

# Price Competition in the Presence of a Web Aggregator\*

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## Abstract

In this paper we examine the impact of a web aggregator on firms and consumers in a horizontally differentiated market. When a firm pays a fee to be listed on the aggregator's website, its location and price become observable to e-users (consumers who visit the website). We consider two settings, depending on the possibility for online firms to offer discounts to e-users. In equilibrium, not all firms will go online – some will choose to remain offline. Online firms attract more customers due to the higher level of information, but face a tougher price competition. When the proportion of e-users is relatively low, price discrimination may hurt the firms. Therefore, less of them can afford to go online. The opposite holds when e-users predominate; price discrimination yields a higher number of online firms than uniform pricing. Finally, we evaluate the aggregator's optimal policy regarding the fee and whether to impose uniform pricing or to allow price discrimination. We discover that, unless the proportion of e-users is relatively low, the aggregator induces only a few firms to go online.

Keywords: online reviews aggregators, price discrimination, competition.

JEL codes: C72, D43, D61, L11, L13, M31.

## 1 Introduction

Recent years have been characterized by the flourishing of online sales. At the beginning, merchants created and used their own web pages both to increase their customer base and to facilitate sales to existing customers. Then, we observed the proliferation of e-commerce companies, web platforms, comparison shopping sites and online auctions through which customers and merchants were able to buy and sell a broad variety of products and services worldwide (*e.g.*,

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eBay, Amazon, Expedia, PriceGrabber). The next step was to gather consumers. Group buying became a common practice thanks to the extraordinary expansion of Groupon, followed by several other similar websites. Each of these online selling methods has been analyzed in the economics and management literature, in order to understand the reasons of their success or failure, and their impact on market competition and welfare.<sup>1</sup>

However, a key player in this market has been partially neglected, at least by the mainstream theoretical literature: online reviews aggregators and reservation services, (web) aggregators for short.<sup>2</sup> They started to operate in the second part of 2000s and enjoyed a rapid expansion ever since.<sup>3</sup> We can distinguish aggregators that are open, such as Booking.com and OpenTable, from those that are closed, such as Relais & Chateaux and Gourmet Society. The latter are exclusive clubs which select their members based on income, education, and social status.<sup>4</sup> Aggregators also differ in the quality/amount of information they provide to customers. The source of information may be mainly left to consumer reviews (TripAdvisor, OpenTable, Zomato), or it can be a combination of both the aggregator itself and peer reviews (Booking.com, TheFork, RatesToGo, HotelClub). In addition to information, many aggregators offer the possibility to book/purchase when the customer finds the desired product (Booking.com, RatesToGo, HotelClub, OpenTable, Gourmet Society, TheFork, Relais & Chateaux). This service is particularly convenient in the lodging and catering sectors, for it allows to reduce consumer search costs by restricting the database to firms that are available (have a free room or a table) on the required day. Often, consumers who book online enjoy a discount, which could be either imposed by the aggregator or decided by the producer. Discounts are more frequent in closed aggregators. All restaurants that appear on Gourmet Society's website, for example, have agreed to offer a discount (25% off the total bill). On the contrary, restaurants listed on TheFork are free to decide their discounting policy (about 20%-30% offer a lower price to their online clients). Several aggregators, however, prohibit price discrimination. Diners using OpenTable and Zomato, for example, pay the same price as those who do not reserve through the aggregator.

The rise of online reviews aggregators is strictly related to the persistence of asymmetric information in markets characterized by experience goods. Starting from the seminal contribution developed by Akerlof (1970), such asymmetries have been widely studied especially for the case of vertically differentiated products. Insurances and warranties are commonly accepted solutions in the market for lemons. However, the same toolkit cannot be easily implemented in the case of horizontally differentiated products, as the perception of quality is subjective.

Our paper delivers a theoretical model to investigate the impact of web aggregators in horizontally differentiated markets. Through the paper we focus on aggregators that provide in-

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<sup>1</sup>Seminal contributions by Alba *et al.* (1997) on consumer incentives to shift to new distribution channels and Bakos (1997) on electronic marketplaces that reduce search and increase competition among retailers have been followed by a proliferation of papers touching different aspects of online commerce. The literature on group buying, although pioneered by Che and Gale (1997), has only recently been developed, following the success of Groupon.

<sup>2</sup>In the next section we provide a brief description of the most relevant empirical papers that focus on the role of online reviews aggregators.

<sup>3</sup>Online review aggregators differ from marketplaces such as Amazon as they do not ship goods to consumers. Rather, they facilitate the match between consumers and producers by providing information about a product's quality and price, and by allowing for online booking.

<sup>4</sup>Membership requires relatively expensive annual fees and it is usually awarded through a combination of different channels, such as private invitation from other members and interviews.

formation and booking services in the catering sector, and we illustrate our model as a stylized version inspired by OpenTable and TheFork.<sup>5</sup> They both provide detailed information about different restaurants, including location, type of cuisine, menu, prices, and dining atmosphere. Together with consumer reviews, this information helps the customer to narrow down her search to a few options that best match her tastes. As pointed out earlier, TheFork allows restaurants to offer online discounts, while OpenTable imposes uniform pricing. This leads us to the following questions. When should an aggregator allow price discrimination? How does the choice of a pricing policy affect social welfare?

We adopt Salop (1979) circular city model to analyze the equilibrium behavior of consumers, restaurants and a web aggregator. We assume that initially consumers do not know the locations of the restaurants in the product space. The restaurants decide whether or not to be listed on the aggregator's website and set their prices. When a restaurant goes online, its location and price become observable to the consumers that visit the website. Consumers who book a table through the aggregator are called *e-users*, while those who choose a restaurant in a more traditional way (yellow pages or randomly walking down the street) are called *walkers*. The proportion of e-users in the population is exogenously fixed. Finally, we assume that the aggregator charges restaurants a fixed fee.<sup>6</sup>

We consider two settings. In the first one the aggregator imposes uniform pricing, while in the second it allows restaurants to price discriminate between walkers and e-users. In both settings, we compute the equilibrium prices, profits, and the number of online restaurants. We show that in equilibrium not all restaurants will go online – some will remain offline. Online restaurants attract more customers than their offline counterparts due to reduced mismatch costs, but they face a tougher price competition. We find that in the first setting (uniform pricing), online restaurants charge a lower price than offline restaurants when the proportion of e-users is sufficiently high. In the second setting (price discrimination), online restaurants set the same price for walkers as offline restaurants, while offering a discount to e-users.

Although simple, our model allows us to derive interesting conclusions both from the private and the social standpoints. In particular, the analysis that we carry out brings forth three interesting results. First, for a given number of online restaurants, we find that allowing discounts may hurt the restaurants. Specifically, when the proportion of e-users in the population is relatively low, online restaurants are trapped in the prisoner's dilemma. Each restaurant has an incentive to offer a discount to capture more e-users; however, as they all offer a discount, the final price may be too low and the restaurants end up losing in comparison to uniform pricing.

Second, we calculate how many restaurants will go online in each of the two settings and compare these numbers with what is best for society. (In our theoretical framework, the social optimum is achieved when all restaurants go online.) We show that uniform pricing induces more restaurants to go online when walkers predominate. When e-users predominate, price discrimination yields a higher number of online restaurants. The socially optimal number of online restaurants can always be reached under price discrimination, provided that the fixed

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<sup>5</sup>TheFork operates in the restaurant market. It is currently available in France, Monaco, Spain, Belgium, Italy, and Switzerland under the names LaFourchette, ElTenedor and TheFork.

<sup>6</sup>In reality, some aggregators charge a fixed per-year fee (Zomato), some a per-client/reservation fee (TheFork), and others a setup fee combined with a monthly fee and/or per-reservation fee (OpenTable). TripAdvisor charges businesses for clickable ads and business listings.

online fee is sufficiently low. In the first setting, that of uniform pricing, the first best outcome can only be achieved if the proportion of e-users is relatively low.

Third, we evaluate the aggregator’s optimal policy regarding the fee to be charged and whether to impose uniform pricing or to allow discounts for e-users. We find that when the proportion of e-users is relatively high, the aggregator optimally allows discounts and charges a rather high fee, inducing only a few restaurants to go online. A similar situation occurs when the proportion of e-users is intermediate, although the aggregator imposes uniform pricing. Also in this case, the profit-maximizing fee results in less than the socially optimal number of online restaurants. Only when the proportion of e-users is relatively low, the aggregator’s optimal policy – uniform pricing and a low fee – yields the socially optimal outcome. Our analysis therefore reveals that possible negative effects for society are not only caused by the aggregator’s choice of the pricing policy, but also (and mainly) by the amount of the fee it charges the restaurants. Hence, the possible intervention by the policy maker to restore efficiency should carefully consider the interplay between the fee and the pricing policy adopted by the aggregator.

The paper is structured as follows. In the next section we review the literature. The model is presented in Section 3. The two settings mentioned above are analyzed in Sections 4 and 5, respectively. In Section 6 we report the choice of the aggregator. Concluding remarks appear in Section 7.

## 2 Related Literature

Our paper contributes to explaining the private and social impacts of online selling when merchants resort to intermediaries in order to disclose certain product features to final consumers. The economic role played by intermediaries in general has been widely studied in the literature (see Gehrig, 1993; Spulber, 1999; Watanabe, 2010, *inter alia*).<sup>7</sup> However, the specific role of online reviews aggregators has so far received attention mainly in empirical studies. Ghose *et al.* (2012) show that consumers base their purchases on the available reviews. Anderson and Magruder (2012) suggest a positive relation between reviews and profits, thus confirming previous results on the effect of word-of-mouth (Chevalier and Mayzlin, 2006) and reviews by professional critics (Reinstein and Snyder, 2005; Hilger *et al.*, 2011) on sales. Clearly, the effectiveness of online reviews depends on their credibility.<sup>8</sup> In our model we abstract from the risk of fake reviews and assume that consumers trust the aggregator.<sup>9</sup>

The literature on two-sided markets – markets in which two distinct user groups interact via a platform – offers another potential research field in which our paper can be positioned. Indeed, in our model firms (one side of the market) can join the aggregator (a platform) to

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<sup>7</sup>In particular, see Spulber (1999) and the references therein for intermediaries that “make the market” by setting input and output prices to maximize their profits.

<sup>8</sup>To increase their credibility, aggregators can allow only certified consumers to write a review. Brynjolfsson and Smith (2000) argue that branding awareness and trust are important sources of heterogeneity among Internet retailers. However, a 2012 Nielsen poll of more than 28,000 Internet consumers from 56 countries revealed that online consumer reviews are the second most trusted source of brand information after suggestions from friends and family. See <http://www.nielsen.com/us/en/press-room/2012/nielsen-global-consumers-trust-in-earned-advertising-grows.html>

<sup>9</sup>Dellarocas (2006), Mayzlin (2006) and Mayzlin *et al.* (2013), among others, study the phenomenon of manipulated reviews.

become “visible” to those consumers (the other side of the market) who visit the aggregator’s website. Note that in contrast to the established models of two-sided markets introduced by Caillaud and Jullien (2003), Rochet and Tirole (2003), and Armstrong (2006), in our model the size of only one side of the market is endogenously determined, as we assume that the fraction of e-users in the population is fixed. Moreover, differently from these and some other more recent contributions, we do not consider competition between platforms.<sup>10</sup> Our aim is to focus on the private and social consequences of the decision taken by the aggregator regarding the fee and whether to impose uniform pricing or to allow discounts for e-users.

The literature on platform pricing overlaps with the literature on two-sided markets, as intermediaries often own a platform and can charge access or usage fees. Following the seminal work by Baye and Morgan (2001) on information gatekeepers in homogeneous product markets, recent contributions investigated platform pricing for differentiated products. In Galeotti and Moraga-González (2009) horizontally differentiated firms decide whether to join a monopoly platform. Participating consumers learn the prices and characteristics of the products of the firms that joined the platform. Differently from our model, the platform charges both firms and consumers. The authors show that in equilibrium all firms and consumers participate in the platform, which does not introduce any additional distortions. In our paper we show that the pricing policy adopted by the platform/aggregator generates market inefficiencies. De Cornière (2016) presents a model of search engine advertising. Like in our model, firms are horizontally differentiated *à la* Salop (1979) and consumers do not know their positions on the circle. The search engine plays the role of a matchmaker: firms select the set of keywords they want to target and consumers enter keywords and then search sequentially through the sponsored links that appear. Firms incur a fixed cost to be registered on the search engine and then pay on a per-click basis. One of the main findings is that in equilibrium the engine imposes a per-click fee that is too high, negating the benefits of targeted advertising.

We also extend the literature that examines the impact of online sales on offline prices. Empirical papers analyzing this issue are, among others, Brown and Goolsbee (2002) on individual insurance policies, and Morton *et al.* (2001) on pricing behavior of car dealerships. On the theoretical side, Lal and Sarvary (1999) and Zettelmeyer (2000) both consider firms competing through alternative distribution channels such as online shopping. They find that online shopping does not necessarily increase price competition. Similar results can be found in Logiнова (2009), who provides a theoretical framework to study competition between online and offline retailers. She shows that there exist circumstances under which conventional stores may increase their prices when online rivals enter the market. One common feature of such theoretical models is that online shopping has pros and cons: on the one hand, it reduces travelling costs; on the other hand, consumers prefer to inspect a specific item before purchasing it. In our model, on the contrary, going online reduces mismatch costs for consumers without causing any discontent.

A stream of literature in economics and marketing has examined information disclosure in monopolistic and competitive settings. The seminal work of Lewis and Sappington (1994) considers a firm’s incentive to reveal its private information to two different types of consumers

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<sup>10</sup>Nocke *et al.* (2007) compared for-profit to open platforms that do not charge access fees, and Belleflamme and Peitz (2010) investigated the seller’s incentive to invest in different activities (cost reduction, consumer targeting, and price discrimination) in the presence of for-profit platforms.

in order to facilitate price discrimination. Anand and Shachar (2011) examine television networks which advertise their own show. This results in a trade-off as it increases the demand for some viewers, but reduces that for others. In our paper price discrimination between e-users and walkers may lead to a similar trade-off, although we focus mainly on price effects as demands remain unchanged. Recent papers by Sun (2010) and Gu and Xie (2013) study the incentives for producers to reveal multiple (vertical and horizontal) product attributes.

Our paper is also related to the practice of selective discounting, which provides opportunities for price discrimination.<sup>11</sup> The conventional result is that price discrimination adopted by producers can be welfare improving, as it allows to serve weaker markets which would otherwise be cut out (see Schmalensee, 1981; Varian, 1985; Stigler, 1987, *inter alia*). Regarding the specific case of third-degree price discrimination, it may decrease welfare when the demand function is linear (Aguirre *et al.*, 2010; Aguirre, 2012). In our paper we consider a scenario in which the aggregator allows restaurants to price discriminate between e-users and walkers. Although the total quantity is constant regardless of the presence of the aggregator, the society at large may benefit if the number of firms going online increases when price discrimination is allowed.

Finally, our paper adds to the current discussion about potential anticompetitive effects of the Most Favored Nation clauses (MFNs) adopted by hotel booking portals such as Booking.com and Expedia.<sup>12</sup> Recent contributions by Johnson (2017) and Edelman and Wright (2015) analyze the effect of similar pricing policies. The first paper focuses on retail price-parity restrictions, according to which the retail price fixed by one supplier through one retailer has to be less or equal to the retail price fixed by that supplier through a competing retailer. The second paper provides a very exhaustive account of different markets characterized by “price coherence,” where the purchase of a given good via an intermediary has to occur at the same price as a purchase of the same good directly from the retailer. The authors also report case studies in which antitrust authorities intervened to stop intermediaries from imposing price coherence. The third paper considers the anticompetitive effect of price-parity restrictions imposed by platforms on participating sellers, but it also shows that platforms resort to such practices to prevent a free-riding problem. In fact, consumers may search on the platform and then switch to direct selling to obtain a discount. Our paper differs from these contributions as we specifically focus on online discounting, while MFNs restrict hotels from offering better deals through other booking portals, on their own websites, or to walk-in customers.

### 3 The Model

The players in our model are uninformed consumers, restaurants and a web aggregator. (Think of tourists visiting Paris and looking for a place to have a nice dinner.) Consumers differ in their preferences for cuisine and dining atmosphere. For example, some customers prefer a more authentic food experience, while others enjoy more mainstream dining. For some customers, presentation is important, whereas for others it is the size of a meal that matters. Depending on the occasion, customers may prefer a more intimate ambience, or a more vibrant one where

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<sup>11</sup>This has been studied in seminal contributions by Varian (1980) and Narasimhan (1984, 1988).

<sup>12</sup>Often, travel professionals refer to this practice as the rate parity policy.

patrons can engage in dynamic conversation. Families may want to avoid student hangouts, and vice versa. We use Salop circular city to model consumer heterogeneity. Consumers of total mass one are uniformly distributed on a circle of circumference one.  $N$  restaurants are located equidistantly around the circle; each produces a meal at the constant marginal cost  $c$ . When a consumer dines in a restaurant located at distance  $x$  from her, her utility from consuming a meal (her valuation) is

$$v - tx,$$

where  $t$  represents the intensity of consumer tastes.

We assume that consumers do not know the locations of the restaurants on the Salop circle nor their prices. Only when a consumer enters a restaurant, she observes its price, and only after the consumer has a meal there, she learns its location on the circle. Since the expected distance between the consumer and a randomly chosen restaurant is  $1/4$ , the consumer's expected utility equals  $v - t/4$ .<sup>13</sup> We also assume that as long as the restaurant's price does not exceed the consumer's expected utility, the consumer dines at that restaurant. Thus, in equilibrium each restaurant will set its price equal to

$$v - \frac{t}{4},$$

leaving all consumers with zero expected payoffs.<sup>14</sup>

We now introduce the online reviews aggregator, such as OpenTable, Zomato, or TheFork. Suppose – just for a moment – that all restaurants appear on the aggregator's website. This means that a consumer can visit the aggregator's website and learn the locations and prices of the  $N$  restaurants. If all consumers visit the website, then we have Salop's circular city model (Salop, 1979), and, therefore, in equilibrium each restaurant will set its price equal to

$$c + \frac{t}{N}.$$

To keep the equilibrium analysis as simple as possible, we assume that  $v$  is sufficiently high,

$$v - c > \frac{3t}{4}. \tag{1}$$

This assumption guarantees that the consumer located halfway between two adjacent restaurants (having to travel  $1/(2N)$  along the circle) receives a strictly positive payoff:

$$v - \frac{t}{2N} > c + \frac{t}{N}$$

holds for any  $N \geq 2$ .

To make the model more realistic, we assume that only some consumers visit the aggregator's website. Let  $\alpha \in [0, 1]$  denote the fraction of consumers who find it worth their time and effort to check out the restaurants online. We will refer to these consumers as *e-users*, and to the rest (who choose a restaurant by simply walking down the street) as *walkers*.

<sup>13</sup>Indeed, the distance is a random draw from  $[0, 1/2]$ . With consumers uniformly distributed on the circle, the expected distance is  $1/4$ .

<sup>14</sup>Alternatively, we can assume that there is a cost  $s$  to search another restaurant. This leads to the well-known Dimond (1971) result. As long as the search cost is strictly positive, the restaurants in equilibrium will charge the monopoly price. That is,  $v - \frac{t}{4}$ .

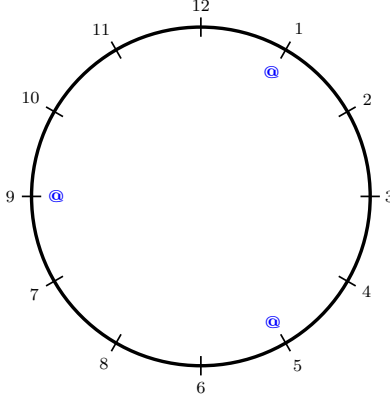


Figure 1: Online and Offline Restaurants on the Salop Circle

Next, we assume that in order to appear on the aggregator's website, a restaurant should pay a fixed fee  $k > 0$  to the aggregator. Let  $n$  denote the number of restaurants that go online (endogenously determined). Again, we assume that these  $n$  restaurants are located equidistantly around the circle. In Figure 1,  $N = 12$  and  $n = 3$ . Restaurants 1, 5 and 9 are on the aggregator's website, while the other restaurants are not. We will call the restaurants on the aggregator's website *online* restaurants and the rest – *offline* restaurants.

In the next section we study a setting in which online restaurants charge the same price to all their customers (walkers and e-users). In Section 5 we allow online restaurants to offer discounts to those consumers who book a table through the aggregator (*i.e.*, to e-users).

#### 4 Setting I: Online Restaurants Charge Uniform Prices

In this setting we assume that online restaurants do not price discriminate and charge the same price to walkers and e-users. Let  $p_{\text{off}}^*$  be the price that offline restaurants charge in a symmetric Nash equilibrium, and let  $p_{\text{on}}^*$  be the price of online restaurants. Since we have assumed that a walker dines at the first restaurant she walks in (as long as the price does not exceed his expected utility),

$$p_{\text{off}}^* = v - \frac{t}{4}.$$

Each of  $N - n$  offline restaurants will attract  $(1 - \alpha)/N$  customers ( $1/N$  of all walkers) and earn

$$\pi_{\text{off}}^* = \frac{1 - \alpha}{N} (p_{\text{off}}^* - c) = \frac{1 - \alpha}{N} \left( v - \frac{t}{4} - c \right).$$

Now let us find  $p_{\text{on}}^*$  and  $\pi_{\text{on}}^*$ . Suppose that there are at least two restaurants on the aggregator's website,  $n \geq 2$ , and  $p_{\text{on}}^* \leq p_{\text{off}}^* = v - t/4$  (see the proof of Lemma 1 in the Appendix). Then in equilibrium each online restaurant will attract  $(1 - \alpha)/N + \alpha/n$  customers ( $1/N$  of all walkers and  $1/n$  of all e-users).



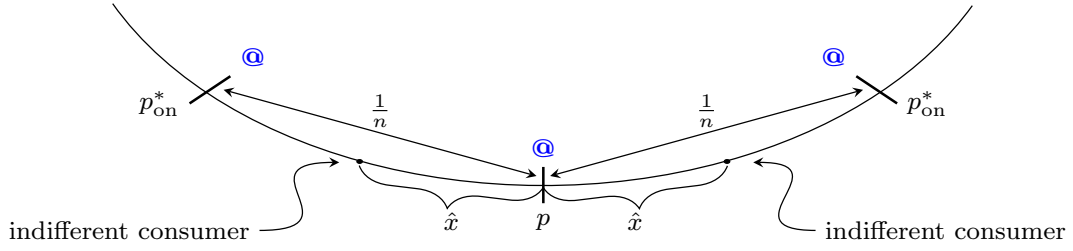


Figure 2: Determining the Equilibrium Price

Consider a deviation by an online restaurant from  $p_{\text{on}}^*$  to  $p$ . The restaurant will attract  $(1 - \alpha)/N + 2\alpha\hat{x}$  customers, where  $\hat{x}$  is determined by

$$v - p - \hat{x}t = v - p_{\text{on}}^* - \left(\frac{1}{n} - \hat{x}\right)t,$$

$$\hat{x} = \frac{1}{2n} + \frac{p_{\text{on}}^* - p}{2t}.$$

(See Figure 2.) Hence, the profit of this restaurant will be

$$\left(\frac{1 - \alpha}{N} + 2\alpha\left(\frac{1}{2n} + \frac{p_{\text{on}}^* - p}{2t}\right)\right)(p - c).$$

The equilibrium requires

$$p_{\text{on}}^* = \arg \max_{p \leq v - \frac{t}{4}} \left(\frac{1 - \alpha}{N} + \alpha\left(\frac{1}{n} + \frac{p_{\text{on}}^* - p}{t}\right)\right)(p - c). \quad (2)$$

Differentiating the objective function with respect to  $p$  and setting the derivative to zero leads to

$$-\frac{\alpha}{t}(p - c) + \frac{1 - \alpha}{N} + \alpha\left(\frac{1}{n} + \frac{p_{\text{on}}^* - p}{t}\right) = 0.$$

Substituting  $p = p_{\text{on}}^*$  into the above equation and solving for  $p_{\text{on}}^*$  yield

$$p_{\text{on}}^* = \min \left\{ c + \frac{(1 - \alpha)t}{\alpha N} + \frac{t}{n}, v - \frac{t}{4} \right\}. \quad (3)$$

In equilibrium, each online restaurant earns

$$\pi_{\text{on}}^* = \left(\frac{1 - \alpha}{N} + \frac{\alpha}{n}\right)(p_{\text{on}}^* - c). \quad (4)$$

Thus, we have the following lemma.

**Lemma 1** (Prices and Profits in Setting I, for a Given Value of  $n$ ). *Consider Setting I and suppose there are  $n$  online restaurants. Then in equilibrium each offline restaurant will charge*

$$p_{\text{off}}^* = v - \frac{t}{4}$$

and earn

$$\pi_{\text{off}}^* = \frac{1 - \alpha}{N} \left( v - \frac{t}{4} - c \right).$$

Online restaurants will set their prices equal to

$$p_{\text{on}}^*(n) = \begin{cases} c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n} & \text{if } \alpha \geq \alpha_0(n) \\ v - \frac{t}{4} & \text{if } \alpha < \alpha_0(n), \end{cases}$$

earning

$$\pi_{\text{on}}^*(n) = \begin{cases} \left( \frac{1-\alpha}{N} + \frac{\alpha}{n} \right)^2 \frac{t}{\alpha} & \text{if } \alpha \geq \alpha_0(n) \\ \left( \frac{1-\alpha}{N} + \frac{\alpha}{n} \right) \left( v - \frac{t}{4} - c \right) & \text{if } \alpha < \alpha_0(n), \end{cases}$$

where

$$\alpha_0(n) = \frac{t}{t + N \left( v - \frac{t}{4} - \frac{t}{n} - c \right)}.$$

The results of this lemma are intuitive. When more consumers visit the aggregator's website,  $\alpha \geq \alpha_0(n)$ , the e-users become the primary target for online restaurants. This leads the restaurants to charge

$$p_{\text{on}}^*(n) = c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n} < v - \frac{t}{4}.$$

Ceteris paribus,  $\alpha$  is more likely to fall into the region  $[\alpha_0(n), 1]$  for larger values of  $n$  ( $\alpha'_0(n) < 0$ ). As  $n$  increases,  $p_{\text{on}}^*(n)$  decreases.

On the contrary, when  $\alpha < \alpha_0(n)$ , the walkers become the primary target for online restaurants, and they charge

$$p_{\text{on}}^* = v - \frac{t}{4}.$$

This is more likely to happen when  $n$  is small.

Next, we find how many restaurants will go online in equilibrium. Recall that in order to appear on the aggregator's website, a restaurant should pay  $k$  to the aggregator. Hence, the equilibrium number of online restaurants is determined by

$$\pi_{\text{off}}^* = \pi_{\text{on}}^*(n) - k. \quad (5)$$

The equilibrium profit of an offline restaurant,  $\pi_{\text{off}}^*$ , does not depend on  $n$ , while the equilibrium profit of an online restaurant,  $\pi_{\text{on}}^*(n)$ , decreases in  $n$  and approaches  $\pi_{\text{off}}^*$  as  $n \rightarrow \infty$  (see the proof of Proposition 1 in the Appendix). Thus, (5) has a unique solution as long as  $k$  is not prohibitively large.

**Proposition 1** (Equilibrium Number of Online Restaurants in Setting I). *In Setting I, the equilibrium number of online restaurants is given by*

$$n^* = \begin{cases} N & \text{if } k \leq k_1 \\ \frac{\alpha}{\sqrt{(k + \frac{1-\alpha}{N}(v - \frac{t}{4} - c))^{\frac{\alpha}{t} - \frac{1-\alpha}{N}}}} & \text{if } k \in (k_1, k_2] \\ \frac{\alpha(v - \frac{t}{4} - c)}{k} & \text{if } k > k_2, \end{cases}$$

where

$$k_1 = \frac{\alpha(v - \frac{t}{4} - c)}{N} - \max \left\{ \frac{v - \frac{t}{4} - c}{N} - \frac{t}{N^2\alpha}, 0 \right\}$$

and

$$k_2 = \frac{\alpha(v - \frac{t}{4} - c)}{N} + \max \left\{ \frac{\alpha(v - \frac{t}{4} - c)^2}{t} - \frac{v - \frac{t}{4} - c}{N}, 0 \right\}.$$

Observe that when

$$\alpha \leq \frac{t}{N(v - \frac{t}{4} - c)} = \alpha_1,$$

$$\max \left\{ \frac{\alpha(v - \frac{t}{4} - c)^2}{t} - \frac{v - \frac{t}{4} - c}{N}, 0 \right\} = \max \left\{ \frac{v - \frac{t}{4} - c}{N} - \frac{t}{N^2\alpha}, 0 \right\} = 0.$$

Hence,  $k_1 = k_2$  and the set  $(k_1, k_2]$  is empty for these values of  $\alpha$ . When  $\alpha > \alpha_1$ ,  $k_1 < k_2$ . While  $k_2$  is always positive,  $k_1$  may be negative. If this happens, then  $k \leq k_1$  in the above proposition is an empty set. That is, even when the fixed online fee is zero, not all restaurants will choose to go online.

Recall that our equilibrium analysis goes through when there are at least two online restaurants,  $n \geq 2$ . This assumption imposes an upper limit to the value of  $k$ . It follows from Proposition 1 that  $n^* \geq 2$  if and only if

$$k \leq \frac{\alpha}{2} \left( v - \frac{t}{4} - c \right). \quad (6)$$

Finally, we find the socially optimal number of online restaurants and compare it with  $n^*$ . The socially optimal number, denoted by  $n^{\text{so}}$ , minimizes the mismatch costs incurred by the e-users:

$$\min_n \alpha \frac{t}{4n}.$$

(As  $k$  represents a transfer from an online restaurant to the aggregator, it is not a cost to the society. That is why  $nk$  does not enter the objective function.) Hence,

$$n^{\text{so}} = N.$$

While efficiency requires all restaurants to go online, in equilibrium  $n^* \leq N$ , with strict inequality when  $k > k_1$ . The private and the social interests are, therefore, not aligned when  $k$  is sufficiently high.

## 5 Setting II: Online Restaurants Charge Different Prices to Walkers and E-users

Suppose that online restaurants can offer discounts to those consumers who book a table through the aggregator. It is easy to see that in equilibrium they will charge walkers the price

$$\bar{p}_{\text{on}}^\dagger = v - \frac{t}{4}$$

and e-users

$$\underline{p}_{\text{on}}^\dagger(n) = c + \frac{t}{n}.$$

This implies the discount of

$$v - \frac{t}{4} - \frac{t}{n} - c,$$

which is positive for all  $n \geq 2$  due to the assumption (1). In equilibrium each online restaurant earns the profit

$$\pi_{\text{on}}^\dagger(n) = \frac{1-\alpha}{N}(\bar{p}_{\text{on}}^\dagger - c) + \frac{\alpha}{n}(\underline{p}_{\text{on}}^\dagger - c) = \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) + \frac{\alpha t}{n^2}.$$

This, we have the following lemma.

**Lemma 2** (Equilibrium Prices and Profits in Setting II, for a Given Value of  $n$ ). *Consider Setting II and suppose there are  $n$  online restaurants. Then the equilibrium prices and profits are given by:*

$$\begin{aligned} p_{\text{off}}^\dagger &= \bar{p}_{\text{on}}^\dagger = v - \frac{t}{4}, \\ \underline{p}_{\text{on}}^\dagger(n) &= c + \frac{t}{n}, \\ \pi_{\text{off}}^\dagger &= \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right), \end{aligned}$$

and

$$\pi_{\text{on}}^\dagger(n) = \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) + \frac{\alpha t}{n^2}.$$

Let us compare the equilibrium prices and profits of online restaurants across the two settings for a given value of  $n$ . In Setting I, an online restaurant serves  $(1-\alpha)/N + \alpha/n$  customers at price  $p_{\text{on}}^*$ . In Setting II, it serves  $(1-\alpha)/N$  customers at price  $\bar{p}_{\text{on}}^\dagger$  and  $\alpha/n$  customers at price  $\underline{p}_{\text{on}}^\dagger$ . When  $\alpha < \alpha_0(n)$ ,

$$\underline{p}_{\text{on}}^\dagger(n) < p_{\text{on}}^*(n) = \bar{p}_{\text{on}}^\dagger,$$

which immediately implies

$$\pi_{\text{on}}^*(n) > \pi_{\text{on}}^\dagger(n).$$

In other words, discounting hurts online restaurants. When  $\alpha \geq \alpha_0(n)$ ,

$$\underline{p}_{\text{on}}^\dagger(n) < p_{\text{on}}^*(n) < \bar{p}_{\text{on}}^\dagger$$

and the comparison between  $\pi_{\text{on}}^*(n)$  and  $\pi_{\text{on}}^\dagger(n)$  can go in either direction. Specifically,

$$\pi_{\text{on}}^*(n) = \left( \frac{1-\alpha}{N} + \frac{\alpha}{n} \right)^2 \frac{t}{\alpha} > \pi_{\text{on}}^\dagger(n) = \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) + \frac{\alpha t}{n^2}$$

if and only if

$$\alpha < \alpha_2(n) = \frac{t}{t + N \left( v - \frac{t}{4} - \frac{2t}{n} - c \right)}.$$

Since  $\alpha_2(n) > \alpha_0(n)$  for any value of  $n$ , we have the following result.

**Lemma 3** (Comparing Equilibrium Profits of Online Restaurants in Settings I and II, for a Given Value of  $n$ ). *Suppose there are  $n$  online restaurants. The restaurants lose from discounting,  $\pi_{\text{on}}^*(n) > \pi_{\text{on}}^\dagger(n)$ , when  $\alpha < \alpha_2(n)$ .*

We would like to stress that in the above lemma the number of online restaurants is fixed. Consider Setting I and assume that there are  $n$  online restaurants. Now suppose that the aggregator allows discounting. Lemma 3 indicates that the ability to price discriminate between walkers and e-users benefits online restaurants only when  $\alpha$  – the fraction of e-users in the population – is sufficiently high. Low values of  $\alpha$  give rise to the prisoner’s dilemma. We demonstrate its existence in the next two paragraphs.

In the absence of discounts, the equilibrium prices and profits are as in Lemma 1. If we allow one restaurant to price discriminate, will it choose to do so? In the Appendix we show that when  $\alpha \geq \alpha_0(n)$ , the deviating restaurant will charge walkers  $v - t/4$  and e-users

$$p^{\text{dev}} = c + \frac{(1-\alpha)t}{2\alpha N} + \frac{t}{n} < p_{\text{on}}^* = c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n}, \quad (7)$$

thereby increasing its profit. However, when all online restaurants are allowed to price discriminate, they charge e-users

$$\underline{p}_{\text{on}}^\dagger = c + \frac{t}{n} < p^{\text{dev}}.$$

When  $\alpha \in [\alpha_0(n), \alpha_2(n))$ , online restaurants lose compared to Setting I where discounts are prohibited.

We also show in the Appendix that when  $\alpha < \alpha_0(n)$ , the deviating restaurant will charge walkers  $v - t/4$  and e-users

$$p^{\text{dev}} = \frac{c+v}{2} - \frac{t}{8} + \frac{t}{2n}. \quad (8)$$

We verify that, again,

$$\underline{p}_{\text{on}}^\dagger = c + \frac{t}{n} < p^{\text{dev}} < p_{\text{on}}^* = v - \frac{t}{4}.$$

The result is, as before, intuitive. As more online restaurants adopt the discount strategy for e-users, the price that e-users pay drops, and the restaurants get trapped in the prisoner’s dilemma for all  $\alpha < \alpha_0(n)$ .

Next, we calculate the equilibrium number of online restaurants:

$$\pi_{\text{off}}^\dagger = \pi_{\text{on}}^\dagger(n) - k,$$

$$\frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) = \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) + \frac{\alpha t}{n^2} - k,$$

$$n^\dagger = \min \left\{ \sqrt{\frac{\alpha t}{k}}, N \right\}.$$

**Proposition 2** (Equilibrium Number of Online Restaurants in Setting II). *In Setting II, the equilibrium number of online restaurants is given by*

$$n^\dagger = \begin{cases} N & \text{if } k \leq \frac{\alpha t}{N^2} \\ \sqrt{\frac{\alpha t}{k}} & \text{if } k > \frac{\alpha t}{N^2}. \end{cases}$$

We see that  $n^\dagger \leq n^{\text{so}} = N$ , with strict inequality for  $k > \alpha t/N^2$ . Put differently, the social welfare is maximized only when the fixed cost  $k$  is sufficiently small,  $k \leq \alpha t/N^2$ . Hence, also in this setting, the private and the social interests fail to coincide when  $k$  is sufficiently high.

It is straightforward to show that  $n^\dagger \geq 2$  if and only if

$$k \leq \frac{\alpha t}{4}.$$

Comparing this condition with (6), we notice that

$$\frac{\alpha t}{4} < \frac{\alpha}{2} \left( v - \frac{t}{4} - c \right)$$

holds under the assumption (1). The maximum value of  $k$  compatible with the equilibrium number of online restaurants being at least two in both settings is, therefore,

$$k_{\max} = \frac{\alpha t}{4}.$$

For the rest of our analysis we will assume that  $k$  does not exceed  $k_{\max}$ .

Finally, we compare the equilibrium number of online restaurants in this setting with that in Setting I. It immediately follows from Propositions 1 and 2 that when  $k \leq \min \{ \alpha t/N^2, k_1 \}$ ,  $n^\dagger = n^* = n^{\text{so}} = N$ . When  $k > \min \{ \alpha t/N^2, k_1 \}$ , either  $n^* < n^\dagger \leq n^{\text{so}}$  or  $n^\dagger < n^* \leq n^{\text{so}}$ .

Note that

$$\frac{\alpha}{\sqrt{\left( k + \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) \right) \frac{\alpha}{t} - \frac{1-\alpha}{N}}} > \sqrt{\frac{\alpha t}{k}}$$

(the left-hand side is the equilibrium number of online restaurants in Setting I when  $k \in (k_1, k_2]$  and the right-hand side is the equilibrium number of online restaurants in Setting II when  $k > \alpha t/N^2$ ) if and only if

$$k > \frac{t}{4\alpha} \left( \frac{\alpha \left( v - \frac{t}{4} - c \right)}{t} - \frac{1-\alpha}{N} \right)^2 = k_3,$$

and

$$\frac{\alpha \left( v - \frac{t}{4} - c \right)}{k} > \sqrt{\frac{\alpha t}{k}}$$

(the left-hand side the equilibrium number of online restaurants in Setting I when  $k > k_2$  and the right-hand side is the equilibrium number of online restaurants in Setting II when  $k > \alpha t/N^2$ ) always holds under the assumption (1) and for  $k \leq k_{\max}$ .

Tedious algebra (see the Appendix) reveals that the thresholds  $k_1$ ,  $k_2$  and  $\alpha t/N^2$  and the critical value  $k_3$  (obtained above) line up as

$$k_3 < \frac{\alpha t}{N^2} < k_1 \leq k_2 \quad (9)$$

when

$$\alpha < \frac{t}{N \left( v - \frac{t}{4} - c \right) - t} = \alpha_3$$

and as

$$k_1 < \frac{\alpha t}{N^2} < k_3 < k_2 \quad (10)$$

when  $\alpha > \alpha_3$ . Thus, we have the following proposition.

**Proposition 3** (Equilibrium Number of Online Restaurants in Settings I and II and Efficiency). *Let  $\alpha < \alpha_3$ . In this case the socially optimal number of online restaurants is reached in both settings only when*

$$k \leq \frac{\alpha t}{N^2}.$$

*That is,  $n^\dagger = n^* = N$ . Otherwise, the equilibrium number of online restaurants is higher under Setting I than under Setting II,  $n^\dagger < n^* \leq N$ .*

*When  $\alpha > \alpha_3$ , the socially optimal number of online restaurants is reached in both settings only when  $k \leq k_1$ . (This interval region is an empty set when  $k_1$  is negative.) For high values of  $k$ ,  $k > k_3$ , Setting I yields a higher number of online restaurants:  $n^\dagger < n^* < N$ . On the contrary, when  $k \in (k_1, k_3)$ , Setting II induces more restaurants to go online:  $n^* < n^\dagger \leq N$ .*

Figure 3 provides a useful representation of the above results.<sup>15</sup> Apart from the case in which the fee is very low,  $k \leq \min \{ \alpha t/N^2, k_1 \}$ , the equilibrium number of restaurants that decide to go online depends on the interplay between  $\alpha$  and  $k$ . Consider the admissible parametric region in which  $k \leq k_{\max}$ . If the fraction of e-users is relatively low, then uniform pricing (Setting I) allows more restaurants to go online when  $k > \alpha t/N^2$ . Moreover, for  $k \in (\alpha t/N^2, k_1)$  the efficient solution is reached, as  $(n^\dagger < )n^* = n^{\text{so}}$ . The intuition goes as follows. From the discussion after Lemma 3 we know that online restaurants lose from price discrimination (Setting II), as they get trapped in the prisoner's dilemma. A lower number of restaurants would, therefore, be able to afford the fee for being listed on the aggregator's website. On the contrary, for relatively high values of  $\alpha$ , online restaurants can charge higher prices for walkers while at the same time competing for e-users (their main target). In this region more restaurants can afford the fee to go online when discounts are allowed. Notice also that when  $k \in (k_1, \alpha t/N^2)$  the efficient outcome is guaranteed under price discrimination, as  $(n^* < )n^\dagger = n^{\text{so}}$ . Finally, observe that the higher is the proportion of e-users in the population, the larger is the region where price discrimination yields a higher number of online restaurants than uniform pricing.

<sup>15</sup>In drawing the picture, we used the following parameter values:  $v = 2$ ,  $c = 0$ ,  $t = 1$ , and  $N = 12$ .

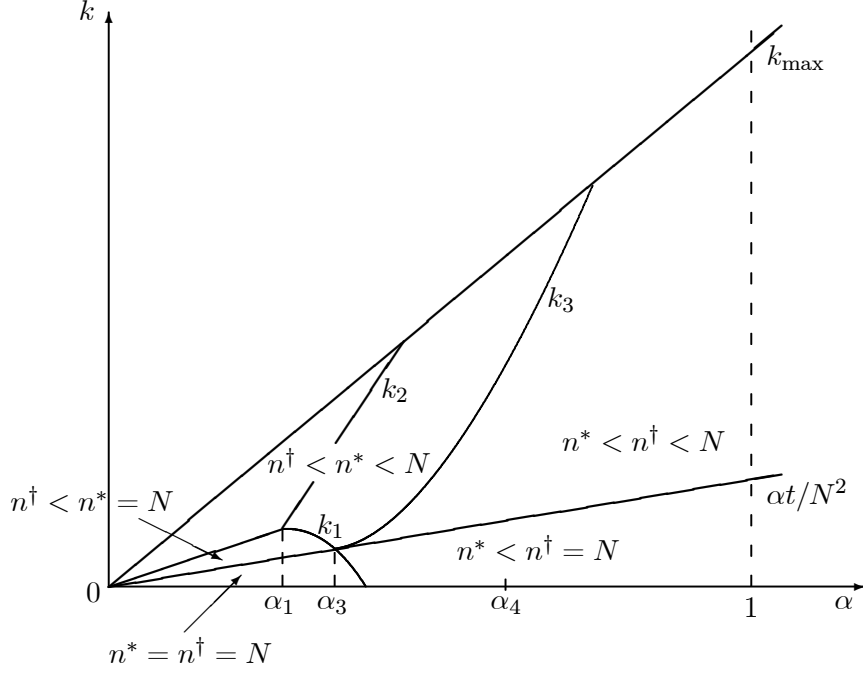


Figure 3: Equilibrium Number of Online Restaurants in Settings I and II

## 6 The Aggregator's Optimal Policy

In this section we make the aggregator an active player. Suppose for a moment that the aggregator can decide whether to prohibit (Setting I) or to allow discounts (Setting II). If the aggregator's profit is simply the product of the equilibrium number of online restaurants and the fee  $k$ , then it will choose the setting that yields the highest number of online restaurants.

To make our model more realistic, we assume that the aggregator not only decides whether to prohibit discounts, but also chooses the online fee  $k$ . Will the aggregator's optimal policy result in  $N$  restaurants (the socially optimal number) going online? Consider Setting I. The aggregator's profit is  $kn^*$ , where  $n^*$  is a decreasing function of  $k$ . As it was noted right after Proposition 1,  $\alpha \leq \alpha_1$  implies

$$k_1 = k_2 = \frac{\alpha \left( v - \frac{t}{4} - c \right)}{N}.$$

If this is the case, then

$$kn^*(k) = \begin{cases} kN & \text{if } k \leq k_1 \\ \alpha \left( v - \frac{t}{4} - c \right) & \text{if } k > k_1. \end{cases}$$

Obviously, the aggregator's profit attains its maximum,

$$\alpha \left( v - \frac{t}{4} - c \right),$$



at any  $k \geq k_1$ . Note that  $k = k_1$  induces all restaurants to go online,  $n^*(k_1) = N$ . This is socially optimal.

When  $\alpha > \alpha_1$ ,  $k_1 < k_2$ , so

$$kn^*(k) = \begin{cases} kN & \text{if } k \leq k_1 \\ \frac{k\alpha}{\sqrt{(k + \frac{1-\alpha}{N})(v - \frac{t}{4} - c))^{\frac{\alpha}{t} - \frac{1-\alpha}{N}}} & \text{if } k \in (k_1, k_2] \\ \alpha(v - \frac{t}{4} - c) & \text{if } k > k_2. \end{cases}$$

Tedious algebra reveals that  $kn^*(k)$  increases on the interval  $(k_1, k_2]$ . Hence, the aggregator's profit is maximized at  $k = k_2$  if  $k_2 < k_{\max}$ , and at  $k = k_{\max}$  if otherwise. In either case the resulting number of online restaurants is below the socially optimal number.

In Setting II, the aggregator's profit is

$$kn^\dagger(k) = \begin{cases} kN & \text{if } k \leq \frac{\alpha t}{N^2} \\ \sqrt{\alpha t k} & \text{if } k > \frac{\alpha t}{N^2}. \end{cases}$$

It is easy to see that  $kn^\dagger(k)$  is increasing in  $k$ , which immediately implies that the optimal  $k$  is  $k_{\max}$ . The aggregator's maximum profit in Setting II equals

$$\frac{\alpha t}{2}.$$

Let

$$\alpha_4 = \frac{t}{t + N(v - \frac{5t}{4} - c)}.$$

Please see the Appendix for the proof of the following proposition.

**Proposition 4** (The Aggregator's Optimal Policy). *When it comes to shaping its policy, the aggregator will allow discounts when the proportion of e-users is sufficiently high,  $\alpha > \alpha_4$ . In this case the aggregator will choose a high fee inducing only a few restaurants to go online. The aggregator will impose uniform pricing when  $\alpha \leq \alpha_4$ . The aggregator's optimal choice of  $k$  will result in all restaurants going online only for sufficiently low values of  $\alpha$ ,  $\alpha \leq \alpha_1$ .*

The intuition is as follows. When the proportion of e-users is large, price discrimination benefits online restaurants. It hurts online restaurants when  $\alpha$  is low (Lemma 3). Thus, allowing discounts strengthens (weakens) the restaurants' incentives to go online when  $\alpha$  is large (low). (See Figure 3 that compares the equilibrium number of online restaurants across the two settings for different values of  $\alpha$  and  $k$ .) Not surprisingly, for large values of  $\alpha$  the rent that the aggregator can capture is higher when online discounts are allowed. For small values of  $\alpha$ , the aggregator is better off prohibiting discounts.

This has relevant policy implications, as it is possible to identify the interval regions in which the interests of the aggregator are not aligned with those of the society. When  $\alpha > \alpha_1$ , the fee charged by the aggregator induces a suboptimal number of restaurants to go online. In order to induce all restaurants to go online, the policy maker should monitor the level of the online fee. Such a policy remedy also requires a careful examination of the price regime imposed by

the aggregator. Consider again Figure 3. While in the region  $\alpha \in (\alpha_1, \alpha_3]$  the policy maker achieves efficiency only by restricting the online fee to  $k \leq k_1$ , in the region  $\alpha \in (\alpha_3, \alpha_4]$  it should both restore price discrimination and limit the fee to  $k \leq \alpha t/N^2$ . Finally, when  $\alpha > \alpha_4$ , in which the aggregator's optimal policy allows discounts, the policy maker should just restrict the fee to  $k \leq \alpha t/N^2$ .

## 7 Concluding Remarks

In this paper we used a model of horizontal product differentiation *à la* Salop (1979) to analyze the impact of a web aggregator on the equilibrium behavior of consumers and producers in a market with asymmetric information. In particular, we imagined a situation where uninformed consumers (tourists visiting a new city) were looking for a place to eat. The aggregator was assumed to provide accurate information about the restaurants listed on its website (online restaurants), therefore helping consumers to find their best match. The population of consumers was divided into those willing to spend time on the Internet (e-users) and those enjoying roaming around the city while looking for a place to eat (walkers).

We considered two different settings, depending on whether online restaurants can offer discounts to e-users. We showed that in the first setting (that of uniform pricing), online restaurants charge a lower price than their offline counterparts only when the proportion of e-users is sufficiently high. In the second setting, in which discounting is allowed, we found that e-users always pay less than walkers. For a given number of online restaurants, we highlighted that the profitability of price discrimination crucially depends on the composition of consumers. If the population of consumers is mainly composed of walkers, then online restaurants do not gain from discounting, as they end up being trapped in the prisoner's dilemma. As compared to uniform pricing, they cannot increase the price for walkers, but they are pressured by competition to offer a discount to e-users. On the contrary, price discrimination pays off when the proportion of e-users is relatively high. This was the first important finding of our analysis.

Next, we computed the equilibrium number of online restaurants in both settings and compared it with the first best for the society, which in our model simply requires all restaurants to go online. It is, again, the composition of consumers that plays a major role. When walkers predominate, then uniform pricing allows more restaurants to go online, as under discounting online restaurants are trapped in the prisoner's dilemma. On the contrary, when the proportion of e-users is relatively high, then online restaurants gain from price discrimination, so a higher number of restaurants can afford to pay the fee for being listed online. The first best can always be reached under price discrimination, provided that the fee charged by the aggregator is sufficiently low. Under uniform pricing, however, even zero fee may fail to induce all restaurants to go online. This happens when the proportion of e-users is close to one.

Finally, we considered the aggregator's optimal policy regarding both the fee and whether to impose uniform pricing or to allow discounts for e-users. We discovered that the aggregator allows discounts when the proportion of e-users is sufficiently high. Otherwise, the aggregator is better off imposing uniform pricing. Only when the proportion of e-users is close to zero the fee chosen by the aggregator induces all restaurants to go online. In all other cases the policy maker should intervene to achieve efficiency. Just limiting the online fee might not be enough.

In some cases it has to be coupled with prohibiting the aggregator to impose uniform pricing.

The analysis carried out in this paper relied on many simplifying assumptions. First of all, we considered the presence of only one aggregator, and supposed that it conveys reliable information at almost zero cost for consumers. This can be justified by the fact that consumers tend to resort to just a few trusted sources of information, those who successfully win the race to become the reference points for consumers unfamiliar with certain product characteristics. For this reason, aggregators usually do not charge final users, but compete for rents coming from the firms that want to get online visibility. Therefore, introducing competition among aggregators would result in lower fees being charged to firms, without changing the qualitative results of our analysis.

A second limitation of our model is that it focuses only on uninformed consumers. In an extension of the basic setting, we introduced local consumers, who previously had the opportunity to try different restaurants in the city.<sup>16</sup> Once a local consumer finds a restaurant that satisfies her tastes, she dines only there, provided that the price does not exceed her valuation. Assuming a proportion  $\beta$  of locals in the population of consumers, we found that our results are still valid. Indeed, by substituting the fraction  $\alpha$  of e-users with the fraction of e-users among the population of tourists, *i.e.*  $\tilde{\alpha} \equiv \alpha(1 - \beta)$ , one can notice that it is possible to reproduce the whole analysis carried out in the paper.

A third criticism that can be made to our model is that it assumed a fixed fraction of e-users in the population of consumers. Although very recent, the proliferation of online reviews is not a new phenomenon. Consumers already got used to online information provided by review aggregators. We are mainly interested in the consequences of allowing or not discounts in terms of how many merchants can afford to go online rather than in the rise of online aggregators.

We also acknowledge that our analysis has been performed under some specific parametric conditions. However, such conditions have been always justified not only for algebraic tractability, but also for being well suited to the specific market case that we wanted to study. All in all, we are also convinced that the basic model that we provided allows to capture the impact of web aggregators in a simple but significant way.

## References

- [1] Aguirre, I., 2012. Welfare Effects of Third Degree Price Discrimination: Ippolito Meets Schmalensee and Varian, in R. Laratta (ed) *Social Welfare*, InTech, ISBN 978-953-51-0208-3. Graphic Analysis of Third-Degree Price Discrimination.
- [2] Aguirre, I., Cowan, S., Vickers, J., 2010. Monopoly Price Discrimination and Demand Curvature. *American Economic Review*, **100**, 1601-1615.
- [3] Akerlof, G.A., 1970. The Market for “Lemons”: Qualitative Uncertainty and the Market Mechanism. *Quarterly Journal of Economics*, **84**, 488-500

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<sup>16</sup>The extension with local consumers is available upon request.

- [4] Alba, J., Lynch, J., Weitz, B., Janiszewski, C., Lutz, R., Sawyer, A., Wood, S., 1997. Interactive Home Shopping: Consumer, Retailer and Manufacturer Incentives to Participate in Electronic Marketplaces. *Journal of Marketing*, **61**, 38–53.
- [5] Anand, B. N., Shachar, R., 2011. Advertising, the Matchmaker. *RAND Journal of Economics*, **42**, 205-245.
- [6] Anderson, M., Magruder, J., 2012. Learning from the Crowd: Regression Discontinuity Estimates of the Effects of an Online Review Database. *The Economic Journal*, **122**, 957-989.
- [7] Armstrong, M., 2006. Competition in Two-Sided Markets. *RAND Journal of Economics*, **37**, 668-691.
- [8] Bakos, J. Y., 1997. Reducing Buyer Search Costs: Implications for Electronic Marketplaces. *Management Science*, **43**, 1676-1692.
- [9] Baye, M. R., Morgan, J., 2001. Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets. *American Economic Review*, **91**, 454-474.
- [10] Belleflamme, P., Peitz, M., 2010. Platform Competition and Seller Investment Incentives. *European Economic Review*, **54**, 1059-1076.
- [11] Brown, J. R., Goolsbee, A., 2002. Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry. *Journal of Political Economy*, **110**, 481–507.
- [12] Brynjolfsson, E., Smith, M., 2000. Frictionless Commerce? A Comparison of Internet and Conventional Retailers. *Management Science*, **46**, 563-585.
- [13] Caillaud, B., Jullien, B., 2003. Chicken & Egg: Competition among Intermediation Service Providers. *RAND Journal of Economics*, **34**, 309-28.
- [14] Che, Y.-K., Gale, I., 1997. Buyer Alliances and Managed Competition. *Journal of Economics & Management Strategy*, **6**, 175-200.
- [15] Chevalier, J. A., Mayzlin, D., 2006. The Effect of Word of Mouth on Sales: Online Book Reviews. *Journal of Marketing Research*, **43**, 345-354.
- [16] De Cornière, A., 2016. Search Advertising. *American Economic Journal: Microeconomics*, **8**, 156-188.
- [17] Dellarocas, C., 2006. Strategic Manipulation of Internet Opinion Forums: Implications for Consumers and Firms. *Management Science*, **52**, 1577-1593.
- [18] Diamond, P. A., 1971. A Model of Price Adjustment. *Journal of Economic Theory*, **3**, 156-168.
- [19] Edelman, B., Wright, J., 2015. Price Coherence and Excessive Intermediation. *Quarterly Journal of Economics*, **130**, 1283-1328.

- [20] Galeotti, A., Moraga-González, J.L., 2009. Platform Intermediation in a Market for Differentiated Products. *European Economic Review*, **53**, 417-428.
- [21] Gehrig, T., 1993. Intermediation in Search Markets. *Journal of Economics & Management Strategy*, **2**, 97-120.
- [22] Ghose, A., Ipeirotis, P. G., Li, B., 2012. Designing Ranking Systems for Hotels on Travel Search Engines by Mining User-Generated and Crowdsourced Content. *Marketing Science*, **31**, 493-520.
- [23] Gu, Z., Xie, J., 2013. Facilitating Fit Revelation in the Competitive Market. *Management Science*, **59**, 1196-1212.
- [24] Hilger, J., Rafert, G., Villas-Boas, S., 2011. Expert Opinion and the Demand for Experience Goods: An Experimental Approach in the Retail Wine Market. *Review of Economics and Statistics*, **93**, 1289-1296.
- [25] Johnson, J. P., 2017. The Agency Model and MFN Clauses. Available at SSRN: <http://ssrn.com/abstract=2217849>.
- [26] Lal, R., Sarvary, M., 1999. When and How is the Internet Likely to Decrease Price Competition? *Marketing Science*, **18**, 485-503.
- [27] Lewis, T., Sappington, D., 1994. Supplying Information to Facilitate Price Discrimination. *International Economic Review*, **35**, 309-327.
- [28] Loginova, O., 2009. Real and Virtual Competition. *Journal of Industrial Economics*, **57**, 319-342.
- [29] Mayzlin, D., 2006. Promotional Chat on the Internet. *Marketing Science*, **25**, 155-163.
- [30] Mayzlin, D., Dover, Y., Chevalier, J., 2014. Promotional Reviews: An Empirical Investigation of Online Review Manipulation. *American Economic Review*, **104**, 2421-2455.
- [31] Morton, S. F.; Zettelmeyer, F., Silva-Risso, J., 2001. Internet Car Retailing. *Journal of Industrial Economics*, **49**, 501-519.
- [32] Narasimhan, C., 1984. A Price Discrimination Theory of Coupons. *Marketing Science*, **3**, 128-147.
- [33] Narasimhan, C., 1988. Competitive Promotional Strategies. *Journal of Business*, **61**, 427-449.
- [34] Nocke, V., Peitz, M., Stahl, K., 2007. Platform Ownership. *Journal of the European Economic Association*, **5**, 1130-1160.
- [35] Reinstein, D. A., Snyder, C. M., 2005. The Influence of Expert Reviews on Consumer Demand for Experience Goods: a Case Study of Movie Critics. *Journal of Industrial Economics*, **53**, 27-51.

- [36] Rochet, J., Tirole, J., 2003. Platform Competition in Two-Sided Markets. *Journal of the European Economic Association*, **1**, 990-1029.
- [37] Salop, S. C., 1979. Monopolistic Competition with Outside Goods. *Bell Journal of Economics*, **10**, 141–156.
- [38] Schmalensee, R., 1981. Output and Welfare Implications of Monopolistic Third Degree Price Discrimination. *American Economic Review*, **71**, 242-247.
- [39] Spulber, D., 1999. *Market Microstructure: Intermediaries and the Theory of the Firm*, Cambridge University Press.
- [40] Stigler, G. J., 1987. *The Theory of Price*. Macmillan USA.
- [41] Sun, M., 2010. Disclosing Multiple Product Attributes. *Journal of Economics & Management Strategy*, **20**, 195 - 224.
- [42] Varian, H. R., 1980. A Model of Sales. *American Economic Review*, **70**, 651-59.
- [43] Varian, H. R., 1985. Price Discrimination and Social Welfare. *American Economic Review*, **75**, 870-875.
- [44] Wang, C., Wright, J., 2016. Search Platforms: Showrooming and Price Parity Clauses, working paper.
- [45] Watanabe, M., 2010. A Model of Merchants. *Journal of Economic Theory*, **145**, 1865-1889.
- [46] Zettelmeyer, F., 2000. Expanding to the Internet: Pricing and Communications Strategies When Firms Compete on Multiple Channels. *Journal of Marketing Research*, **37**, 292-308.

## Appendix

### Proof of Lemma 1.

In the derivations preceding Lemma 1 we have guessed that  $p_{\text{on}}^* \leq v - t/4$ . That is, the price that the online restaurants charge in equilibrium does not repel walkers. We also did not check deviations to prices above  $v - t/4$ .

1. First, we will show that whenever

$$p_{\text{on}}^* = v - \frac{t}{4},$$

an online restaurant will not benefit from deviating to a price  $p \in (v - t/4, v]$ . Indeed, the deviating restaurant will only attract  $2\alpha\hat{x}$  customers, where

$$\hat{x} = \frac{1}{2n} + \frac{v - \frac{t}{4} - p}{2t}.$$

Hence, its profit will be

$$2\alpha \left( \frac{1}{2n} + \frac{v - \frac{t}{4} - p}{2t} \right) (p - c).$$

Differentiating with respect to  $p$  and setting the derivative to zero yields

$$-\frac{1}{t}(p - c) + \frac{1}{n} + \frac{v - t/4 - p}{t} = 0,$$

$$p = \frac{c + \frac{t}{n} + v - \frac{t}{4}}{2}.$$

Observe that this price is below  $v - t/4$ :

$$\frac{c + \frac{t}{n} + v - \frac{t}{4}}{2} < v - \frac{t}{4},$$

$$v - c > \frac{t}{n} + \frac{t}{4},$$

which holds for any  $n \geq 2$  under the assumption (1). This confirms that deviations to prices above  $v - t/4$  are not profitable.

2. Next, we will show that the equilibrium price  $p_{\text{on}}^*$  cannot exceed  $v - t/4$ . Suppose the contrary,  $p_{\text{on}}^* > v - t/4$ . The walkers are not served by the online restaurants, hence

$$p_{\text{on}}^* = c + \frac{t}{n}.$$

However,

$$c + \frac{t}{n} < v - \frac{t}{4}$$

holds for any  $n \geq 2$  under the assumption (1). This contradicts the supposition.

3. For the threshold  $\alpha_0(n)$  we solve

$$c + \frac{(1 - \alpha)t}{\alpha N} + \frac{t}{n} = v - \frac{t}{4},$$

$$\alpha_0(n) = \frac{t}{t + N \left( v - \frac{t}{4} - \frac{t}{n} - c \right)}.$$

Observe that  $\alpha_0(n) < 1$ , as

$$v - \frac{t}{4} - \frac{t}{n} - c > 0$$

for all  $n \geq 2$  due to the assumption (1).

**Proof of Proposition 1.**

That  $\pi_{\text{on}}^*(n)$  decreases in  $n$  can be easily seen from (3) and (4). Indeed, since

$$c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n}$$

decreases in  $n$ ,

$$p_{\text{on}}^*(n) = \min \left\{ c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n}, v - \frac{t}{4} \right\}$$

decreases in  $n$ . Next, because both

$$\frac{1-\alpha}{N} + \frac{\alpha}{n}$$

and

$$p_{\text{on}}^*(n) - c$$

decrease in  $n$ ,

$$\pi_{\text{on}}^*(n) = \left( \frac{1-\alpha}{N} + \frac{\alpha}{n} \right) (p_{\text{on}}^*(n) - c)$$

decreases in  $n$ . The rest of the proof is completed in three steps:

1. If  $k$  is small,

$$k \leq \pi_{\text{on}}^*(N) - \pi_{\text{off}}^* = \begin{cases} \frac{\alpha(v - \frac{t}{4} - c)}{N} + \frac{t}{N^2\alpha} - \frac{v - \frac{t}{4} - c}{N} & \text{if } \alpha \geq \frac{t}{N(v - \frac{t}{4} - c)} \\ \frac{\alpha(v - \frac{t}{4} - c)}{N} & \text{if otherwise,} \end{cases}$$

then  $n^* = N$ .

2. If  $k > \pi_{\text{on}}^*(N) - \pi_{\text{off}}^*$ , the equilibrium number of restaurants is determined by

$$\frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) = \left( \frac{1-\alpha}{N} + \frac{\alpha}{n} \right)^2 \frac{t}{\alpha} - k$$

for large values of  $\alpha$ , so

$$n^* = \frac{\alpha}{\sqrt{\left( k + \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) \right) \frac{\alpha}{t} - \frac{1-\alpha}{N}}}.$$

If  $\alpha$  is small, (5) becomes

$$\frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) = \left( \frac{1-\alpha}{N} + \frac{\alpha}{n} \right) \left( v - \frac{t}{4} - c \right) - k,$$

hence

$$n^* = \frac{\alpha \left( v - \frac{t}{4} - c \right)}{k}.$$



The threshold value for  $\alpha$  can be found from

$$\frac{\alpha}{\sqrt{\left(k + \frac{1-\alpha}{N} \left(v - \frac{t}{4} - c\right)\right)^{\frac{\alpha}{t} - \frac{1-\alpha}{N}}}} = \frac{\alpha \left(v - \frac{t}{4} - c\right)}{k},$$

$$\alpha = \frac{t + \frac{tNk}{v - \frac{t}{4} - c}}{t + N \left(v - \frac{t}{4} - c\right)}.$$

Thus, we have

$$n^* = \begin{cases} \frac{\alpha}{\sqrt{\left(k + \frac{1-\alpha}{N} \left(v - \frac{t}{4} - c\right)\right)^{\frac{\alpha}{t} - \frac{1-\alpha}{N}}}} & \text{if } \alpha \geq \frac{t + \frac{tNk}{v - \frac{t}{4} - c}}{t + N \left(v - \frac{t}{4} - c\right)} \\ \frac{\alpha \left(v - \frac{t}{4} - c\right)}{k} & \text{if } \alpha < \frac{t + \frac{tNk}{v - \frac{t}{4} - c}}{t + N \left(v - \frac{t}{4} - c\right)}, \end{cases}$$

or

$$n^* = \begin{cases} \frac{\alpha}{\sqrt{\left(k + \frac{1-\alpha}{N} \left(v - \frac{t}{4} - c\right)\right)^{\frac{\alpha}{t} - \frac{1-\alpha}{N}}}} & \text{if } k \leq \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \frac{\alpha \left(v - \frac{t}{4} - c\right)^2}{t} - \frac{v - \frac{t}{4} - c}{N} \\ \frac{\alpha \left(v - \frac{t}{4} - c\right)}{k} & \text{if } k > \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \frac{\alpha \left(v - \frac{t}{4} - c\right)^2}{t} - \frac{v - \frac{t}{4} - c}{N}. \end{cases}$$

3. Combining the results in steps 1 and 2 yields:

$$n^* = \begin{cases} N & \text{if } k \leq \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \frac{t}{N^2 \alpha} - \frac{v - \frac{t}{4} - c}{N} \\ \frac{\alpha}{\sqrt{\left(k + \frac{1-\alpha}{N} \left(v - \frac{t}{4} - c\right)\right)^{\frac{\alpha}{t} - \frac{1-\alpha}{N}}}} & \text{if } k \in \left( \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \frac{t}{N^2 \alpha} - \frac{v - \frac{t}{4} - c}{N}, \right. \\ \left. \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \frac{\alpha \left(v - \frac{t}{4} - c\right)^2}{t} - \frac{v - \frac{t}{4} - c}{N} \right] \\ \frac{\alpha \left(v - \frac{t}{4} - c\right)}{k} & \text{if } k > \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \frac{\alpha \left(v - \frac{t}{4} - c\right)^2}{t} - \frac{v - \frac{t}{4} - c}{N} \end{cases}$$

if

$$\alpha > \frac{t}{N \left(v - \frac{t}{4} - c\right)} = \alpha_1,$$

and

$$n^* = \begin{cases} N & \text{if } k \leq \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} \\ \frac{\alpha \left(v - \frac{t}{4} - c\right)}{k} & \text{if } k > \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} \end{cases}$$

if  $\alpha \leq \alpha_1$ .

**Derivation of (7).**

When  $\alpha \geq \alpha_0(n)$ ,

$$p_{\text{on}}^* = c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n}.$$

The deviating restaurant will charge walkers  $v - t/4$  and e-users

$$p^{\text{dev}} = \arg \max_{p \leq v - \frac{t}{4}} \left( \frac{1 - \alpha}{N} \left( v - \frac{t}{4} - c \right) + 2\alpha \hat{x}(p - c) \right),$$

where  $\hat{x}$  is determined by

$$v - p - \hat{x}t = v - p_{\text{on}}^* - \left( \frac{1}{n} - \hat{x} \right) t,$$

$$\hat{x} = \frac{1}{2n} + \frac{p_{\text{on}}^* - p}{2t} = \frac{1}{2n} + \frac{c + \frac{(1-\alpha)t}{\alpha N} + \frac{t}{n} - p}{2t}.$$

That is,

$$p^{\text{dev}} = \arg \max_{p \leq v - \frac{t}{4}} \left( \frac{1 - \alpha}{N} \left( v - \frac{t}{4} - c \right) + \frac{2\alpha}{t} \left( \frac{c}{2} + \frac{(1 - \alpha)t}{2\alpha N} + \frac{t}{n} - \frac{p}{2} \right) (p - c) \right).$$

Differentiating

$$\left( \frac{c}{2} + \frac{(1 - \alpha)t}{2\alpha N} + \frac{t}{n} - \frac{p}{2} \right) (p - c)$$

with respect to  $p$  and setting the derivative to zero yield

$$p^{\text{dev}} = c + \frac{(1 - \alpha)t}{2\alpha N} + \frac{t}{n}.$$

### Derivation of (8).

When  $\alpha < \alpha_0(n)$ ,

$$p_{\text{on}}^* = v - \frac{t}{4}.$$

The deviating restaurant will charge walkers  $v - t/4$  and e-users

$$p^{\text{dev}} = \arg \max_{p \leq v - \frac{t}{4}} \left( \frac{1 - \alpha}{N} \left( v - \frac{t}{4} - c \right) + 2\alpha \hat{x}(p - c) \right),$$

where

$$\hat{x} = \frac{1}{2n} + \frac{p_{\text{on}}^* - p}{2t} = \frac{1}{2n} + \frac{v - \frac{t}{4} - p}{2t}.$$

That is,

$$p^{\text{dev}} = \arg \max_{p \leq v - \frac{t}{4}} \left( \frac{1 - \alpha}{N} \left( v - \frac{t}{4} - c \right) + \frac{2\alpha}{t} \left( \frac{v}{2} - \frac{t}{8} + \frac{t}{2n} - \frac{p}{2} \right) (p - c) \right).$$

Differentiating

$$\left( \frac{v}{2} - \frac{t}{8} + \frac{t}{2n} - \frac{p}{2} \right) (p - c)$$

with respect to  $p$  and setting the derivative to zero yield

$$p^{\text{dev}} = \frac{c + v}{2} - \frac{t}{8} + \frac{t}{2n}.$$

**Proof of (9) and (10).**

The proof is completed in five steps:

1. First, we show that  $k_2 < k_3$  for all values of  $\alpha$ . When  $\alpha \leq \alpha_1$ ,

$$k_2 = \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N}.$$

The inequality  $k_2 < k_3$  can be written as

$$\frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} < \frac{\alpha \left(v - \frac{t}{4} - c\right)^2}{t},$$

or

$$v - c > \frac{t}{4} + \frac{t}{N},$$

which holds for  $N \geq 2$  under the assumption (1). When  $\alpha > \alpha_1$ ,

$$k_2 = \frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} + \left( \frac{\alpha \left(v - \frac{t}{4} - c\right)^2}{t} - \frac{v - \frac{t}{4} - c}{N} \right).$$

The inequality  $k_2 < k_3$ , obviously, holds.

2. Next, we show that  $k_1 = \alpha t/N^2$  if and only if  $\alpha = \alpha_3$ . Indeed, when  $\alpha \leq \alpha_1$ ,

$$k_1 = \frac{\alpha_3 \left(v - \frac{t}{4} - c\right)}{N}.$$

It is easy to see that the equation  $k_1 = \alpha t/N^2$  does not have a solution. When  $\alpha > \alpha_1$ ,

$$k_1 = \frac{\alpha_3 \left(v - \frac{t}{4} - c\right)}{N} - \left( \frac{v - \frac{t}{4} - c}{N} - \frac{t}{N^2 \alpha_3} \right).$$

Solving  $k_1 = \alpha t/N^2$  yields

$$\frac{\alpha \left(v - \frac{t}{4} - c\right)}{N} - \left( \frac{v - \frac{t}{4} - c}{N} - \frac{t}{N^2 \alpha} \right) = \frac{\alpha t}{N^2},$$

$$\frac{(1 - \alpha^2)t}{N^2 \alpha} = \frac{(1 - \alpha) \left(v - \frac{t}{4} - c\right)}{N},$$

$$\frac{(1 + \alpha)t}{N \alpha} = v - \frac{t}{4} - c,$$

$$\alpha = \frac{t}{N \left(v - \frac{t}{4} - c\right) - t} = \alpha_3.$$

3. It immediately follows from the previous step that  $k_1 > \alpha t/N^2$  when  $\alpha < \alpha_3$  and  $k_1 < \alpha t/N^2$  when  $\alpha > \alpha_3$ .

4. It is left to show that  $k_4 = \alpha t/N^2$  if and only if  $\alpha = \alpha_3$ . We have:

$$\frac{t}{4\alpha} \left( \frac{\alpha \left( v - \frac{t}{4} - c \right)}{t} - \frac{1 - \alpha}{N} \right)^2 = \frac{\alpha t}{N^2},$$

$$\left( \frac{\alpha \left( v - \frac{t}{4} - c \right)}{t} - \frac{1 - \alpha}{N} \right)^2 = \frac{4\alpha^2}{N^2},$$

$$\frac{\alpha \left( v - \frac{t}{4} - c \right)}{t} - \frac{1 - \alpha}{N} = \frac{2\alpha}{N},$$

$$\alpha = \frac{t}{N \left( v - \frac{t}{4} - c \right) - t} = \alpha_3.$$

5. It immediately follows from the previous step that  $k_4 < \alpha t/N^2$  when  $\alpha < \alpha_3$  and  $k_4 > \alpha t/N^2$  when  $\alpha > \alpha_3$ .

#### Proof of Proposition 4.

The results of the proposition will follow immediately from the analysis of the two cases below.

1. Suppose  $\alpha \leq \alpha_1$ . The aggregator's profit under Setting I is

$$\alpha \left( v - \frac{t}{4} - c \right)$$

when  $k$  is chosen optimally. It is higher than under Setting II since

$$\alpha \left( v - \frac{t}{4} - c \right) > \frac{\alpha t}{2}$$

always holds due to the assumption (1).

2. Suppose  $\alpha > \alpha_1$ . If  $k_2 > k_{\max}$ , then the aggregator's profit under Setting I is, again,

$$\alpha \left( v - \frac{t}{4} - c \right),$$

higher than under Setting II. Otherwise, the aggregator's profit under Setting I is

$$\frac{k_{\max} \alpha}{\sqrt{\left( k_{\max} + \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) \right) \frac{\alpha}{t} - \frac{1-\alpha}{N}}}.$$

It is higher than under Setting II if and only if

$$\frac{k_{\max} \alpha}{\sqrt{\left( k_{\max} + \frac{1-\alpha}{N} \left( v - \frac{t}{4} - c \right) \right) \frac{\alpha}{t} - \frac{1-\alpha}{N}}} > \frac{\alpha t}{2},$$

$$\frac{\frac{\alpha^2 t}{4}}{\sqrt{\left(\frac{\alpha t}{4} + \frac{1-\alpha}{N} \left(v - \frac{t}{4} - c\right)\right) \frac{\alpha}{t} - \frac{1-\alpha}{N}}} > \frac{\alpha t}{2},$$

$$\frac{\alpha}{2} + \frac{1-\alpha}{N} > \sqrt{\frac{\alpha^2}{4} + \frac{\alpha(1-\alpha)}{Nt} \left(v - \frac{t}{4} - c\right)},$$

$$\frac{\alpha^2}{4} + \frac{\alpha(1-\alpha)}{N} + \frac{(1-\alpha)^2}{N^2} > \frac{\alpha^2}{4} + \frac{\alpha(1-\alpha)}{Nt} \left(v - \frac{t}{4} - c\right),$$

$$\alpha + \frac{1-\alpha}{N} > \frac{\alpha}{t} \left(v - \frac{t}{4} - c\right),$$

or

$$\alpha < \frac{t}{t + N \left(v - \frac{5t}{4} - c\right)} = \alpha_4.$$