

# Credible Signaling via Transfers, Job Application Fees

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How low might be the resource costliness of making signals credible? Using a job market as an example, We build a signaling model to determine the extent to which a transfer from an applicant might replace a resource cost as an equilibrium method of achieving signal credibility. As long as a firm's claim to be hiring for an open position is credible, and profitability of the hiring process per se is limited to an application fee, the firm has an incentive to use the properly calibrated fee to implement a separating equilibrium. Applicant risk aversion does not necessarily discourage a monopsonist potential employer from using an application fee, but a firm hiring in a competitive labor market with risk-averse applicants may prefer a pooling equilibrium, hiring all applicants at their average productivity. Partial extension to a model with third-party assistance (a headhunter or a job board) is possible. (*JEL* D82, J24, C72, J31)

Adverse selection becomes a concern when a party A faces a decision based on information possessed by a party B, whose utility is also affected by A's decision. That is, under what circumstances can party A rely on information communicated by party B?

Spence's 1973 paper introduced a model in which employers may use education as a screening device. In a related context, Akerlof (1970) provided perhaps the most widely taught adverse-selection example. Stiglitz (1975) discussed the

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concept of screening in the context of employment and education. All the above mechanisms are costly ways of solving an adverse-selection problem by creating an incentive to self-select. By contrast, in cheap-talk games (Crawford and Sobel, 1982; Chakraborty and Harbaugh, 2010), where communication is privately and socially costless, information that can credibly be transmitted is limited, usually severely. This paper asks, since a sender must incur a cost of transmitting if the message is to be credible (for present purposes, the cost of obtaining, say, an MBA degree, is here labeled a cost of transmitting), to what extent can the cost be reduced for society by using a transfer instead of a pure resource cost?

We address this question not to explain common occurrences in markets, but to better understand the foundations of the economics of transacting under asymmetric information. To explore these foundations, imagine that a firm can credibly commit to considering only those applicants who pay an application fee that might be substantial, and also credibly commit to not collecting application fees as a profitable activity without an appropriately compensated job waiting for the chosen applicant. A test that might distinguish between applicants in some aspect of their suitability could still be conducted, but only if the firm's resource costs of administering the test and evaluating the effectiveness shown are quite small, and an applicant's resource costs of preparing for and taking the test are negligible compared both to the resource costs of a usual signal and to the size of the application fee. With this setup, the question becomes whether a suitably calibrated application fee can achieve the same types of signaling equilibria that are accomplished by calibrating the resource cost of the usual sort of signal, such as obtaining a particular level of education. That substantial application fees are not a common element of any labor markets of acquaintance does not bear on the relevance of this research.

Two papers touch on this question, both less directly, and subject to clear objections. Wang (1997) introduces an employment model in which only if the firm commits to a wage schedule before the applicants pay the fee might an application-fee equilibrium be possible. Though set-of-wages, positive-application-fee equilibria may be possible below, it is not because the firm has ex ante to be committed to a set of wages before applicants pay the fee (see the headhunter model in Section III). As to using necessity of commitment to explain why no application fee is

observed in reality as Wang (1997) does, no practical reason can be given why a firm could not commit to a schedule of multiple wages corresponding to multiple estimated productivities (indeed, this is a feature of nearly every job posting seen in the economics new-Ph.D. market). Also, the pre-commitment argument is based on the assumption that firms have full control over wages. If the wage is instead determined through, say, Nash bargaining, it is obvious that applicants can still expect to share some surplus, making a positive application-fee possible.

Guasch and Weiss (1981) suggest that applicants' risk aversion and an assumption that applicants do not have perfect information about themselves may prevent a positive-application-fee equilibrium. As shown below, risk aversion alone is insufficient to prevent a positive-application-fee equilibrium. The Guasch/Weiss model requires the assumption that the labor-supply constraint is not binding, which is problematic: if there are more than enough high types applying, why test all of them? Where do the "extra" high-type applicants go? Firm profit maximization implies that they are not hired while applicants' expected returns show that they get paid and hired.

The models used in this paper are similar to those of Guasch and Weiss (1981). In fact, the one-firm, multiple-applicant case can be regarded as a simpler version of their model, while avoiding the "labor-supply constraint not binding" problem.

The models below are based on the assumption that the firm genuinely wishes to hire someone and this is believed by the applicant. Indeed, a rather convincing explanation of why job application fees are not observed is that with a fee, the applicant is no longer certain whether the advertised vacancy exists, or if the firm is simply trying to collect application fees. The credibility of many firms, especially prestigious ones, is in fact too costly to risk fraudulently collecting application fees for nonexistent jobs. Also, with a third party collecting fees, as in the headhunter model below, firms' credibility is not an issue.

Although the models use job-application settings, they can, to varying degrees, apply to other contexts as well. For example, the job vacancy can easily be interpreted as a promotional opportunity within a firm. The fee may not necessarily represent a cash transfer from applicant to the firm, but may also represent, say, payment below productivity during a required internship period. Another possibil-

ity is that a firm may attempt to credibly signal quality of a product or product line or a service by a donation to charity that it knows will be given publicity at no cost to the firm.<sup>1</sup>

In the following models, as long as there is a separating equilibrium, there is always a positive fee. Whether there is a separating equilibrium depends entirely on the firm's (or the headhunter's, in the headhunter model) incentives. If there is no separating equilibrium, everyone is hired with no fee, and no testing occurs, which is hardly a tenable condition. With a zero test cost, there is almost always a separating equilibrium, except a very specific case under the risk-averse-applicant assumption.

## I. THE BASE MODEL

Consider a game between a profit-seeking monopsony employer and a potential applicant. The applicant is either type 1 or type 2, and knows her own type. The firm does not know the type but correctly knows the distribution of types (probability  $p \in (0, 1)$  of being type 1;  $1 - p$  of being type 2). Type  $t$  worker has productivity  $t$  if working for the firm ( $t = 1, 2$ ). Both types can produce  $k \in (1, 2)$  at home if not hired.<sup>2</sup> At cost  $c \geq 0$ , the firm can conduct a small test, with probability  $q \in (0.5, 1)$  of correctly revealing the applicant's type and probability  $1 - q$  of being misleading, thus possibly of very little reliability. The hiring game is played in the following way:

**Step 1** The firm chooses its strategy  $s = (w_L, w_H, f)$ , where  $w_L$  is the wage offered to an applicant with test result 1,  $w_H$  is the wage offered to an applicant with test result 2, and  $f$  is the application fee.

**Step 2** The potential applicant sees the wage/fee schedule  $s$  and decides whether or not to apply for the position. If she applies, she must pay the application

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<sup>1</sup>For a context yielding several more examples, see Spence (2002).

<sup>2</sup>A common default productivity accords with Spence's 1973 assumptions, and fits reasonably a case in which the differential productivity the firm seeks to uncover is firm-specific, or perhaps industry-specific, rather than yielding a similarly large productivity difference to most potential employers.

fee.

**Step 3** If the applicant has applied in step 2, she takes the test and the result is revealed for both the firm and the applicant. The applicant then decides whether to accept the wage offer.

For the above defined game, the firm's strategy space is  $\mathbb{R}^3$ . The applicant chooses  $(App(f, w_L, w_H, t), Acc(f, w_L, w_H, t, x))$ , in which  $t$  is her type and  $x$  is the realized test result. "App" can be either "apply" or "not"; "Acc" can be either "accept" or "not".

To avoid trivialities, assume

$$c < 2 - k. \quad (1)$$

That is, the cost cannot be so large that the firm would not make an offer to a known high type. For simplicity, also assume that the applicant accepts the offer if she is indifferent in Step 3, and that she applies if she is indifferent in Step 2. Assume, of course, that both the firm and the applicant play to maximize their expected payoff. The above specifications yield the following Theorem, proved in Appendix.A.

**Theorem 1** (Main Theorem). *A strategy profile satisfies subgame-perfect equilibrium of the above defined game if and only if:*

*In step 1, the firm implements a separating equilibrium in which the potential applicant applies if and only if she is a high type, and hires anyone that applies while setting  $w_L$ ,  $w_H$  and  $f$  such that*

$$w_H > w_L \geq k, \quad (2)$$

$$qw_H + (1 - q)w_L - k = f. \quad (3)$$

*In step 2, a type  $t$  potential applicant applies if and only if*

$$[(3 - 2t)q + t - 1] * \max\{w_L, k\} + [(2t - 3)q + 2 - t] * \max\{w_H, k\} - k \geq f. \quad (4)$$

*In step 3, the applicant accepts the offer if and only if the wage is no less than  $k$ . That is, for an applicant with test result  $x$ , accept if and only if  $w_x \geq k$ .*

Equation (4) has the potential applicant apply if the expected value added by applying is no less than the application fee  $f$ .

From (3),  $w_L \geq k$  and  $w_H \geq k$  ensures that if the applicant applies, she is hired, at a test-dependent wage level.  $w_H > w_L$  separates the value of applying for different types, in favor of the high type, given  $q > 0.5$ .

The fee determined by equation (3) leads the high type to apply, though indifferent. Any higher fee prevents the high type from applying. The low type does not apply because, compared to the high type, she has a lower chance of receiving the high wage, but faces the same application fee. For given  $w_L, w_H$  satisfying (2),  $f' = (1 - q)w_H + qw_L - k$  is the highest fee that induces the low type to apply; fees in the interval  $(f', f)$  reduce fee income, and lead the high type to strictly prefer applying, without otherwise affecting the outcome.

For example, setting  $w_L = k$ ,  $w_H > k$  and let  $f$  be determined by equation (3) yields a subgame-perfect equilibrium. In such an equilibrium, both types are in their most productive positions (firm for high types and home for low types), and perfect separation is achieved without testing the low type, thus saving on testing cost. The application fee serves to make an imperfect sorting device (the test) perfect, even though the fee is purely a private cost rather than a social cost.

Interestingly, the actual cost of the test  $c$  does not enter equation (3) in determining the fee. Indeed, as long as  $c$  is nonnegative and satisfies equation (1) the theorem holds. Specifically,  $c$  can be 0. A nonnegative  $c$  does not play a role in separation, only provides an incentive for the firm to separate. A negative  $c$  may make it optimal for the firm to test everyone (the last inequality in case 3 of the proof may not hold if  $c < 0$ ). For some situations, applying this model would naturally suggest a negative  $c$ . If the test represents some form of internship or other productive activity, and the fee as the reduced pay in this activity, there is a legitimate reason to claim that  $c$  can be negative, meaning the interns are producing more than the funds it took the firm to set up such a program.

Similar to the discussion about the internship, if an employee's type may be (imperfectly) revealed only after some periods of employment, the employment period

before such revelation can be considered a test to determine the wage afterwards. Aside from the possibility that  $c$  is negative, there are two differences to the base model: the “test” cost  $c$  now is less for a high than for a low type, and the “test” is possibly perfect. There will still be a set of separating equilibria if  $c$  is still positive.

Note that the base model is a screening game rather than a signaling game. That is, the firm makes all the decisions first, and then lets nature and the applicant do all the separating, rather than observing some signals sent by the applicant, and then make decisions based on updated information about applicant type. Note also that, while the fee is an effective screening device, it cannot be made into a signaling device simply by moving the wage decision to the last step. Subgame perfection would require that the firm pay no more than  $k$  in the last stage, and as a result no applicant would pay a positive fee to apply. For an example in which a separating equilibrium is reached with a positive fee and firm decisions after signaling by an applicant, see the headhunter model in Section III.

## II. MODIFICATIONS TO MODEL

This, and the following four sections, examine the robustness of Theorem 1, with eight alterations to or extensions of the model. In the base model’s separating equilibrium, the firm seeks to exclude the low type, has no option but to test an applicant, and the cost  $c$  of conducting a test of limited reliability is unavoidable. That a separating equilibrium results may be no surprise. This section considers three ways in which the firm might hire without necessarily testing.

### II.A. *Mod 1*

Suppose, instead of only being considered if the applicant pays the fee, she is considered automatically, but to be tested requires paying the fee. In step 1, the firm, in addition to  $(w_L, w_H, f)$ , now chooses  $w_N$ , the wage for an applicant not paying the fee.

For this first modification, there can be pooling equilibria. Specifically, if  $\frac{c(1-p)}{p(k-1)} > 1$ , the firm decides that the adverse selection problem is not big enough to justify

spending so much resources on finding out who is the high type.<sup>3</sup> It offers a fee so high or  $w_H$  and  $w_L$  so low that the applicant does not take the test and then hires both types at wage  $w_N = k$  without testing. Therefore, if the above inequality is met, there is a set of pooling equilibria but no separating equilibria.

## II.B. Mod 2

For either model (Base Model or Mod 1), instead of testing everyone who paid the fee, suppose the firm gives fee payers a random chance  $m$  of actually being tested. In addition to  $(w_L, w_H, f)$ , the firm also selects  $w_M$ , the wage offered if the applicant paid to take the test but was not randomly selected to take it, and  $w_N$ , the wage offered if the applicant applied but did not pay for a chance to take the test. The firm can set  $m$  as close to 0 as possible and can still implement a separating equilibrium. It accomplishes this by setting wage/fee schedule with  $w_H > w_L \geq k > w_N$ ,  $w_M \geq k$  and so that the high type pays the fee though indifferent, and the low type does not pay. This produces a result approaching the full-information labor allocation, while the firm extracts all the surplus. Therefore there cannot be a pooling equilibrium.

## II.C. Mod 3

As in Mod 2, the firm always considers the applicant, and an applicant can decide to pay the fee and request to be tested, with the firm randomly administering the test with chosen probability  $m$ . Now, however, suppose the application fee is refunded unless the test is actually administered. The firm can again approach the full-information optimum, as in Mod 2 by setting  $w_H > w_L \geq k > w_N$ ,  $w_M \geq k$  and so that high type pays the fee though indifferent, and let  $m$  approach 0.

The next two sections respond to the interesting questions raised by Wang (1996) and Guasch and Weiss (1981), respectively. The following section considers

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<sup>3</sup>In a pooling equilibrium where the firm hires without testing, its profit is  $p + 2(1 - p) - k = 2 - p - k$ , in a separating equilibrium, its profit is  $(1 - p)(2 - c - k)$ . Comparing the two:  $2 - p - k > (1 - p)(2 - c - k) \iff c(1 - p) - p(k - 1) > 0 \iff \frac{c(1 - p)}{p(k - 1)} > 1$ .

whether salary commitment is a necessary condition for a positive application fee, by introducing a third party screener. Section IV considers risk-averse applicants.

### III. APPLICATION FEE IN A SIGNALING GAME: ADDING A THIRD PARTY

As discussed in Section I, the application fee in the base model cannot be converted into a signaling device directly. Suppose there is perfect competition by firms hiring in this labor market, but an applicant can only be considered by a firm after she pays a fee to that particular firm. Once an applicant has paid the fee to a particular firm, that firm no longer faces any competition in hiring that worker, and so offers at most wage  $k$ . Any positive fee is then impossible.

This issue may be resolved by having a third party. The applicant must pay a fee to this third party to enter the market; upon entry, all firms in this market can compete for her employment. In this variation, the firms do not receive the fee, therefore their statements that they have an actual job opening to fill will be more credible.

Consider a job market with multiple firms competing with each other, while still only having one applicant, with type assumptions as before. Now assume there is a headhunter, who holds some monopoly power in the market: firms can only hire a job applicant through the headhunter, who may demand a fee for the applicant to be available for hire.<sup>4</sup>

The hiring game is played in sequence as follows:

1. The headhunter sets a fee  $f$ .
2. The applicant decides whether to pay the fee to enter the market.
3. The firms quote wages.

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<sup>4</sup>A frequent example is a government agency that has to specify that an applicant meets certain criteria before she can be hired into a particular field or for a particular job. This model considers the agency possibly setting the fee way in excess of their cost of the certification, which may sometimes be realistic.

4. If she has entered the market, the applicant chooses a firm and applies.
5. The applicant is tested, costing the firm  $c$ ; she signs a waiver ceding the right to apply to or negotiate with any other firm.<sup>5</sup>
6. Applicant and firm learn the test results; previously set wages are offered to the applicant; she decides whether to accept or not; if she accepts, she is hired. If not, she returns home and produces  $k$ .

The waiver is a convenient way to [i] keep both  $w_L$  and  $w_H$  wage quotations relevant to applicant decisions, and [ii] prevent the applicant from applying to another firm if she tests low at the current firm. It yields the most straightforward comparison to analysis in prior sections.

Lexicographic tiebreakers: as before, [i] the applicant is assumed to apply and work if indifferent. Also, [ii] if two firms quote wage offers with the same expected wage, it is convenient to break the tie by assuming the high type applies to the one quoting a higher  $w_H$  (thus a lower  $w_L$ ), and the low type applies to the one quoting a higher  $w_L$  (lower  $w_H$ ). This tie-breaker follows trembling-hand considerations (Selten, 1975). [iii] Across firms quoting identical wages, she randomizes equiprobably.

Of course, it does not matter to the applicant to whom she pays the fee. So if equation (4) from section I holds, the high-type applicant continues to apply though indifferent, and the low type continues to strictly prefer not applying.

In a separating equilibrium, each firm for whom the probability of receiving an application is positive must set  $(w_L, w_H)$  so that  $w_H > w_L$  and (3) is satisfied. Despite introducing the headhunter, (3) still makes the high type indifferent over entering the market and applying, and yields a strict preference for the low type to stay home.

Suppose the headhunter sets  $f \leq 2 - k - c$ , the high type applies, and the low type may or may not apply. Were a given firm to face a pattern of wage offers in which every other firm offered the high type an expected wage below  $2 - c$ , its best response would be to offer slightly higher wages.

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<sup>5</sup>Having the headhunter cover the testing cost out of fee revenue only yields obvious adjustments to the equilibria.

Thus, for the headhunter to set  $f = 2 - k - c$ , firms to set  $(w_L, w_H)$  so that  $w_H > w_L$  and (3) and (4) are satisfied, high type to apply, and the low type not to apply, constitutes a separating equilibrium. In such an equilibrium, the headhunter expropriates all the social surplus, and the high-type expected wage is  $2 - c$ , leaving firms with 0 profit in this labor market. The high type applies to the firm whose wage offer has the largest difference  $w_H - w_L$ , but the expected wage being driven up to productivity removes any incentive for other firms to deviate to attract the high type.

Are these the only equilibria? The headhunter can be shown, without doing algebra, to set too high a fee to allow a pooling equilibrium in which both types enter. In separating equilibrium, the headhunter extracts all surplus of an efficient labor market. The low type only enters if the chance of being hired justifies paying the fee, and hiring the low type reduces surplus. Wages yielding a high enough expected wage to yield low-type entry must pay some surplus to the high type, who has a greater probability of being offered  $w_H$ . So the headhunter would receive only a portion of the smaller surplus. Details are provided in Appendix C.

If application-fee revenue is used by the headhunter for some social purpose with social marginal valuation approximately dollar-for-dollar (or better), then the applicant's private cost of signaling is nearly a transfer, at most a negligible social cost.<sup>6</sup>

#### IV. RISK-AVERSE APPLICANTS

This section returns to the model with only one firm. A natural question to raise is why the model's predicted job application fees are rarely a transparent occurrence (how often some implicit transfer occurs, especially relying on a third party providing a connection, is unknown). One explanation may be that applicants are risk-averse; so far, both the firm and the applicant have been risk-neutral. As Guasch and Weiss (1981) pointed out, when the applicant becomes risk-averse, she demands more than an average return of  $k$  to be willing to take on the risk of an uncertain wage due to the uncertain test result. Intuitively, if the risk premium de-

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<sup>6</sup>For a model using a similar setup to discuss the status-seeking motive of charitable donations, see Glazer and Konrad (1996).

manded is high enough, and the cost of hiring a low type is low enough, the firm may be unwilling to pay the risk premium as the cost of separating equilibrium, and hire everyone without testing instead. Assume both types of applicant have the same pattern of risk tolerance.

Under what conditions can a separating equilibrium be preserved? Instead of positing a particular risk-averse utility function, consider a wage/fee schedule and ask how high a risk premium is needed for a high type to accept. Specifically, in the base model, the firm can choose to have the two wages to be arbitrarily close, the focal issue is the risk premium required if  $w_L$  is quite close to  $w_H$ .

Above, as is usual, the applicant is indifferent between a wage of 12 and fee of 2, and a wage of 13 and fee of 3. However, this section's analysis of risk aversion is clarified by generalizing to a utility function  $U(w - f, f)$ , for either type of applicant, with the usual concavity maintained via assuming  $w \geq f \implies \frac{\partial U}{\partial (w-f)}(w - f, f)$  is decreasing in  $w - f$  for any  $f$ .

Let  $w > k$ ; there exists an  $\varepsilon > 0$  small enough such that  $w - \varepsilon \geq k$ . Then the wage/fee schedule  $w - \varepsilon, w + \frac{1-q}{q}\varepsilon$  and  $f = w - k$  is a viable schedule to implement separating equilibrium in the risk-neutral case. As above,  $q$  is the probability the test correctly identifies the applicant's type. Since this involves risk, a risk-averse high type would demand a risk premium to accept such an offer; for clarity, treat the risk premium as being subtracted from  $f$ .<sup>7</sup>

Naturally, assume the risk premium increases with  $\varepsilon$ . Therefore the firm would prefer to offer wages as close to each other as possible.

A schedule  $s$  that makes the high type indifferent over accepting would not be accepted by the low type, who would end up receiving the low wage with a greater probability than the high type. So the firm only has to make sure that high types are indifferent in order to implement a separating equilibrium.

Thus  $s^* = [w - \varepsilon, w + \frac{1-q}{q}\varepsilon, f - RP(f, \varepsilon)]$ —where  $RP$  is a function that maps  $f$  and  $\varepsilon$  into the amount of risk premium that makes the high type indifferent—is a potential schedule in a separating equilibrium. The applicant types separate under this wage/fee schedule, provided the firm is willing.

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<sup>7</sup>The risk could be addressed by increasing  $w_H$ , but as a high type cannot ensure the high wage, subtracting from  $f$  is more straightforward.

Note that all possible sets of wages satisfying equation (2) can be represented by the above wage schedule, via changing  $\varepsilon$ . Since any separating equilibrium must satisfy equation (2), the wage schedule can be represented as above. Given the above wage schedule, the fee must be  $f - RP(f, \varepsilon)$ , as the high type would not accept any higher fee, and the firm's profit is suboptimal for any lower fee. So any separating equilibrium takes this form.

First consider, for later comparison, the extreme case in which the applicant is absolutely risk-averse. That is, the applicant would always value a lottery at the lowest possible payoff. For this case,  $RP(f, \varepsilon) = k - (w - \varepsilon) + f = \varepsilon$ ,<sup>8</sup> the upper bound of the  $RP$  function.  $RP = \varepsilon$  guarantees a payoff of at least  $k$ , even if the test result suggests a low type. So this case yields the low and high types evaluating the wage/fee schedule identically, preventing a separating equilibrium.

In less extreme cases, the firm can choose  $f$  to minimize  $RP$ . Suppose

$$\inf_{f \in (0, \infty)} \lim_{\varepsilon \rightarrow 0} RP(f, \varepsilon) = 0; \quad (5)$$

this is equivalent to a continuous utility function at  $k$ . A separating equilibrium can be implemented by choosing the right  $f$  and setting  $\varepsilon$  as close to 0 as possible to pay almost no risk premium. This is different from Guasch and Weiss (1981), because in their model, the high type has a higher reserve wage than the low type, thus selecting  $w_L$  and  $w_H$  arbitrarily close is not an option.<sup>9</sup>

Our third risk-aversion model assumes a strictly positive risk premium for any distance between  $w_H$  and  $w_L$  (less extreme than absolute risk aversion). Let

$$0 < z = \inf_{f \in (0, \infty)} \lim_{\varepsilon \rightarrow 0} RP(f, \varepsilon), \quad (6)$$

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<sup>8</sup>The applicant is only willing to apply if the difference between the low wage and the fee is at least  $k$ , therefore  $(w - \varepsilon) - (f - RP(f, \varepsilon)) = k$ , rearrange to get the first equation. Replace  $f$  with  $w - k$  to get the second equation.

<sup>9</sup>The Guasch and Weiss assumption, though the reverse of Spence's assumed common default productivity, could be apt for a situation in which, without applying, a high type would be able to obtain a significantly greater wage in other industries than would a low type.

i.e., for any  $f$ , applicant utility function is discontinuous at net income  $k$ .<sup>10</sup>

Consider this third case of risk aversion. Instead of production of 1 and 2, consider production level  $l$  for low type and  $h$  for high type. Compare firm's surplus in the separating equilibrium and in the pooling equilibrium in which the firm hires both types without testing.

The firm's surplus in separating equilibrium is

$$(1 - p)(h - c - k - z), \quad (7)$$

and in pooling equilibrium is

$$p(l - k) + (1 - p)(h - k). \quad (8)$$

Subtracting (8) from (7)

$$(1 - p)(-c - z) - p(l - k), \quad (9)$$

or,

$$-c - z + p(c + z - l + k). \quad (10)$$

The firm only seeks a separating equilibrium if expression (10) is positive. The base model assumes  $1 < k$  to give a welfare motivation to not hire the low types. With a similar assumption that  $l < k$ , separating equilibrium becomes more likely as  $p$  goes up (the low type becomes more likely, so hiring without testing becomes more harmful), as  $c$  or  $z$  goes down (cost of separating equilibrium becomes lower), as  $l$  goes down (the cost of hiring the low type becomes more harmful), as  $k$  goes up (hiring becomes more costly). Note that  $h$  is not in expression (9), since in both separating and pooling equilibria, high types are hired.

This analysis applies anytime a strictly positive risk premium is required. For example, if equation (5) holds, but for any reason  $\varepsilon$  cannot approach zero—that is,  $w_L$  and  $w_H$  cannot be arbitrarily close—a risk premium bounded above zero may be needed. A possible reason for  $\varepsilon$  not to approach zero is at-home productivity  $k$

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<sup>10</sup>Since all concave functions on real open intervals are continuous, there does not exist a utility function fitting this third case; its analysis is distinct, and is used in the next section.

differing with type. The next section provides another.

## V. MULTIPLE FIRMS COMPETING FOR APPLICANT

This section examines whether multiple firms competing for one risk-neutral hire can affect realization of separating equilibrium. If separating equilibrium is still achievable, it must be allowing the high type to get all the surplus, since otherwise another firm would offer a higher wage and attract the high-type worker away. On the other hand, the low type must be getting less than  $k$  if she took the firm's separating offer. This immediately means that, any wage/fee schedule that implements separating equilibrium with multiple firms needs to separate the two types sufficiently far. Opportunities to separate with a small fee, or with  $w_L$  closely below  $w_H$  are more restricted, perhaps preventing separating equilibrium were this section blending firm competition and applicant risk aversion.

For separation, the wage/fee schedule must satisfy:

$$w_H q + w_L(1 - q) - f = 2 - c, \quad (11)$$

$$w_H(1 - q) + w_L q - f < k. \quad (12)$$

Equation (11) ensures the high type's expected wage minus fee equals social surplus; equation (12) discourages the low type from applying. Subtracting (12) from (11) yields

$$(2q - 1)(w_H - w_L) > 2 - c - k. \quad (13)$$

Equation (2) is still needed to ensure hiring all that applied. Any set of  $w_H$ ,  $w_L$  and  $f$  which satisfies equations (2), (11) and (13) can be a wage/fee schedule for a separating equilibrium.

Having specified what a separating equilibrium must be like, can it be achieved? Yes, unless there is an arrangement that can provide an expected wage minus fee for high types higher than  $2 - c$ , while keeping the firm's return non-negative. Since

the firms and the low type are already getting their reservation level, allowing the high type to get even more requires higher social surplus than separating equilibrium can attain. Since the only deviation from the full-information optimum in the separating equilibrium comes from testing the high types, if randomizing tests are disallowed, the only possible way to achieve higher surplus is by hiring everyone without testing, which can be checked by:

$$2 - c \geq 2(1 - p) + p = 2 - p. \quad (14)$$

Separating equilibrium is not possible if  $p$  is so low that the firm can hire without testing while offering a sizable wage (close to 2), or if  $c$  is so high that testing is too costly to be justified.

As in sections I, II.A, and III, even when  $c$  is 0 (costless test administration), separating equilibrium may still be achieved simply by having a positive application fee.

## VI. CONTINUOUS TYPES

Returning to one firm, is separating equilibrium robust to the applicant having continuous types?

Let the applicant's type be any real number in  $[1, 2]$ , and type  $t$  generates output worth  $t$  if hired by the firm. The test still only produces two possible results: a high test result and a low test result. Let the test accuracy be  $q$ ,  $1 > q > 0.5$  as before, with  $(2q - 1)t + 2 - 3q$  the probability type  $t$  attains a high test result. Thus, the probability of a high test result increases linearly with  $t$ , from  $1 - q$  for  $t = 1$  to  $q$  for  $t = 2$ . Initially, assume  $k$ , the applicant's home production level, is the same for all types, and that the firm can only hire an applicant after she is tested. (So, for now, the model is more similar to the Base Model than to Mod 1.) For the first part of this section, also assume there is a smallest monetary unit  $0 < \delta < (2q - 1)^{-1}$ .

Hiring necessarily costs at least  $k$  in salary plus  $c$  for the test, so the firm has no interest in types below  $k + c$ , but wishes to hire types above  $k + c$  if cheap enough. Observing only a high or a low test result, but not observing  $t$ , limits what is attain-

able.

For any wage/fee schedule  $s$ , a type  $t$  applicant's expected net wage is:

$$W(t | s) = [(2q - 1)t + 2 - 3q]w_H + [1 - ((2q - 1)t - 2 + 3q)]w_L - f, \quad (15)$$

which is linear in  $t$  with slope  $(2q - 1)(w_H - w_L)$ .

Consider a separating equilibrium where only types no less than a certain threshold apply. As (by assumption) both productivity and the chance of testing high increase linearly with type, so will the expected wage. The firm prefers to hire a higher type if and only if the slope of productivity, which is 1, exceeds the expected wage slope:

$$1 > (2q - 1)(w_H - w_L). \quad (16)$$

Consider the case in which the firm chooses  $s$  satisfying the following:

$$w_H > w_L \geq k, \quad (17)$$

$$W(k + c | s) = f, \quad (18)$$

$$w_H - w_L = \delta. \quad (19)$$

The first two equations simply mirror firm behavior in the base model specified by (2) and (3). Equation (17) ensures that the gain from applying increases with type, and the offer for testing low is accepted. Equation (18) has type  $k + c$  apply though indifferent. Equation (19) (which implies (16)) minimizes the surplus paid to types above  $k + c$ , allowing the firm the maximum attainable surplus. Note that the distribution of types does not enter the equations characterizing separating equilibrium.

To consider pooling equilibrium, enable the firm to hire an applicant not taking the test. (Now the model is more similar to the one in Mod 1 rather than the Base Model.) A pooling equilibrium is implemented if the firm hires everyone without testing at wage  $k$ . Equilibrium profit depends on the distribution of types.

A direct comparison of a separating equilibrium (where the hiring of an appli-

cant not taking the test is disallowed) and a pooling equilibrium (where such hiring is allowed) would be unconvincing. So consider a separating equilibrium assuming any type applicant can decline to pay the fee in order to take the test, and may still be hired. Interestingly, an equilibrium schedule  $s$  leads to an interval of types applying and choosing to take the test, and the firm does not make an offer to non-test-taking types. To see this, suppose the firm hires the applicant even if test-taking is declined. In equilibrium, there must exist a type  $t^* < 2$  such that [i] types  $t \geq t^*$  apply, take the test, and are hired at wage  $w_H$  if testing high,  $w_L$  if testing low, and [ii] types  $t < t^*$  apply, decline to be tested, and are hired at wage  $k$ . If  $t^* \leq k$ , the firm is better off not hiring non-test-takers, so an equilibrium requires  $t^* > k$ . However, in this case, all types are hired, all are paid at least  $k$ , and the firm incurs test-administration cost  $(2 - t^*)c > 0$ , so it is strictly better off pooling than separating and hiring non-test-takers.

Therefore, the separating equilibrium specified by equations (17) (18) and (19) still stands. The firm compares the loss of hiring types below  $k + c$  at wage  $k$  (could be negative depending on distribution of types) with the cost of testing types above  $k + c$ , if the former is greater the firm implements a separating equilibrium, otherwise it implements the pooling equilibrium.<sup>11</sup>

Now discard the assumption about the smallest monetary unit  $\delta$ , and the possibility of hiring a non-test-taking applicant. Suppose types are uniformly distributed between 1 and 2, and let  $k(t)$  be a continuous increasing function distributed on  $[1, 2]$ , with  $k(1) > 1$  and  $k(2) < 2$ , so that the firm still might profitably hire type 2, but not type 1. Also, assume the firm is again not allowed to hire an applicant not taking the test. The resulting model offers some interesting cases, illustrated in figures I, II and III below. Note that in those figures  $c$  is set very large, so that offering a flat wage without testing may yield a higher profit; as  $c$  approaches 0 if  $c$  is set smaller, testing becomes more desirable.

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<sup>11</sup>A zero-cost test would make no qualitative difference.

## VII. ONE FIRM, FINITELY MANY POTENTIAL APPLICANTS

This section returns to the two-types world. Instead of one potential applicant, there are  $n$ , each is independently a low type with probability  $p$ ;  $n$  and  $p$  are assumed common knowledge. The firm can only use one worker productively.<sup>12</sup>

Seeking a separating equilibrium, the firm has neither the desire nor the need to test all applicants. Let it adopt the strategy of testing one randomly selected applicant, hiring her if her test result is high, and otherwise hiring a second randomly selected applicant (possibly the same applicant as the first) without conducting even a second test.

This testing strategy can support a separating equilibrium. If there is no high type in the pool, no one applies and the firm receives no profit. As long as there is at least one high type, there are fee-paying applicants, the firm tests and hires someone. Therefore the firm maximizes expected payoff conditioning on at least one high type in the pool.

Let  $m_n$  be the realization of number of high types in a pool of  $n$ ,  $w_u$  be the wage offered to the applicant getting high test result, and  $w_d$  be the wage offered to the applicant selected through the second random draw.<sup>13</sup> The firm maximizes the payoff:

$$E[2 - c + m_n f - q w_u - (1 - q) w_d \mid n, m \geq 1]. \quad (20)$$

Which can be simplified to (ignoring the constant  $2-c$ ):

$$E[m_n \mid n, m \geq 1] f - q w_u - (1 - q) w_d. \quad (21)$$

A high type, competing with  $n - 1$  rival potential applicants who are each a low type with probability  $p$ , though indifferent, will apply if facing a wage/fee schedule

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<sup>12</sup>Note that number of workers being finite is important because with an infinite number of applicants, no matter how the firm sets up the hiring scheme, all applicants face a 0 chance of being hired, therefore no separation can occur.

<sup>13</sup>When  $n = 1$ , this model reduces to the model in Mod 1, with  $w_u$  and  $w_d$  playing the role of  $w_H$  and  $w_L$ , respectively.

satisfying:

$$E\left[\frac{w_d}{m_{n-1}+1} + \frac{q(w_u - w_d)}{m_{n-1}+1} - f \mid n\right] = k, \quad (22)$$

or

$$f = E\left[\frac{1}{m_{n-1}+1} \mid n\right] * (qw_u + (1-q)w_d) - k. \quad (23)$$

Facing the same wage/fee schedule, a low type has the same expected payoff if randomly selected second, but a lower payoff if randomly selected first, as the probability of testing high is less. So this wage/fee schedule accomplishes separation.

Substituting (23) into the firm's expected profit (21) yields:

$$(E[m_n \mid n, m \geq 1] * E\left[\frac{1}{m_{n-1}+1} \mid n\right] - 1) * (qw_u + (1-q)w_d) - kE[m_n \mid n, m \geq 1]. \quad (24)$$

The firm does not separately care about  $w_u$  and  $w_d$ , so long as  $w_d \geq k$  so that the wage offer is accepted, and  $w_u > w_d$  to give high types greater incentive to apply than low types. Only their weighted sum, with test-reliability weights, enters (24).

For all positive integers, the first term in parentheses in (24),  $E[m_n \mid n, m \geq 1] * E\left[\frac{1}{m_{n-1}+1} \mid n\right] - 1$ , equals 0 for any  $p$ .<sup>14</sup> Therefore, similar to the model in Mod 1, the firm can implement a separating equilibrium with any wage/fee schedule that satisfy  $w_u > w_d \geq k$  and (23).

In a pooling equilibrium, the firm simply hires the first of the applicants at wage  $k$  without testing, which is identical to the model in Mod 1. While the solution is more complicated than the solution to the model in Mod 1, unlike the introduction of more firms, introducing more applicants does not seem to qualitatively change the feasibility of using  $f$  and a test to create separating equilibria.

## VIII. DISCUSSION

The models presented yield the following conclusions:

- It is possible to use a transfer to implement a separating equilibrium; in that

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<sup>14</sup>An intuitive argument is provided in the supplemental materials here.

sense, the private cost of signaling need not be a social cost.

- Commitment to a wage by the firm is not necessary to use a transfer as a signaling device.
- Applicant risk aversion alone is normally insufficient to prevent existence of a separating equilibrium. Considerable differences in home productivity across types may increase the likelihood that equilibrium requires pooling.
- Firms' credibility concerns can be avoided by having a centralized third party collect the application fee.

Are results affected if the firm has to spend money advertising jobs in order to attract applicants? Add to base model (or Mod 1), an assumption that the firm needs to incur a fixed cost in order to let the applicant be aware of the opportunity, i.e., to enter the market. However, once paid it becomes a sunk cost, so it should not affect the firm's choice of wage and fee. It can affect the firm's choice of whether to enter the market.

The continuous-types case is difficult to generalize. In our model, having continuous types without varying  $k$  does not affect the separating equilibrium. However, by simply allowing the variation of reservation wages across types, a multitude of possibilities become available. A more thorough study of this case will be worthwhile.

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## APPENDIX.A PROOF OF THE MAIN THEOREM

(For online publication) Step 3 is trivial. For step 2, the left-hand side of (4) is the expected value of applying for  $t = 1, 2$ , while the right-hand side is the cost of applying. It remains to show that in Step 1 the firm prefers the wage/fee schedules defined by equations (2) and (3) to all other schedules. In effect, the firm can decide who gets hired in Step 3 by changing  $w_L$  and  $w_H$ . Given  $w_L$  and  $w_H$ , the firm can decide who applies by changing  $f$ . Let  $s^* = (w_L^*, w_H^*, f^*)$  be an arbitrary schedule satisfying (2) and (3). Facing  $s^*$ , a high type by assumption applies though indifferent, while a low type's expected payoff is  $qw_L^* + (1 - q)w_H^* - f^* < k$ , preventing applying. An applicant under  $s^*$  is thus a high type. With the wage determined by the test result, the firm's expected profit is

$$(f^* + 2 - c)(1 - p) - w_H^*q(1 - p) - w_L^*(1 - q)(1 - p) = (1 - p)(2 - k - c). \quad (25)$$

With probability  $1 - p$ , the potential applicant is a high type, who applies, pays the fee  $f^*$ , costs the firm  $c$  to be tested, is hired, has productivity 2, and is, in expectation, paid  $w_L^*(1 - q) + w_H^*q = f^* + k$  [from (3)], attaining the strictly positive [from (1)] right-hand side of (25).

It is trivial to dismiss as suboptimal any schedule  $s$  that [a] leads to only low types applying, [b] leads to neither type applying, or [c] leads to hiring only those who test low. Nontrivial alternatives fall into the following three cases.

### ***Case 1: Only high types apply, only high-result applicant is hired***

All such possibilities can be dealt with as if  $w_H \geq k > w_L$ . Then, adjusting (3), the highest fee acceptable for a high type to apply becomes

$$f^{**} = q(w_H - k) > (1 - q)(w_H - k). \quad (26)$$

The equality yields high-types applying though indifferent, the inequality low types not applying. Compared to  $s^*$ , the firm's profit has fallen by  $(1 - p)(1 - q)(2 - k)$ ,

as high types who tested low were profitably hired in  $s^*$ . Reducing the fee to  $f < f^{**}$  at best allows increasing  $w_H$  by  $\frac{(f^{**}-f)}{q}$ , which cannot yield an increase in expected profit, so offering no advantage. Same can be argued for increasing the fee to  $f > f^{**}$ .

### ***Case 2: All types apply, all are hired***

An applicant is always tested (as required in the base model). Initially assuming  $w_H \geq w_L \geq k$ , sets the highest acceptable fee to  $f^{**} = (1 - q)(w_H - k) + q(w_L - k)$ . With both types hired, expected productivity is  $2 - p$ , so expected profit is at most  $2 - p - k - (1 - p)(w_H - w_L)(2q - 1) - c \leq 2 - k - p - c$   
 $= (1 - p)(2 - k - c) + p(2 - k - c) - p < (1 - p)(2 - k - c)$ ,  
 which is expected profits for  $s^*$ , as  $k > 1, c \geq 0$ . Next, reverse the initial assumption:  $w_L \geq w_H \geq k$ , the analysis corresponds:

$$2 - p - k - p(w_L - w_H)(2q - 1) - c \leq 2 - k - c - p$$

$$= (1 - p)(2 - k - c) + p(2 - k - c) - p < (1 - p)(2 - k - c),$$

Again yielding lower expected profit than  $s^*$ .

### ***Case 3: All types apply, only a high-result applicant is hired***

Hiring only an applicant who tests high, as in case 1, it suffices to consider  $w = w_H \geq k > w_L$ . However, to get the low types to apply, the highest fee becomes  $f^{**} = (1 - q)(w_H - k)$ , which has the low type apply though indifferent (and the high type strictly prefer applying). As no type offered wage  $w_L$  accepts, expected profit at  $f^{**}$  is

$$f^{**} - c + [(1 - p)q(2 - w_H)] + \{p(1 - q)(1 - w_H)\}$$

$$= (1 - q)(w_H - k) - c + [(1 - p)q(2 - w_H)] + \{p(1 - q)(1 - w_H)\} \quad (27)$$

$$= (1 - 2q)(1 - p)w_H - c + 2(1 - p)q + p(1 - q) - (1 - q)k.$$

where the term in  $[\ ]$  is productivity less wage for a high type who tests high, that in  $\{ \}$  is the same difference for a low type who tests high, the first equality substitutes

for  $f^{**}$ , the second collects terms in  $w_H$ . As  $q > \frac{1}{2}$ , the coefficient of  $w_H$  is negative, so expected profit is maximized at  $w_H = k$ , which sets  $f^{**} = 0$ . Substituting these values of  $w_H$  and  $f^{**}$  into the left-hand side of (27) yields

$$[(1-p)q(2-k)] + \{p(1-q)(1-k)\} - c \leq [(1-p)q(2-k)] - c < (1-p)(2-k) - (1-p)c = (1-p)(2-c-k),$$

where dropping the nonpositive term in provides the weak inequality, and substituting the larger 1 for  $q$  and the smaller  $(1-p)$  for 1 provides the strict inequality, again yielding lower expected profit than  $s^*$ .

Thus, an arbitrary wage/fee schedule  $s^*$  satisfying (2) and (3) attains a positive expected profit that exceeds all alternative schedules. Q. E. D.

## APPENDIX B. SEPARATING EQUILIBRIA FOR MOD 2 AND MOD 3

(For online publication) Here  $w_M$  is used to denote the wage for someone who signs up for the test but not receiving a test,  $w_N$  is for someone not signing up for the test. For II.B., to achieve separation, the firm makes the high type indifferent over paying the application fee:

$$\begin{aligned} -f + mq(w_H) + m(1-q)(w_L) + (1-m)w_M &= k \\ &> -f + m(1-q)(w_H) + mq(w_L) + (1-m)w_M. \end{aligned} \tag{28}$$

The inequality ensures that the low type does not pay the fee, and is achieved as long as  $w_H > w_L$ . Conditioning on separation, the firm wants to hire the fee-payer for sure, so  $w_L$ ,  $w_H$  and  $w_M$  are all no less than  $k$ . Even if the firm is allowed to hire a non-fee payer, it does not wish to do so, which yields  $w_N < k$ . The fee achieves separation. Then firm's profit is

$$(1-p)(2+f-mq(w_H)-m(1-q)(w_L)-(1-m)w_M) = (1-p)(2-mc-k), \tag{29}$$

with equality due to the expected wage being  $k+f$  [from (28)]. As  $m$  goes to 0

this approaches the full-information optimum, so pooling equilibrium can never be more profitable even allowing  $w_N$  as in Mod 1.

For II.C., the separating condition becomes:

$$\begin{aligned} -fm + mq(w_H) + m(1-q)(w_L) + (1-m)(w_M) &= k \\ &> -fm + m(1-q)(w_H) + mq(w_L) + (1-m)w_M, \end{aligned} \quad (30)$$

and the firm's profit becomes:

$$(1-p)[2 + fm - mc - (k + fm)] = (1-p)(2 - mc - k). \quad (31)$$

This, again, approaches the full-information optimum.

## APPENDIX C. OTHER EQUILIBRIA WITH A THIRD PARTY

(For online publication) First, there is no equilibrium in which only the high type enters, but is hired only with a high test result. Were that the situation, the head-hunter would get no revenue with a fee higher than  $q(2-k-c)$ . Competition forces the firms to zero profit, but a firm deviating to hire a low-test-result applicant at wage  $k$  attains a positive profit.

Suppose a situation in which both types enter with positive probability. Initially suppose firms 1 and 2 set wages  $(w_{L1}, w_{H1})$  and  $(w_{L2}, w_{H2})$ , with (by labeling choice)  $w_{H1} > w_{H2}$ . For firm 2 to attract the high type requires  $w_{L2} > \frac{q(w_{H1}-w_{H2})}{(1-q)} + w_{L1}$ . For firm 2 to avoid hiring the low type requires  $w_{L2} < \frac{(1-q)(w_{H1}-w_{H2})}{q} + w_{L1}$ .<sup>15</sup> Recalling that  $q > \frac{1}{2}$  is the probability the test correctly reveals type, there is no value of  $w_{L2}$  at which firm 2 is best responding to firm 1.

If both types enter, and all firms except firm 1 offer the same wage schedule  $(w_L, w_H)$  with  $w_L > k$ , then firm 1 gains by deviating to  $w_{L1}$  in  $(k, w_L)$  and  $w_{H1} = w_H +$

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<sup>15</sup>Tiebreaker [ii] above generates the strict inequalities.

$(1-q)\frac{(w_L-w_L)}{q}$  (by tiebreaker [ii], firm 1 attracts the high but not the low type).

This leaves two types of candidate equilibria as follows. First, where both types enter and low scores are hired: [a]: each firm offers the same  $(w_L, w_H)$  with  $w_H > w_L = k$  (so that both types accept an offer), [b]:  $f = (1-q)w_H + qk - k = (1-q)(w_H - k)$  (so that the low type applies), and [c]:  $2(1-p) + p - [q(1-p) - p(1-q)]w_H - [pq + (1-p)(1-q)]k - c = 0$  (which sets  $w_H$  to compete away firm profits). In the second type, both types enter and low scores are not hired: [a']: each firm offers the same  $(w_L, w_H)$  with  $w_H \geq k > w_L$ , [b']:  $f = (1-q)w_H - k$ , [c']:  $2q(1-p) + p(1-q) - [p(1-q) + q(1-p)]w_H - c = 0$  (for the same reasons).

Solving [c] for  $w_H$ :  $w_H = \frac{2-p-c-[pq+(1-p)(1-q)]k}{p(1-q)+(1-p)q}$ . Substituting into [b]:

$$\begin{aligned}
 f &= (1-q)\left(\frac{2-c-p-[pq+(1-p)(1-q)]k}{q(1-p)+p(1-q)} - k\right) \\
 \implies [q(1-p)+p(1-q)]f &= (1-q)\left(\{2-c-p-[pq+(1-p)(1-q)]k\} - k[q(1-p)+p(1-q)]\right) \\
 &= (1-q)\{(2-c-p)-k[pq+(1-p)(1-q)+q(1-p)+p(1-q)]\} \\
 &= (1-q)[2-c-p-k(pq+1+pq-p-q+q-pq+p-pq)] \\
 &= (1-q)(2-c-p-k)
 \end{aligned} \tag{32}$$

So the maximum application fee if low types apply and low scores are hired:  $f = \frac{(1-q)(2-c-p-k)}{q(1-p)+p(1-q)}$ .

As mentioned in text, surplus is reduced. To see this, notice that the  $(2-c-p-k)$  term is multiplied by a coefficient less than 1 [subtract the  $(1-q)$  term in the numerator from the denominator:  $q(1-p)+p(1-q)-(1-q) = q(1-p)-(1-p)(1-q) = (2q-1)(1-p) > 0$ , so the ratio  $< 1$ ]. Then, conditioning on  $2-c-p-k > 0$  yields:

$$\frac{(1-q)(2-c-p-k)}{q(1-p)+p(1-q)} < 2-c-p-k < 2-c-k - (2-c-k)p = (1-p)(2-c-k), \tag{33}$$

which is the headhunter's surplus in a separating equilibrium.

Solving [c'] for  $w_H$ :  $w_H = \frac{2q(1-p)+p(1-q)-c}{p(1-q)+q(1-p)}$  Substituting into [b']:

$$f = (1-q) \frac{2q(1-p) + p(1-q) - c}{p(1-q) + q(1-p)} - k, \quad (34)$$

which is the maximum application fee if low types apply and low scores are not hired. Then the headhunter's surplus in this case:

$$\begin{aligned} (1-q) \frac{2q(1-p) + p(1-q) - c}{p(1-q) + q(1-p)} - k &< 2q(1-p) + p(1-q) - c - k \\ &< 2 - p - c - k \\ &< (1-p)(2 - c - k) \end{aligned} \quad (35)$$

The first inequality is based on the already established  $q(1-p) + p(1-q) > 1 - q$ ; the second inequality is based on  $2 - p > 2q(1-p) + p(1-q)$ .<sup>16</sup> Therefore, the equilibria described in the text are the only equilibria for this model.

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<sup>16</sup> $[2 - p] - [2q(1-p) + p(1-q)] = 2(1-p) + p - 2q(1-p) - p(1-q) = 2(1-p)(1-q) + pq > 0$

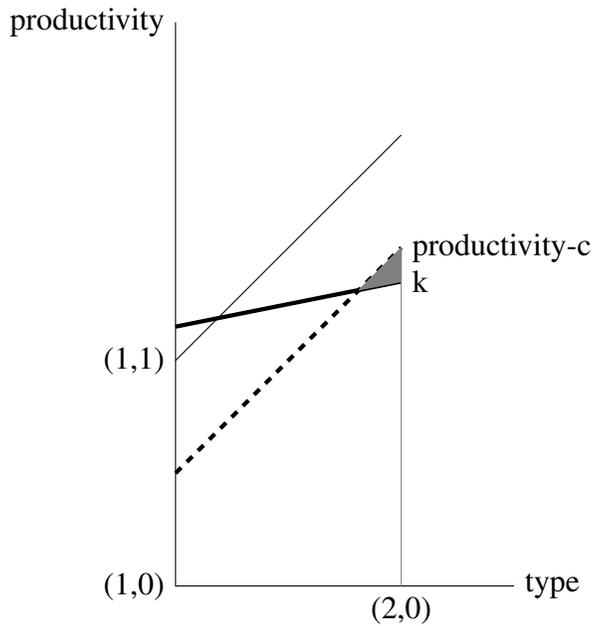


Figure I: A simple case of variable at-home productivity,  $k$ . The thin line,  $y(t) = t$ , represents the gross productivity for each type. The dashed line shows net productivity, productivity reduced by test-administration cost  $c$ , a downward shift (0.5 in the graph). The thick line represents  $k(t)$ . By changing  $s = (w_L, w_H, f)$ , the firm can generate as  $W(t | s)$ , net expected wage, any line with nonnegative slope, obtaining the employ of any types for which  $W$  exceeds  $k$ , profiting by the height (net productivity -  $W$ ). For the case shown, matching  $W(t | s) = k(t)$  is the equilibrium, with the firm obtaining all the surplus (the shaded area).

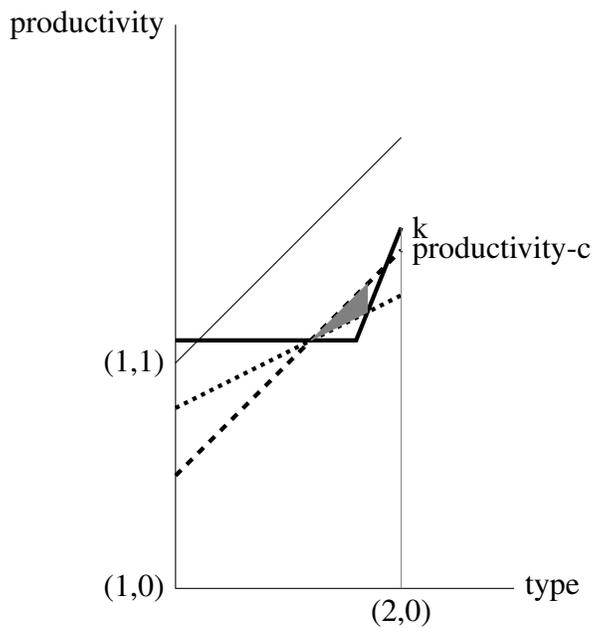


Figure II: Another case of variable  $k$ . this time flat until  $t = 1.8$ , and then steep. A possible (not necessarily optimal)  $W$  is the dotted line in the figure, which yields the shaded area as surplus for the firm, while the triangle below the shaded region is applicant surplus. An interval of intermediate types is hired, while lower types are insufficiently productive, higher types overly expensive.

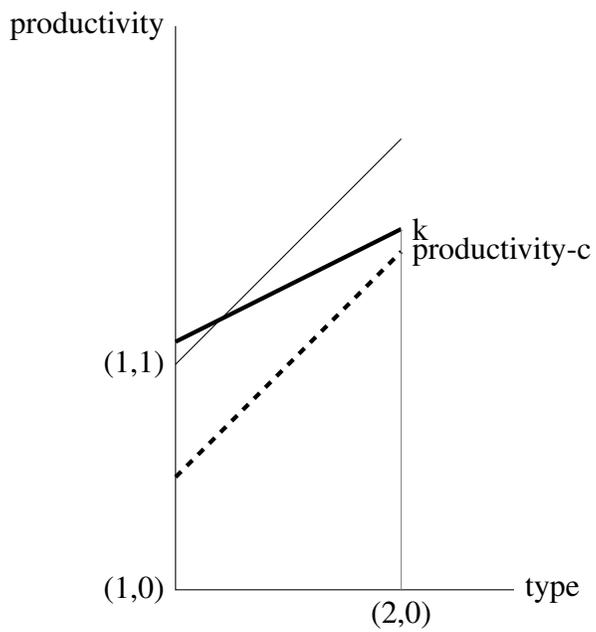


Figure III: In this case  $c$  is so high that testing guarantees a loss, thus neither testing nor hiring occurs. Were hiring without testing possible, the firm would set  $f = 0$ ,  $w_H = w_L = 1.2$ , with the flat wage separating at the intersection of the two solid lines.