Credit Constraints, House Prices, and the Impact of Life Cycle Dynamics

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Abstract

How does the life cycle—namely, mortality risk and the expectation at birth of a rising age-profile of income and assets—impact house price dynamics? This paper investigates how equilibrium house prices respond to a tightening in credit constraints under two different but similarly calibrated models: one an infinite-horizon setting and the other a life-cycle environment. The main conclusion is that house price dynamics are magnified by the presence of life cycle features. Two primary explanations stand out: the distinction between stocks and flows of mortgage debt in the cross-section and the importance of gross housing tenure flows, i.e. churn.

Keywords: House Prices, Mortgage Debt, Credit Constraints, Life Cycle Models

JEL Classification Numbers: D15, D31, E21, E44, G11, G12, G21, R21, R31

1 Introduction

Events in global housing markets over the past 15 years have sparked intense interest in the determinants of house prices and their macroeconomic spillovers. However, with researchers coming at these issues from a variety of angles, no consensus has emerged regarding the choice between which of two canonical classes of model to employ: the infinite-horizon framework or a life-cycle environment. For topics that deal explicitly with age or demographics, life-cycle models are clearly necessary. However, for the

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broad umbrella of macroeconomics and housing as a whole, the question remains regarding how much the life cycle impacts housing dynamics, particularly prices.

Undoubtedly, many papers utilize a life-cycle macro-housing model, such as Corbae and Quintin (2015), Chambers, Garriga and Schladenhauf (2009), Li, Liu, Yang and Yao (2016), Bajari, Chan, Krueger and Miller (2013), Hatchondo, Martinez and Sánchez (2015), Ortalo-Magné and Rady (2006), and Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012), Favilukis, Ludvigson and Van Nieuwerburgh (2017). In some cases, the authors even study housing demand or other housing-related individual behavior over the life cycle. However, this paper sets out to study the impact of the life cycle itself—namely, mortality risk and the expectation at birth of a rising age-profile of income and assets—on the dynamic house price response to a tightening in borrowing limits relative to an infinite-horizon model. In that sense, this paper is most similar in spirit to Peterman and Sager (2018), who study the implication of life cycle motives for the optimal level of public debt. Also related is Wong (2018), who examines how demographics alter the transmission of monetary policy. Wong (2018) takes house prices as given, whereas the focus here is on equilibrium price dynamics.

The main finding in this paper is that life-cycle features magnify the decline in house prices after a tightening in down payment requirements relative to an infinite-horizon model, even when the two economies feature a nearly identical stock of housing wealth, liquid assets, and distribution of outstanding mortgage debt. However, behind these similarities, important differences between the two environments give clues about the source of amplification. First, the life cycle model features a thicker right tail in the flow of high-leverage new loans among borrowers lower down on the income scale. Because these borrowers are closest to the margin of renting and buying, their housing demand is most sensitive to credit constraints. Second, the life cycle model is characterized by significantly greater housing tenure churn between owning and renting. When credit tightens, rent-to-own transitions in the life-cycle model decline especially severely, which depresses housing demand and prices.
2 The Model Economy

The life-cycle and infinite-horizon economies both feature heterogeneous households, rental and owner-occupied markets for obtaining shelter, and competitive banks.

2.1 Households

Households have utility over consumption \( c \) and shelter \( s \) given by

\[
U(\{(c_t, s_t)\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} (\beta \rho)^t \left( c_t^{\omega} s_t^{1-\omega} \right)^{1-\sigma},
\]

where \( \rho \) is the survival probability. Shelter can be purchased as apartment space \( s = a \in [0, \bar{a}] \) each period at unit price \( p_a \) or enjoyed as a dividend from owner-occupied housing, \( s = h \in H \), which exists in fixed supply. Segmentation by quality, \( \bar{a} \leq \min H \equiv \bar{h} \), partially motivates the decision to buy rather than rent. Houses are traded in a decentralized market with search frictions, as in Hedlund (2016). Sellers of house \( h \) choose list price \( x_s \) and sell with decreasing probability \( \eta_s(x_s, h) \), while buyers choosing house \( h \) and bid price \( x_b \) buy with increasing probability \( \eta_b(x_b, h) \). In other words, there is a trade off between the gains from trade and expected search time. Households receive a stochastic income endowment \( e \cdot z \) with transitory \( e \in E \) drawn from \( F(e) \) and persistent \( z \in Z \) that follows \( \pi_z(z'|z) \). All households can save in one-period bonds \( b \) traded at price \( q \), while homeowners can also borrow using defaultable mortgages. In the stylized life cycle economy, households who die are replaced by newborns at the lowest state \( z \in Z \) with zero assets, debt, and housing.

After learning their endowment \( (e, z) \) at the beginning of the period, homeowners with cash at hand \( y \) (the sum of income and bonds \( b \)), house \( h \), and mortgage \( m \) first decide whether to list their house on the market and at which price \( x_s \). Selling outcomes are then realized, and remaining homeowners with mortgages decide whether to make a regular payment, refinance, or default. Besides losing their house,
defaulting borrowers also receive a credit flag \( f = 1 \) that excludes them from future borrowing until the flag disappears with probability \( 1 - \lambda_f \).

Agents entering the period without a house choose whether to continue renting or to search for a house to purchase. Prospective buyers with access to credit choose how much of the purchase to finance with debt vs. accumulated savings, while buyers with bad credit flags must pay entirely out of their existing cash at hand. The next section describes the mortgage lending environment and structure of loan contracts. Sections B.2 and B.3 provide the value functions and equilibrium conditions, respectively.

### 2.2 Banks

Competitive banks have access to external financing at interest rate \( r \). Mortgages are long-term contracts where default occurs in equilibrium. Thus, banks price each new mortgage \( m' \) individually according to the default risk of each borrower with state vector \( X = (m', b', h, z) \). At origination, the borrower receives resources \( q_m(X)m' \), and subsequently, the borrower makes payments \( l \leq l \equiv \frac{r_m}{1+r_m}m \) in excess of a minimum, interest-only payment each period. Borrowers roll over unpaid balances at rate \( r_m \). Borrowers can also refinance subject to a proportional origination cost \( \zeta \).

Banks repossess and sell the houses of defaulting borrowers, losing a proportion \( \chi \) of the proceeds to foreclosure costs. In the life cycle economy, if a borrower dies, the bank receives proceeds from an estate sale up to the value of outstanding debt. Section B.1 describes the recursive equilibrium mortgage pricing equation.

### 3 Calibration

The infinite horizon and life cycle calibrations both target the same key features of the mid-2000s U.S. economy, with some parameters set externally and the remainder determined jointly. The calibration details are provided in Hedlund (2018). Table 1 briefly compares the infinite horizon and life cycle economies.
Table 1: Steady State Model Comparison

<table>
<thead>
<tr>
<th>Description</th>
<th>Infinite Horizon</th>
<th>Life Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeownership Rate</td>
<td>68.1%</td>
<td>68.0%</td>
</tr>
<tr>
<td>Owner Housing Wealth/Income</td>
<td>3.95</td>
<td>3.99</td>
</tr>
<tr>
<td>Med Owner Liq Assets/Income</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td>Median Loan-to-Value (LTV)</td>
<td>60.3%</td>
<td>57.5%</td>
</tr>
<tr>
<td>Share with $LTV \geq 80%$</td>
<td>26.8%</td>
<td>24.4%</td>
</tr>
<tr>
<td>Share with $LTV \geq 90%$</td>
<td>9.9%</td>
<td>12.9%</td>
</tr>
<tr>
<td>Share with $LTV \geq 95%$</td>
<td>4.8%</td>
<td>8.4%</td>
</tr>
<tr>
<td>Foreclosure Rate</td>
<td>0.68%</td>
<td>0.43%</td>
</tr>
</tbody>
</table>

*Note:* The life cycle model has an annualized survival probability of $\rho = 0.975$, which implies an expected 40-year life span.

4 Results

As figure 1 clearly shows, the quantitative response of house prices to a tightening in credit constraints depends significantly on whether an infinite-horizon or life-cycle model is used. In both cases, house prices predictably decline, but the life cycle model exhibits a steeper initial drop and a lower long-run level. With the imposition of an 80% borrowing limit, the presence of life cycle features nearly doubles the initial fall in house prices from just over 5% to 10%. In the more extreme scenario of a 50% maximum loan-to-value at origination (which was common prior to World War II in the United States and is still present in several countries today), house prices fall by nearly 35% in the life-cycle model compared to just under 25% in the infinite-horizon model. Both declines are significant, but the impact of the life cycle is considerable.

4.1 Borrowing in the Cross Section: Stocks vs. Flows

Closer inspection of borrowing patterns gives some insight into the forces creating house price amplification in the life cycle model. The top left panel of figure 4 in the appendix demonstrates visually what table 1 summarized numerically—namely, that the distribution of debt is similar in the two economies. However, when broken down by borrower income, the distributions exhibit some important differences. Panels 2
and 3 in the top row show that, among low-income and middle-income households, there are a greater number of very highly leveraged borrowers in the life cycle economy.

Switching from stocks to flows of debt—a distinction that is absent in models with one-period loans—the middle and bottom rows reveal even larger contrasts in borrowing patterns between the two economies. Especially for new-purchase loans, the life-cycle economy features a significantly higher fraction of borrowers who take out mortgages with leverage above 90% or even 100% at origination.\(^1\) Precisely because these borrowers are the ones closest to the margin between buying and renting, housing demand is more susceptible to large declines after a tightening of credit constraints in the life-cycle economy compared to the infinite-horizon economy.

The fact that borrowers further down the income distribution are the ones holding higher-leverage mortgages is also important for explaining house price amplification in the life cycle model, as these households are more responsive to income shocks. Figure 2 shows how these households respond directly to tighter borrowing limits, ignoring the equilibrium feedback response from house prices. The panels in the left column show that average leverage at origination exhibits similar patterns between

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\(^1\) In the data, such loans gained considerable popularity during the recent housing boom, as explained by Herkenhoff and Ohanian (2015).
the infinite-horizon and life-cycle economies, with only modest amplification in the latter. However, the middle column shows that low-income households—who are the most responsive to credit when making housing tenure decisions—start with much higher leverage in the life cycle economy and experience a larger decline. In response, housing demand experiences a more significant drop. By contrast, the cutback in borrowing for middle-income homeowners is comparable between the two economies, even though they begin at different initial levels.

### 4.2 Gross Flows vs. Net Flows: The Importance of Churn

Beneath the calm surface of the initial equilibrium, figure 3 reveals significantly greater churn between renting and owning in the life-cycle model relative to the infinite-horizon model. Prior to any credit tightening, the entry of renters into homeownership and exit of owners into renting are both nearly twice as high in the life-cycle
After the imposition of a leverage cap, the flow of renters into owning collapses across the entire income distribution in the life-cycle model, whereas only high-income renters experience such a decline in the infinite-horizon economy. By contrast, the own-to-rent dynamics largely mirror each other between the two economies, with the exception of a temporary rise among middle-income homeowners in the infinite horizon model attributable mostly to higher foreclosures.\(^2\) This marked disruption of renters entering homeownership in the life-cycle model is consistent with the magnified borrowing responses described in the previous section.

5 Conclusions

For researchers interested in using quantitative models to analyze the intersection of macroeconomics and housing, these findings reveal that the choice of whether to utilize an infinite-horizon or life-cycle framework is not innocuous. Calibration

\(^2\)Foreclosures account for 60% of the peak own-rent transition rate in the infinite-horizon economy but only 30% for the life-cycle economy.
strategies that focus only on matching first-moments of household portfolio holdings or even second-moments of the stock of debt still leave room for the two classes of economies to exhibit markedly different dynamic behavior. Looking beyond the mechanics of structural modeling, the importance of life-cycle dynamics in this paper suggests that demographics—both with regard to fertility and longevity—may play a significant role not just in driving long-run housing market behavior, but also in shaping the short-run reaction to credit market shocks.

References


A Supplementary Figures

Figure 4: The loan-to-value distributions for outstanding mortgages, new originations, and new-purchase loans in the infinite-horizon and life-cycle economies.
B Supplemental Equations

B.1 Mortgage Pricing

The value of repossessing house \( h \) is given by

\[
J_{\text{REO}}(h) = \max_{x_s \in \emptyset \cup \mathbb{R}^+} \left[ \frac{(1 - \chi)x_s + [1 - \eta_s(x_s, h)]}{\text{prob of selling}} \right] \left[ -\delta ph \right] + \frac{1}{1 + r} J_{\text{REO}}(h)
\]  

(1)

where \( x_s = \emptyset \) indicates the choice to not list the house this period.

Competition between banks implies that \( q_m \) satisfies the recursive equation

\[
(1 + \zeta)q_m(X) = \frac{\rho}{1 + r_m} \mathbb{E} \left[ \eta_s(x'_s, h) + \left( 1 - \eta_s(x'_s, h) \right) \times \max \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right] + (1 - d')
\]

\times \left\{ \begin{array}{c}
1_{[\text{Refi]}} \\
1_{[\text{No Refi}]}
\end{array} \right\}

\left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\} + \left( 1 - d' \right)

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

\times \left\{ \begin{array}{c}
\left( \frac{1 - \rho}{1 + \rho} \min \left\{ \frac{J_{\text{REO}}(h)}{m'}, 1 \right\} \right)
\end{array} \right\}

(2)

where \( x'_s \) is the household’s list price choice and \( d' \in \{0, 1\} \) their default decision.

B.2 Selected Household Value Functions

Let \( X_{\text{own}} = (y, m, h, z) \) and \( X_{\text{rent}} = (y, z) \). Beginning-of-period owners solve

\[
W_{\text{own}}^{f=0}(X_{\text{own}}) = \max_{x_s \in \emptyset \cup \{x_s + y \geq m\}} \eta_s(x_s, h)W_{\text{rent}}^0(y + x_s - m, z) + [1 - \eta_s(x_s, h)]
\]

\times \max\{V_{\text{debt}}(X_{\text{own}}), V_{\text{own}}^0(y - m, h, z), W_{\text{rent}}^1(y, z)\}.

(3)

where \( x_s + y \geq m \) states that borrowers must pay off their mortgage upon selling.

Owners with \( f = 1 \) face a similar problem but have no debt to constrain their price.
Next, agents without a house decide whether to buy or continue renting,

\[
W^f_{\text{rent}}(X_{\text{rent}}) = \max_{h \in \emptyset \cup H, \ x_b \in B^f(h, z)} \eta_b(x_b, h) V^f_{\text{own}}(y - x_b, h, z) + [1 - \eta_b(x_b, h)] V^f_{\text{rent}}(X_{\text{rent}}). \tag{4}
\]

For buyers with access to credit, the set \(B^0(h, z) = \{x_b \in \mathbb{R}^+ : y - x_b \geq y(h, z)\}\), where \(y(h, z) < 0\) captures the ability to borrow using mortgages. Buyers with no credit access can only buy using cash at hand, i.e. \(B^1(h, z) = \{x_b \in \mathbb{R}^+ : y - x_b \geq 0\}\).

Owners with debt choose payment \(l \geq l^*\), bonds \(b'\), and consumption \(c\),

\[
V_{\text{debt}}(X_{\text{own}}) = \max_{\substack{l \geq l^*, b' \geq 0, \\ c \geq 0}} u(c, h) + \beta \rho \mathbb{E} W^0_{\text{own}}(X'_{\text{own}}) \text{ subject to }
\]

\[
c + \delta ph + qb' + l \leq y
\]

\[
X'_{\text{own}} = (e'z' + b', (m - l)(1 + r_m), h, z').
\]

Owners with \(f = 0\) but no debt choose loan \(m'\), bonds \(b'\), and consumption \(c\),

\[
V^0_{\text{own}}(y, h, z) = \max_{\substack{m' \geq 0, b' \geq 0, \\ c \geq 0}} u(c, h) + \beta \rho \mathbb{E} W^0_{\text{own}}(X'_{\text{own}}) \text{ subject to }
\]

\[
c + \delta ph + qb' \leq y + q_m(m', b', h, z)m'
\]

\[
q_m(m', b', h, z)m' \leq \vartheta ph
\]

\[
X'_{\text{own}} = (e'z' + b', m', h, z').
\]

where \(\vartheta\) is the maximum leverage. Owners with \(f = 1\) can save but not borrow.

Similarly, renters can only save but must also choose apartment \(a \in [0, \bar{a}]\).

### B.3 Equilibrium

An equilibrium consists of value functions and policy functions; mortgage prices \(q_m\); trading probabilities \(\eta_a\) and \(\eta_b\); a house price index \(p\); and household distributions \(\Phi_{\text{rent}}\) and \(\Phi_{\text{own}}\) such that agents optimize and the housing market clears. Given
\( \eta_s(x_s; p) \) and \( \eta_b(x_b; p) \), the price \( p \) equates the flow of housing from sellers to buyers,

\[
\int h^* \eta_b(x_b^*, h^*; p) d\Phi_{\text{rent}} = S_{\text{REO}}(p) + \int h \eta_s(x_s^*, h; p) d\Phi_{\text{own}},
\]

(7)

where \( S_{\text{REO}}(h) \) is the supply of bank-owned REO houses.\(^3\) The mapping from \( p \) to \( \eta_s(\cdot; p) \) and \( \eta_b(\cdot; p) \) emerges from directed search between brokers and sellers/buyers looking to trade. These brokers merely serve as passive conduits but are helpful for computational tractability. Hedlund (2016) provides more details, but here, it suffices to posit \( \eta_s(x_s; p) = \min \left\{ 1, \left( \frac{ph - x_s}{\kappa_s h} \right)^{\gamma_s} \right\} \) and \( \eta_b(x_b; p) = \min \left\{ 1, \left( \frac{x_b - ph}{\kappa_b h} \right)^{\gamma_b} \right\} \), which depend on seller/buyer choices \( x_s \) and \( x_b \) and parameters \( \gamma_s, \gamma_b, \kappa_s, \) and \( \kappa_b \).

\(^3\)As in standard models with capital, houses of different sizes are all lumped into one market and traded at a per-unit price. This assumption simplifies the analysis by allowing one \( p(\Phi) \) to clear the market instead of a separate \( p_h(\Phi) \) for each \( h \in H \).