5823a > 3\alpha_2 + 2a$. $\square$

Proof of Theorem 6: In Allocation III all firms single-home and all consumers multi-home; we have $n_2^M = 0$ and $n_1^A = n_1^B = n_1^M = \overline{N}_1 = \frac{a}{b}$. With $p_1 = 0$ and $p_2 = 0$ we have $n_2^A = n_2^B = (1/2)N_2 = \frac{\alpha_2 a}{bc}$; resulting in

$$\Pi^A = \Pi^B = 0, \tag{28}$$

$$CS^{AIII} = \int_0^{a/b} (a - b\tau)\frac{\alpha_2 a}{bc} d\tau = \frac{a^3\alpha_2}{2(cb^2}, \tag{29}$$

$$FS^{AIII} = \int_0^{\frac{\alpha_2 a}{bc}} \alpha_2\overline{N}_1 - c\theta d\theta = \frac{3a^2\alpha_2^2}{8cb^2}, \tag{30}$$

$$W^{AIII} = \frac{a^2\alpha_2}{8cb^2}(3\alpha_2 + 4a); \tag{31}$$

where the superscript $AIII$ denotes Allocation III.

By comparing equations (19) and (31) we see that $W^{AIII} > W^{m_C}$ always holds. The corner solution for the monopoly platform is implemented when $\alpha_2 \geq a$; thus, $W^{AIII} > W^m$ when $\alpha_2 \geq a$. When an interior solution occurs, equations (23) and (31) imply $W^{AIII} > W^{m_I}$ if and only if $0 > 5a^4 - 44a^3\alpha_2 - 18a^2\alpha_2^2 + 20a\alpha_2^3 + 5\alpha_2^4$. This occurs when $\alpha_2 > (1/4)a$. Thus, $W^{AIII} > W^m$ when $\alpha_2 > (1/4)a$. $\square$

Proof of Theorem 7 The proof follows in two parts:

Lemma 1. *When $f_1 \leq f_1^L \equiv \delta v + \frac{\alpha_2^2\overline{N}_1}{8c}$ we have $p_1^X = p_1^Y = \delta v$, all consumers multi-home, $N_1 = n_1^M = n_1^X = n_1^Y = \overline{N}_1$, $p_2^X = p_2^Y > f_2$, all firms that participate single-home, $n_2^X = n_2^Y = (1/2)N_2$, and profits are zero for each platform.*

Proof of Lemma 1 The fact that neither platform has an incentive to deviate given such prices with profits equal to zero follows in the same manner as Theorem 2. To determine the cutoff point, $f_1^L \equiv \delta v + \frac{\alpha_2^2 \overline{N}_1}{8c}$, we have two equations and two unknowns. First, $u_2(N_2) = 0$ gives the last firm to join a platform. This implies $N_2 = \frac{\alpha_2 \overline{N}_1 - p_2}{c}$ which implies $p_2 = \alpha_2 \overline{N}_1 - cN_2$. The second equation is the zero profit condition, $\Pi^X = \Pi^Y = 0$, which implies $(1/2)N_2 + \overline{N}_1(\delta v - f_1) = 0$. Substituting $N_2(p_2)$ implies p_2 is given by $(\alpha_2 \overline{N}_1 - p_2)p_2 = 2\overline{N}_1 c(f_1 - \delta v)$. Similarly, substituting $p_2(N_2)$ implies $N_2 \geq 0$ is given by $N_2(\alpha_2 \overline{N}_1 - cN_2) = 2\overline{N}_1(f_1 - \delta v)$. The most N_2 can be is $N_2 = \frac{\alpha_2 \overline{N}_1}{2c}$; hence, profits can only be nonnegative when $f_1 \leq \delta v + \frac{\alpha_2^2 \overline{N}_1}{8c} \equiv f_1^L$. $\square$

When the marginal cost gets larger, $f_1 > f_1^L$, the pricing strategy in Lemma 1 is not sustainable as profits are negative if both platforms have participation. Since Platform Y is non-focal, it must charge a higher price while using the divide and conquer strategy and this still affects the focal platforms prices.

Lemma 2. *When f_1 is such that $f_1^L < f_1 < f_1^H \equiv \delta v + \frac{\alpha_2^2 \overline{N}_1}{4c}$ we have $p_1^X = \delta v$, all consumers join Platform X, $N_1 = n_1^X = \overline{N}_1$, and $p_2^X > f_2$, all firms that participate join Platform X, $n_2^X = N_2$, and $0 < \Pi^X < \Pi^m$.*

Proof of Lemma 2 The fact that neither platform has an incentive to deviate given such prices with profits equal to zero follows in the same manner as Theorem 2. To determine the cutoff point, $f_1^H \equiv \delta v + \frac{\alpha_2^2 \overline{N}_1}{4c}$, we have two equations and two unknowns. First, $u_2(N_2) = 0$ gives the last firm to join a platform. This implies $N_2 = \frac{\alpha_2 \overline{N}_1 - p_2}{c}$ which implies $p_2 = \alpha_2 \overline{N}_1 - cN_2$. The second equation is profits which must be maximized by Platform X: $\Pi^X = N_2(\alpha_2 \overline{N}_1 - cN_2) - \overline{N}_1(f_1 - \delta v)$. Maximizing implies which implies $N_2^* = \frac{\alpha_2 \overline{N}_1}{2c}$. Profits are positive with these prices only when $f_1 < \delta v + \frac{\alpha_2^2 \overline{N}_1}{4c} \equiv f_1^H$. $\square$

When the marginal cost on the consumer side is too large, $f_1 > f_1^H$, the divide and conquer strategy is not profitable and the presence of the non-focal platform has no competitive affect on the focal platform. Thus, the focal platform is a monopoly platform. $\square$

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