

Learning in Advance Selling with Heterogeneous Consumers^{*}

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April 14, 2014

Abstract

The advance selling strategy is implemented when a firm offers consumers the opportunity to order its product in advance of the regular selling season. Advance selling reduces uncertainty for both the firm and the buyer and enables the firm to update its forecast of future demand. The distinctive feature of the present study of advance selling is that we divide consumers into two groups, experienced and inexperienced. Experienced consumers know their valuations of the product in advance, while inexperienced consumers learn their valuations only in the regular selling season. The presence of experienced consumers yields new insights. Specifically, pre-orders from experienced consumers lead to a more precise forecast of future demand by the firm. We show that the firm will always adopt advance selling and that the optimal pre-order price may be at a discount or a premium relative to the regular selling price.

Keywords: advance selling, the Newsvendor Problem, demand uncertainty, experienced consumers, inexperienced consumers, learning.

JEL codes: C72, D42, L12, M31.

^{*}Financial support from NET Institute www.NETinst.org is gratefully acknowledged. We thank Stanislav Kolenikov for his help on proofs.

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1 Introduction

Advance selling occurs when firms and retailers offer consumers the opportunity to order a product or service in advance of the regular selling season. Remarkable developments in the Internet and information technology have made advance selling an economically efficient strategy in many product categories.¹ Examples include new books, movies and CDs, software, electronic games, smart phones, travel services and vacation packages.

There are several major advantages of advance selling. First, advance selling reduces uncertainty for both the firm and the buyer, because advance orders are pre-committed. In situations where the firm needs to decide how much to produce (procure) prior to the regular selling season, advance orders reduce demand uncertainty. For the buyer, an advance order guarantees delivery of the product in the regular selling season, possibly at a discount to the retail price. Second, orders from advance selling may provide valuable information for the firm to better forecast the future demand. In particular, the firm may be able to update its forecast of the size of consumer pool and the distribution of consumers' valuations. Finally, advance selling may increase the overall demand. Indeed, when a consumer pre-orders the product, she commits to purchase it. In the absence of advance selling the same consumer will not purchase the product if she learns later her valuation is low.²

The motivation for the present study is based on two observations. First, some products were not made available for pre-orders when they were first introduced, but pre-orders became possible for later versions, with many experienced consumers placing their orders in advance of the regular selling season. Examples include Amazon Kindle, Harry Potter books, iPhone. This observation points to an important role played by experienced consumers. Second, although advance selling discounts are often practiced, some retailers set high prices for pre-orders and then cut their prices after the product release date. For highly sought-after products, some consumers (new technology lovers or loyal fans) are willing to pay a premium price for guaranteed delivery on the release date.

While inexperienced consumers learn more about their valuations of the product (maybe through inspection) when it becomes available, experienced consumers are likely to have a good idea about their valuations of the product in advance. Therefore, experienced consumers have less incentives to wait until the regular selling season. It follows that when there are experienced consumers, advance selling is more likely to be utilized by consumers. In addition, pre-orders from experienced consumers are more informative than those from inexperienced consumers. One can thus conclude that the presence of experienced consumers makes the first two of the

¹See empirical studies by Moe and Fader (2002, 2009), Hui, Eliashberg, and George (2008), and Sainam, Balasubramanian, and Bayus (2010).

²Similar points have been made by other authors, e.g., Xie and Shugan (2001).

aforementioned advantages of advance selling more pronounced.

A number of papers in the literature have emphasized some or all of the three advantages of advance selling (see the literature review in Section 2), but none have modeled experienced consumers and the role they play in advance selling. This paper is the first study of advance selling with inexperienced consumers as well as experienced consumers. Moreover, our paper yields the result that both advance selling at a discount and at a premium can arise in equilibrium.

Our model has two periods. The first is the advance selling season and the second is the regular selling season. In the first period, the firm chooses whether to make its product available for pre-orders, and if so, the pre-order price. There are two groups of consumers – experienced and inexperienced. Experienced consumers know their valuations of the product from the outset, while inexperienced consumers learn their valuations only in the second period. All consumers decide whether to pre-order the product (if this option is available) or wait until the regular selling season, in which they will face a probability of not being able to get the product (the stock-out probability). At the conclusion of the first period, the firm must choose its production quantity, which has to be at least the size of pre-orders. The product is delivered at the end of the second period.

Consumers are heterogeneous in their valuations, which are assumed to follow a normal distribution. The firm does not know the mean of this distribution. The group size of experienced consumers is fixed and known to the firm. However, the firm is uncertain about the number of inexperienced consumers.

In the second period the firm faces the Newsvendor Problem by analogy with the situation faced by a newsvendor who must decide how many copies of the day's paper to stock on a newsstand before observing demand, knowing that unsold copies will become worthless by the end of the day. If the produced quantity is greater than the realized demand, the firm must dispose of the remaining units at a loss (due to the salvage value being below the marginal cost). If the produced quantity is lower than the realized demand, the firm forgoes some profit.³

Our main research questions are the following. Will the firm adopt advance selling? If so, will an advance selling discount or a premium be offered? Can the firm learn from pre-orders? How are the answers to these questions affected by the parameters of the model, such as the salvage value, the demand uncertainty, the variation of consumer valuations, and the composition of experienced/inexperienced consumers in the population?

Our main results are summarized below.

- The firm has three possible pricing strategies for pre-orders: advance selling at the regular selling price (i.e., zero discount), advance selling at a discount price, and advance sell-

³If there is no uncertainty about the second-period demand or if the salvage value equals the marginal cost, then the Newsvendor Problem disappears.

ing at a premium price. Under advance selling at the regular selling price, experienced consumers do not wait until the regular selling season. Those with valuations above the advance selling price pre-order. All inexperienced consumers wait until the regular selling season.

- Under advance selling at a discount, the firm's optimal advance selling price is such that no consumer waits until the regular selling season. Specifically, experienced consumers whose valuations are above the advance selling price and all inexperienced consumers pre-order.
- Under advance selling at a premium, inexperienced consumers do not pre-order. For experienced consumers, those with high valuations pre-order, those with valuations below the regular selling price never buy the product, and the rest wait to purchase in the regular selling season.
- The firm learns from pre-orders. It learns whether there are any consumers who have chosen to wait until the regular selling season. If nobody waits, the firm only needs to fill all pre-orders. In the case when some consumers wait, the firm learns the mean of consumers' valuations.
- For the firm, advance selling is strictly better than no advance selling; the optimal advance selling price can be either at a discount or with a premium. Advance selling at a discount is more likely if the salvage value is small, the proportion of experienced consumers is small, the variation of consumer valuations is small, or the uncertainty about the number of inexperienced consumers is large.

Our paper contains several contributions to the literature on advance selling. First, as mentioned earlier, we study advance selling in a model with inexperienced as well as experienced consumers. We believe our model captures an important aspect of the advance selling phenomena. Second, learning by the firm in our model is not only on the consumer pool but also on the distribution of consumers' valuations of the product. Finally, like in the recent papers by Li and Zhang (2013) and Zhao and Pang (2011), the stock-out probability that consumers face when they wait until the regular selling season is endogenously determined in our model. In most of the literature, the stock-out probability has been modeled as exogenously given.

After the literature review (Section 2), the rest of the paper is organized as follows. In Section 3 we introduce the model. In Sections 4 (Advance Selling at a Discount) and 5 (Advance Selling at a Premium) we derive consumers' optimal purchasing decisions and the retailer's optimal production quantity. In Section 6 we study the optimal advance selling price. Concluding

remarks are provided in Section 7. Proofs of all lemmas and propositions, as well as derivations for some expressions and claims, are relegated to the Appendix.

2 Literature Review

Several strands of the literature have studied advance selling. One deals with advance selling from manufacturers to retailers, e.g., Cachon (2004) and Özer and Wei (2006). Another is on advance selling from firms and retailers to consumers under limited capacity, with applications to the airline and hotel industries (Gale and Holmes, 1993, Dana, 1998, Möller and Watanabe, 2010). The literature that is closest to the present study is on advance selling from firms and retailers to consumers without capacity constraints.

Our review below focuses on the third strand.⁴ Two modeling approaches have been adopted by researchers. In the first approach consumers are non-strategic in their decisions on whether to pre-order the product. In the second approach consumers are strategic.

Papers that model consumers as non-strategic include Weng and Parlar (1999), Tang, Rajaram, Alptekinoğlu, and Ou (2004), McCardle, Rajaram, and Tang (2004), and Chen and Parlar (2005). In all of these papers the fraction of consumers who place advance orders is an exogenously given decreasing function of the advance selling price.⁵ In Tang, Rajaram, Alptekinoğlu, and Ou (2004) and McCardle, Rajaram, and Tang (2004) there are two brands belonging to rival firms. Advance selling by a firm attracts customers of the other brand. The former paper examines the decision on advance selling by a single firm, while the latter focuses on competition between two firms in adopting the advance selling strategy. Another paper is Boyaci and Özer (2010) that considers advance selling in multiple periods and learning by the firm about current and future demands.

Several papers have treated consumers as strategic. Strategic consumers compare the options of ordering in advance and of waiting until the regular selling season. In the model by Xie and Shugan (2001) consumers are uncertain about their valuations in the advance selling season. This uncertainty makes advance selling profitable when it induces consumers who would otherwise not purchase to pre-order. Chu and Zhang (2011) allow the firm to control the release of information about the product at pre-order. They show that the seller may want to release some information or none, but never all. While in Xie and Shugan (2001) and Chu and Zhang (2011) consumers are homogeneous in the advance selling season (before their valuation uncertainty is resolved), in the studies cited below they are heterogeneous. Zhao and Steckel (2010) classify

⁴The main difference between the second and the third strands is that in situations of limited capacity firms mainly choose prices, while without capacity constraints firms choose their production quantities as well as prices.

⁵In Chen and Parlar (2005) an alternative model is considered, in which the probability that each consumer orders in advance is a beta-distributed random variable.

consumers according to whether they are loss averse. A loss averse consumer is more averse to a negative surplus (when the realized valuation is below the advance selling price) than is attracted to the equivalent positive surplus. Prasad, Stecke, and Zhao (2011) and Zhao and Pang (2011) divide consumers into two groups. The informed group consists of consumers who know about the option to buy in advance, while the uninformed group is not aware of this option.⁶ In Nocke, Peitz, and Rosar (2011) consumers differ in their expected valuations. In contrast to the above papers, Li and Zhang (2013) assume deterministic consumer valuations. Consumers with high valuations arrive in the advance selling season, consumers with low valuations arrive in the regular selling season.⁷

The major issues studied in this literature are (i) the Newsvendor Problem, and (ii) learning and updating by the firm about consumer demand. The Newsvendor Problem arises because the firm, facing uncertain demand, has to choose its production quantity prior to the regular selling season. Obviously, learning from pre-orders benefits the firm because it helps to better forecast the demand in the regular selling season. Both issues are also central in our paper. Because we assume that the mean of the distribution of consumers' valuations is unknown to the retailer, learning in our model is not only on the consumer pool, but also on the distribution of consumers' valuations.

Consumers in our model are strategic. The key differences between our paper and extant literature on advance selling with strategic consumers are the introduction of experienced consumers in the model setup and the study of learning from advance orders.

3 Model Setup

Consider a firm or a retailer who sells a product over two periods.⁸ The first period is the advance selling season and the second period is the regular selling season. Any consumer who pre-orders in the first period is guaranteed delivery of the product in the second period. Those who do not pre-order can buy in the regular selling season, but there is a risk that the product will be out of stock. There are two groups of consumers – experienced and inexperienced. Experienced consumers are assumed to know their valuations of the product from the outset, whereas

⁶We would like to emphasize that experienced/inexperienced consumers in the present study do not correspond to informed/uninformed consumers in Prasad, Stecke, and Zhao (2011) and Zhao and Pang (2011). All consumers in the present study – experienced and inexperienced – are informed about the option to buy in advance. Informed consumers in Prasad, Stecke, and Zhao (2011) and Zhao and Pang (2011) are inexperienced, as their valuation uncertainty is resolved only in the regular selling season.

⁷Xie and Shugan (2001), Li and Zhang (2013), and Zhao and Pang (2011) will be further discussed in Section 5 that considers advance selling at a price premium.

⁸From now on we shall speak of the firm as the retailer.

Table 1: Notation

Parameters	
c	marginal cost
s	salvage value
p	price in the regular selling season
m_e	number of experienced consumers
Random variables	
$v \mu \sim N(\mu, \sigma^2)$	distribution of consumers' valuations, cdf $F_{v \mu}(v) = \Phi\left(\frac{v-\mu}{\sigma}\right)$
$\mu \sim N(\mu_0, \psi^2)$	distribution of μ , cdf $F_\mu(\mu) = \Phi\left(\frac{\mu-\mu_0}{\psi}\right)$
$v \sim N(\mu_0, \sigma^2 + \psi^2)$	unconditional distribution of consumers' valuations, cdf $F_v(v) = \Phi\left(\frac{v-\mu_0}{\sqrt{\sigma^2+\psi^2}}\right)$
$M_i \sim \text{LN}(\nu_i, \tau_i^2)$	number of inexperienced consumers, cdf $G(M_i) = \Phi\left(\frac{\ln M_i - \nu_i}{\tau_i}\right)$ and mean $m_i = \exp\left\{\nu_i + \frac{\tau_i^2}{2}\right\}$
Decision variables	
q	quantity produced for the regular selling season
Q	total quantity produced (including pre-orders)
x	advance selling price
Other notation	
D_1, D_2	demands in the advance and regular selling seasons
π	retailer's expected profit from the regular selling season
Π	retailer's total expected profit (includes pre-orders)
η	stock-out probability
β	$= \frac{p-c}{p-s}$
\bar{F}	$= 1 - F$

inexperienced consumers learn their valuations only in the second period.⁹ Each consumer is willing to buy at most one unit of the product.

The number of experienced consumers is exogenously given, denoted by m_e .¹⁰ The number of inexperienced consumers, M_i is a random variable; the distribution of M_i is lognormal

⁹ As discussed in the Introduction, experienced consumers may be taken as those who have purchased earlier versions of the product in the past, so it is reasonable to assume that they know their willingness to pay for the new version.

¹⁰ The non-randomness of m_e follows from footnote 9.

$\text{LN}(\nu_i, \tau_i^2)$ with the mean $m_i = \exp\{\nu_i + \tau_i^2/2\}$.¹¹ Both m_e and the distribution of M_i are common knowledge.

Consumers' valuations of the product are i.i.d. normal with mean μ and variance σ^2 .¹² The retailer and consumers do not know μ . We model this uncertainty by assuming that μ is drawn from the normal distribution with mean μ_0 and variance ψ^2 . That is,

$$v|\mu \sim N(\mu, \sigma^2)$$

and

$$\mu \sim N(\mu_0, \psi^2).$$

In the Appendix we show that the unconditional distribution of v is

$$v \sim N(\mu_0, \sigma^2 + \psi^2). \quad (1)$$

We denote by $F_{v|\mu}(v)$, $f_{v|\mu}(v)$, $F_\mu(\mu)$, $f_\mu(\mu)$, $F_v(v)$, and $f_v(v)$ the corresponding cumulative distribution and density functions.

The marginal production cost is c and the price during the regular selling season is p .¹³ For each unsold unit of the product at the end of the regular selling season the retailer gets its salvage value s . We assume $s < c < p$.

The retailer decides on the advance selling price x in the beginning of the advance selling season. After the conclusion of the advance selling season the retailer must decide how much to produce. Let D_1 denote the number of consumers who buy in the advance selling season. Then the retailer's quantity choice is $Q = D_1 + q$, where D_1 fulfills the pre-orders. Quantity q satisfies the (stochastic) demand during the regular selling season, denoted by D_2 .

Table 1 lists the notation introduced above and also some of the notation introduced later. Figure 1 displays the timeline of the model. In the beginning of the first period, Nature determines μ and all experienced consumers learn their valuations. The retailer announces the advance selling price x (together with the regular selling price p). Each consumer then decides whether to pre-order. At the end of the first period, the retailer observes the number of pre-orders D_1 , updates his forecast of the second-period demand D_2 and chooses his production quantity Q . During the second period, all inexperienced consumers learn their valuations and those consumers who did not pre-order then decide whether to purchase the product at price p .

¹¹We use a lognormal distribution to avoid negative realizations of the number of inexperienced consumers.

¹²One may argue that the valuation distributions are different between experienced and inexperienced consumers. Section 7 (Concluding Remarks) has further discussion of this assumption.

¹³In our model p is exogenously given. In Prasad, Stecke, and Zhao (2010) and Zhao and Stecke (2010), p is called the market (spot) price. Among the papers reviewed in Section 2, most treat the regular selling season price as exogenous. In the Concluding Remarks section we have a discussion on endogenous p .

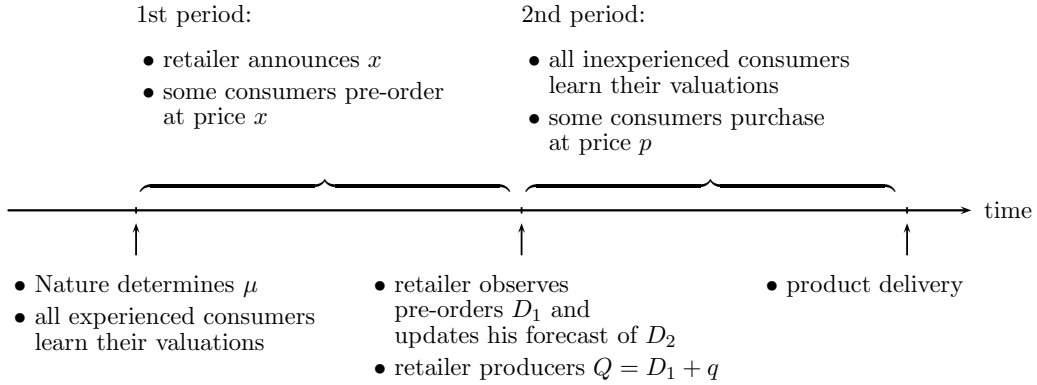


Figure 1: Timeline of the model

The product is delivered at the end of the second period.

4 Advance Selling at a Discount

In this section we consider $x \leq p$ so as to focus on pre-order discounting.¹⁴ We first derive consumers' optimal purchasing decisions. We then examine how the retailer learns from pre-orders and chooses his production quantity. Finally, we calculate the endogenous stock-out probability and present the retailer's expected profit as a function of the advance selling price x .

4.1 Consumers' Optimal Purchasing Decisions

Since experienced consumers know their valuations from the outset, they never wait until the regular selling season. Experienced consumers with valuations above x pre-order the product and pay the discounted price $x \leq p$.

Inexperienced consumers do not know their valuations in the advance selling season. An inexperienced consumer has two options. The first is to pre-order and pay x . In this case the consumer's expected payoff is

$$E[v] - x = \mu_0 - x.$$

The other option is to wait until the regular selling season. The consumer learns her valuation v and purchases the product (provided it is in stock) if $v \geq p$. Her expected payoff is

$$(1 - \eta) \int_p^{+\infty} (v - p) f_v(v) dv,$$

¹⁴We will refer to the case $x = p$ as zero discount.

where η is the stock-out probability. The stock-out probability is the probability that the consumer will not be able to get the product when she actually wants to purchase it. We use the unconditional density of v , $f_v(v)$, in the integration because the consumer does not know μ .¹⁵

Thus, inexperienced consumers pre-order if and only if

$$\mu_0 - x \geq (1 - \eta) \int_p^{+\infty} (v - p) f_v(v) dv,$$

or, equivalently,

$$x \leq \hat{x} \equiv \mu_0 - (1 - \eta) \int_p^{+\infty} (v - p) f_v(v) dv.$$

The explicit expression for the threshold value \hat{x} (derived in the Appendix) is

$$\hat{x} = \mu_0 - (1 - \eta) \left((\mu_0 - p) \bar{F}_v(p) + (\sigma^2 + \psi^2) f_v(p) \right), \quad (2)$$

where $\bar{F}_v = 1 - F_v$.

Lemma 1 (Properties of \hat{x}). *The threshold value \hat{x} , given by (2), has the following properties:*

- (i) $\partial \hat{x} / \partial \sigma < 0$;
- (ii) $\partial \hat{x} / \partial \psi < 0$;
- (iii) $\partial \hat{x} / \partial \eta > 0$.

The properties in this lemma stipulate that inexperienced consumers are less likely to pre-order the more uncertain they are about their valuations (higher σ^2 and/or ψ^2) and the smaller the stock-out probability η is.

For the rest of our analysis we will assume $\hat{x} < p$.¹⁶ We consider the following two regions for the advance selling price x .

- **Region A:** $x \leq \hat{x}$. All inexperienced consumers pre-order.
- **Region B:** $\hat{x} < x \leq p$. All inexperienced consumers wait until the second period.

¹⁵In the next subsection we show that the retailer learns the demand parameter μ from observing the number of pre-orders D_1 . Thus, his optimal choice of q becomes a function of μ (Section 4.3). In Lemma 2 we show that the endogenously determined stock-out probability η ends up being the same for all realizations of μ . This allows us to keep things simple in the above formula.

¹⁶Obviously, if $\mu_0 < p$ then $\hat{x} < p$ holds.

4.2 Learning by the Retailer from Pre-orders

When the advance selling price x is in region A, experienced consumers with valuations above x and all inexperienced consumers pre-order. No consumer will wait until the regular selling season. As the retailer's forecast of the second-period demand is $D_2 = 0$, he produces $Q = D_1$.

When x is in region B, the retailer learns μ from observing the first-period demand. Specifically,

$$D_1 = m_e \text{Prob}(v > x|\mu) = m_e \bar{F}_{v|\mu}(x) = m_e \left(1 - \Phi \left(\frac{x - \mu}{\sigma} \right) \right),$$

from which μ can be deduced. The retailer produces $Q = D_1 + q$, where q satisfies the second-period demand that is comprised of inexperienced consumers with valuations above p ,

$$D_2 = M_i \text{Prob}(v > p|\mu) = M_i \bar{F}_{v|\mu}(p).$$

Because $M_i \sim \text{LN}(\nu_i, \tau_i^2)$, it is straightforward to show that

$$D_2 \sim \text{LN}(\nu_i + \ln \bar{F}_{v|\mu}(p), \tau_i^2).$$

4.3 Optimal Value of q

In light of the results of Subsection 4.2, it remains to find the optimal q for

$$D_2 \sim \text{LN}(\nu_i + \ln \bar{F}_{v|\mu}(p), \tau_i^2).$$

For any D_2 , if q units are produced then $\min\{q, D_2\}$ units are sold and $(q - D_2)^+ = \max\{q - D_2, 0\}$ are salvaged. The retailer's expected profit from the second period, denoted by π , is

$$\pi(q) = p \text{E}[\min\{q, D_2\}] + s \text{E}[(q - D_2)^+] - cq. \quad (3)$$

The problem of maximizing (3) is the Newsvendor Problem, well-known in the operations management literature. Using the fact that $\min\{q, D_2\} = D_2 - (D_2 - q)^+$, we can rewrite the retailer's expected profit as

$$\pi(q) = \text{E}[D_2](p - c) - \text{E}[(D_2 - q)^+](p - c) - \text{E}[(q - D_2)^+](c - s).$$

The optimal value of q , therefore, minimizes the expected underage and overage costs

$$\text{E}[(D_2 - q)^+](p - c) + \text{E}[(q - D_2)^+](c - s).$$

The first-order condition is

$$\text{Prob}(D_2 \leq q^*) = \beta, \quad (4)$$

where

$$\beta \equiv \frac{p - c}{p - s}.$$

It is clear that q^* selected this way increases in β and therefore increases in the per unit underage cost $p - c$ and decreases in the per unit overage cost $c - s$.

For the lognormal distribution $D_2 \sim \text{LN}(\nu, \tau^2)$ the optimal production quantity is given by

$$q^* = \exp\{\nu + \tau z_\beta\} \quad (5)$$

and

$$\pi^* \equiv \pi(q^*) = (p - s) (1 - \Phi(\tau - z_\beta)) \exp\left\{\nu + \frac{\tau^2}{2}\right\}, \quad (6)$$

where z_β is the β -th percentile of the standard normal distribution, $z_\beta \equiv \Phi^{-1}(\beta)$. See the Appendix for derivations of (5) and (6).

Applying (5) and (6) to $D_2 \sim \text{LN}(\nu_i + \ln \bar{F}_{v|\mu}(p), \tau_i^2)$, the optimal q , denoted by $q^*(\mu)$, and the resulting expected profit are

$$q^*(\mu) = \exp\{\nu_i + \tau_i z_\beta\} \bar{F}_{v|\mu}(p) \quad (7)$$

and

$$\pi^*(\mu) \equiv \pi(q^*(\mu)) = (p - s) (1 - \Phi(\tau_i - z_\beta)) m_i \bar{F}_{v|\mu}(p). \quad (8)$$

4.4 Stock-out Probability

In this subsection we determine the equilibrium value of the stock-out probability η .

Given D_2 , the (conditional) probability of any consumer who wants to purchase the product in the regular selling season but is unable to get it is the fraction of excess demand. That is,

$$\text{Prob}(\text{stock-out}|D_2) = \left(\frac{D_2 - q^*}{D_2}\right)^+ = \begin{cases} 0, & D_2 \leq q^*, \\ \frac{D_2 - q^*}{D_2}, & D_2 > q^*. \end{cases}$$

Hence, the stock-out probability is the expected value of this expression over the distribution of D_2 ,

$$\eta = \text{E} \left[\left(\frac{D_2 - q^*}{D_2}\right)^+ \right].$$

For $D_2 \sim \text{LN}(\nu, \tau^2)$,

$$\eta = \int_{q^*}^{+\infty} \frac{D_2 - q^*}{D_2} g(D_2) \, dD_2,$$

where $g(D_2)$ is the density function of $\text{LN}(\nu, \tau^2)$ and q^* is given in (5). The explicit expression for the stock-out probability is obtained in the Appendix:

$$\eta = 1 - \beta - \exp\left\{\tau z_\beta + \frac{\tau^2}{2}\right\} (1 - \Phi(z_\beta + \tau)). \quad (9)$$

Note that this expression is independent of ν . Accordingly, η that results from

$$D_2 \sim \text{LN}(\nu_i + \ln \bar{F}_{v|\mu}(p), \tau_i^2)$$

is the same for all μ . This finding is presented in Lemma 2.

Lemma 2 (Stock-out probability η). *The stock-out probability under the second-period demand $D_2 \sim \text{LN}(\nu_i + \ln \bar{F}_{v|\mu}(p), \tau_i^2)$ is the same for all realizations of μ and is given by*

$$\eta = 1 - \beta - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} (1 - \Phi(z_\beta + \tau_i)).$$

It is worthwhile to note that the stock-out probability in our model is endogenously determined, because q^* is the optimal choice. It follows from Lemma 2 that $\eta < 1 - \beta$. This is expected as the first-order condition (4) implies $1 - \beta = \text{Prob}(D_2 > q^*)$. The stock-out probability η is smaller than $\text{Prob}(D_2 > q^*)$ because when $D_2 > q^*$ any consumer who wants to purchase the product in the regular selling season will get it with a positive probability.

Lemma 3 (Properties of η). *The stock-out probability $\eta = \eta(\beta, \tau_i)$, given in Lemma 2, has the following properties:*

- (i) $\partial\eta/\partial\tau_i > 0$, $\eta(\beta, 0) = 0$, and $\lim_{\tau_i \rightarrow +\infty} \eta(\beta, \tau_i) = 1 - \beta$;
- (ii) $\partial\eta/\partial\beta < 0$, $\eta(0, \tau_i) = 1$, and $\eta(1, \tau_i) = 0$.

Combining the results of Lemma 1(iii) and Lemma 3 we can conclude that the threshold value \hat{x} increases in τ_i and decreases in β .

4.5 The Retailer's Expected Profit

We can now write the retailer's expected total profit Π as a function of the advance selling price x . The part of the retailer's expected profit that comes from experienced consumers equals

$$m_e \mathbb{E}_\mu [\text{Prob}(v > x|\mu)] (x - c) = m_e \mathbb{E}_\mu [\bar{F}_{v|\mu}(x)] (x - c),$$

as experienced consumers never wait until the regular selling season and only those with valuations above x purchase the product in the advance selling season. In the Appendix we show that

$$\mathbb{E}_\mu [\bar{F}_{v|\mu}(x)] = \bar{F}_v(x). \quad (10)$$

The retailer's expected profit from experienced consumers can, therefore, be written as

$$m_e \bar{F}_v(x)(x - c). \quad (11)$$

The purchasing behavior of inexperienced consumers depends on the region x belongs to. If $x \leq \hat{x}$ (region A), then all inexperienced consumers pre-order. Hence,

$$\Pi^A(x) = m_e \bar{F}_v(x)(x - c) + m_i(x - c). \quad (12)$$

Next, consider $\hat{x} < x \leq p$ (region B). All inexperienced consumers wait until the second period. Since the retailer learns μ , his expected profit from inexperienced consumers is $\mathbb{E}_\mu [\pi^*(\mu)]$, where $\pi^*(\mu)$ is given by (8). Hence,

$$\Pi^B(x) = m_e \bar{F}_v(x)(x - c) + \mathbb{E}_\mu [\pi^*(\mu)].$$

Let us calculate $\mathbb{E}_\mu [\pi^*(\mu)]$:

$$\begin{aligned} \pi^* &\equiv \mathbb{E}_\mu [\pi^*(\mu)] = \mathbb{E}_\mu [(p - s)(1 - \Phi(\tau_i - z_\beta)) m_i \bar{F}_{v|\mu}(p)] \\ &= (p - s)(1 - \Phi(\tau_i - z_\beta)) m_i \mathbb{E}_\mu [\bar{F}_{v|\mu}(p)]. \end{aligned}$$

The term $\mathbb{E}_\mu [\bar{F}_{v|\mu}(p)]$ equals $\bar{F}_v(p)$ (see (10)), so we have

$$\pi^* = (p - s)(1 - \Phi(\tau_i - z_\beta)) m_i \bar{F}_v(p).$$

The retailer's expected total profit in region B is, therefore,

$$\begin{aligned} \Pi^B(x) &= m_e \bar{F}_v(x)(x - c) + \pi^* \\ &= m_e \bar{F}_v(x)(x - c) + (p - s)(1 - \Phi(\tau_i - z_\beta)) m_i \bar{F}_v(p). \end{aligned} \quad (13)$$

Combining (12) and (13) yields the retailer's expected total profit as a function of the advance selling price x :

$$\Pi(x) = \begin{cases} \Pi^A(x), & x \leq \hat{x}, \\ \Pi^B(x), & \hat{x} < x \leq p. \end{cases}$$

4.6 Optimal Advance Selling Discount Price

For the rest of our analysis we focus on the case $\hat{x} > c$.¹⁷ Furthermore, we make the following simplifying assumption.

Assumption 1. *The function*

$$\bar{F}_v(x)(x - c)$$

increases in x on $[c, p]$.

This assumption implies that the expected profit from experienced consumers, given by (11), is an increasing function of x for all $x \in [c, p]$.¹⁸ It follows that, as far as experienced consumers are concerned, the retailer has no incentives to offer an advance selling discount. Accordingly, if discounting for pre-orders is offered it must be due to the presence of inexperienced consumers.

It is easy to see that under Assumption 1 the retailer's expected profit $\Pi(x)$ increases in x in each of the two regions A and B. This does not imply, however, that $\Pi(x)$ increases in x on $[c, p]$, as $\Pi(x)$ can jump down at $x = \hat{x}$. A jump down occurs if and only if $\Pi^A(\hat{x}) > \Pi^B(\hat{x})$.¹⁹ That is,

$$m_i(\hat{x} - c) > \pi^* = (p - s)(1 - \Phi(\tau_i - z_\beta))\bar{F}_v(p)$$

(see (12) and (13)). Hence, we have the following two possible patterns for $\Pi(x)$, which are depicted in Figure 2:

- Pattern 1: $\Pi(x)$ jumps down at \hat{x} .
- Pattern 2: $\Pi(x)$ jumps up at \hat{x} .

Regarding the optimal advance selling price, under pattern 1 it is either \hat{x} or p and under pattern 2 it is p . Advance selling at p corresponds to zero discount for pre-orders.

Proposition 1 (Optimal advance selling discount price). *The optimal advance selling discount price is either \hat{x} or p .*

The two potentially optimal advance selling prices reflect two different tradeoffs for the retailer: between low price-high sales and high price-low sales, and between low price-low expected overage and underage costs and high price-high expected overage and underage costs. Under either price, experienced consumers only buy in the advance selling season. Hence, the

¹⁷If $\hat{x} < c$, then the optimal advance selling price will be p , as will become clear from Proposition 1 and the discussion preceding it.

¹⁸As it can be shown that $\bar{F}_v(x)(x - c)$ is single-peaked, Assumption 1 holds as long as p is less than or equal to the value at which this function peaks. The latter value is the optimal price for a monopolist who sells in only one period and faces consumers with valuations that follow the distribution function $F_v(x)$.

¹⁹Since $\Pi_B(x)$ is defined on $(\hat{x}, p]$, $\Pi^B(\hat{x})$ is the limiting value.

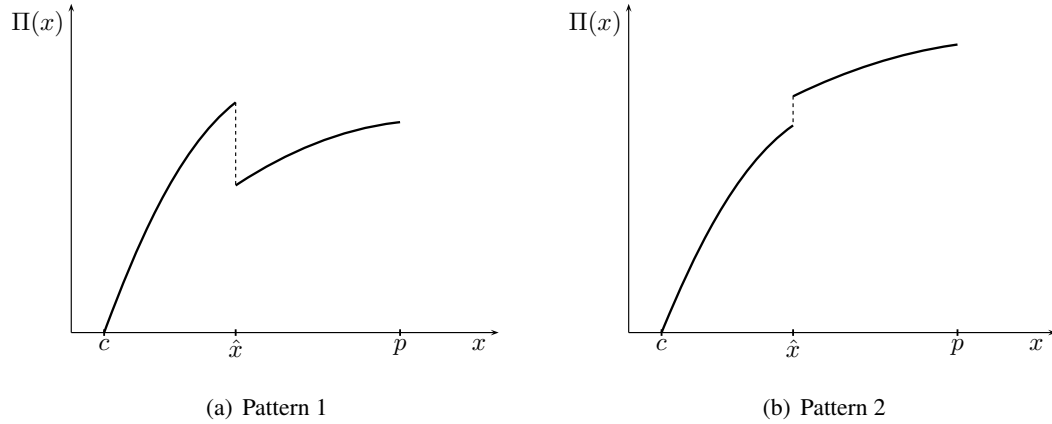


Figure 2: The retailer's expected profit as a function of x

tradeoff for the retailer from this group of consumers is only between a lower price and therefore higher expected sales and a higher price and lower expected sales.

For the inexperienced group of consumers, all pre-order at \hat{x} ;²⁰ none pre-order at p and only those with realized values above p will buy in the regular selling season. Hence, both tradeoffs described above are present here. First, \hat{x} corresponds to higher sales and a lower price, while p corresponds to much lower sales and a higher price. Second, \hat{x} means zero overage and underage costs, while p leads to positive expected overage and underage costs.

It is worth noting that advance selling at p is not equivalent to not offering advance selling, as the former pricing strategy allows the retailer to learn μ . In fact, it is straightforward to show that the retailer's expected profit under advance selling at p is strictly higher than under no advance selling. Hence, advance selling (at a discount) is always superior to no advance selling for the retailer.

5 Advance Selling at a Premium

In this section we consider $x > p$ so as to explore advance selling at a price above the regular selling price.

Since $\hat{x} < p < x$, inexperienced consumers always wait until the regular selling season. Each experienced consumer compares her payoff from pre-order, $v - x$, with her expected payoff from waiting until the regular selling season, $(1 - \eta)(v - p)$. Later in this section we

²⁰The point that the retailer can benefit from selling to inexperienced consumers before they learn their valuations was originally made by Courty (2003).

show that the endogenously determined η depends on μ and x , $\eta = \eta(\mu, x)$.²¹ Experienced consumers observe their valuations, but not μ , so they use $E_{\mu|v}[\eta(\mu, x)]$ in their calculations. In the Appendix we show that the random variable $\mu|v$ is distributed according to

$$\mu|v \sim N\left(\frac{\sigma^2\mu_0 + \psi^2v}{\sigma^2 + \psi^2}, \frac{\sigma^2\psi^2}{\sigma^2 + \psi^2}\right). \quad (14)$$

Let $\hat{v}(x)$ denote the solution to

$$v - x = (1 - E_{\mu|v}[\eta(\mu, x)])(v - p).$$

That is, all experienced consumers with valuations $v \geq \hat{v}(x)$ pre-order the product.

The stock-out probability η is crucial for positive demand at a premium price. In most theoretical papers on advance selling with strategic consumers there is no stock-out risk either because the aggregate market demand is deterministic, or because $s = c$. As a result, the advance selling price should be below the regular selling price to induce some of the consumers to place advance orders. The exceptions are Xie and Shugan (2001), Li and Zhang (2013), and Zhao and Pang (2011). Xie and Shugan (2001) analyze the seller's optimal pricing strategy under limited capacity and show that it is possible to have advance selling prices that are higher than spot prices. The reason is the same as in our paper: advance buyers must consider both the spot price and the likelihood of unavailable capacity. Li and Zhang (2013) assume uncertain market with heterogeneous consumers. Consumers with the high valuation v_H arrive in the advance selling season, while consumers with the low valuation v_L arrive in the regular selling season. In their model the regular selling price is equal to v_L and the advance selling price is at a premium. Advance selling discounts do not arise in equilibrium. In Zhao and Pang (2011) consumers also arrive in two different periods: informed consumers – in the advance selling season, uninformed – in the regular selling season. The two types of consumers draw their valuations from the same distribution. Because the model includes the Newsvendor Problem (stochastic market size and inventory risk), the probability of a stock-out is positive, making advance selling at a price premium possible.²²

The retailer learns μ from observing the first-period demand. Specifically,

$$D_1 = m_e \text{Prob}(v > \hat{v}(x)|\mu) = m_e \bar{F}_{v|\mu}(\hat{v}(x)) = m_e \left(1 - \Phi\left(\frac{\hat{v}(x) - \mu}{\sigma}\right)\right),$$

from which μ can be inferred. The rest of the experienced consumers with valuations above p

²¹This is contrast to the result in Lemma 2 for the case of advance selling at a discount.

²²Prasad, Stecke, and Zhao (2010) and Zhao and Stecke (2010) restrict the retailer's strategy to advance selling discounts, although in these models price premiums can induce strictly positive demand in the advance selling season.

(i.e., experienced consumers with $p < v < \hat{v}(x)$) and inexperienced consumers with valuations above p constitute the demand during the regular selling season:

$$D_2 = m_e (\bar{F}_{v|\mu}(p) - \bar{F}_{v|\mu}(\hat{v}(x))) + M_i \bar{F}_{v|\mu}(p).$$

Because the first term of D_2 is non-random and

$$M_i \bar{F}_{v|\mu}(p) \sim \text{LN}(\nu_i + \ln \bar{F}_{v|\mu}(p), \tau_i^2),$$

D_2 is a shifted lognormal distribution. The retailer produces

$$Q = D_1 + m_e (\bar{F}_{v|\mu}(p) - \bar{F}_{v|\mu}(\hat{v}(x))) + q,$$

where q satisfies the random part of D_2 , $M_i \bar{F}_{v|\mu}(p)$. The optimal value of q is given by (7), that is,

$$q^*(\mu) = \exp\{\nu_i + \tau_i z_\beta\} \bar{F}_{v|\mu}(p).$$

Let us now calculate the stock-out probability. If the realization of $M_i \bar{F}(p)$ is greater than q^* , then $M_i \bar{F}(p) - q^*$ of the consumers who constitute D_2 will not get the product. Hence,

$$\eta(\mu, x) = \mathbb{E}_{M_i} \left[\left(\frac{M_i \bar{F}_{v|\mu}(p) - q^*}{D_2} \right)^+ \right].$$

The system of equations

$$\begin{cases} \eta(\mu, x) = \mathbb{E}_{M_i} \left[\left(\frac{M_i \bar{F}_{v|\mu}(p) - \exp\{\nu_i + \tau_i z_\beta\} \bar{F}_{v|\mu}(p)}{m_e (\bar{F}_{v|\mu}(p) - \bar{F}_{v|\mu}(\hat{v}(x))) + M_i \bar{F}_{v|\mu}(p)} \right)^+ \right] \\ \hat{v}(x) - x = (1 - \mathbb{E}_{\mu|\hat{v}(x)}[\eta(\mu, x)]) (\hat{v}(x) - p) \end{cases}$$

implicitly defines $\hat{v}(x)$ and $\eta(\mu, x)$.

Lemma 4 (Expected profit from a premium price). *Advance selling at a premium price x yields the expected profit*

$$\Pi(x) = m_e \bar{F}_v(\hat{v}(x))(x - p) + \Pi(p) \quad (15)$$

to the retailer.

The derivation of (15) is straightforward. Advance selling at a price premium allows the retailer to learn μ , but uncertainty about the number of inexperienced consumers, M_i , who wait until the regular selling season, remains. The same applies to advance selling at p . Under either

x or p the retailer produces the quantity

$$Q = m_e \bar{F}_{v|\mu}(p) + q^*(\mu).$$

The difference between advance selling at x and at p is that, under x , $m_e \bar{F}_{v|\mu}(\hat{v}(x))$ of the total number of transactions, $m_e \bar{F}_{v|\mu}(p) + \min\{q^*(\mu), M_i \bar{F}_{v|\mu}(p)\}$, occur at a premium price, while under p all transactions occur at price p . Hence,

$$\Pi(x) - \Pi(p) = E_\mu [m_e \bar{F}_{v|\mu}(\hat{v}(x))(x - p)] = m_e \bar{F}_v(\hat{v}(x))(x - p).$$

It follows from (15) that $\Pi(x) > \Pi(p)$ for any $x > p$, i.e, the retailer is better off selling in advance at a price premium than at p . Observe that $\Pi(x)$ converges to $\Pi(p)$ as x approaches p from above, and it can be shown that $\Pi(x)$ converges to $\Pi(p)$ as $x \rightarrow \infty$. Hence, $\Pi(x)$ must attain a maximum value at some x , $p < x < \infty$. Denote this optimal advance selling premium price by x^p . Hence, we have the following proposition.

Proposition 2 (Optimal advance selling premium price). *The optimal advance selling premium price x^p exists and is strictly greater than p .*

Recall the two tradeoffs discussed following Proposition 1. In comparing advance selling at x^p and at p , the first tradeoff goes in favor of advance selling at x^p , while the expected overage and underage costs (the second tradeoff) are the same. As a result, advance selling at x^p dominates advance selling at p .

6 Equilibrium Analysis

In this section we analyze the retailer's optimal advance selling price by combining the separate analyses of advance selling at a discount (Section 4) and at a premium (Section 5).

6.1 Optimal Advance Selling Price

The retailer chooses x that maximizes

$$\Pi(x) = \begin{cases} \Pi^A(x), & x \leq \hat{x}, \\ \Pi^B(x), & \hat{x} < x \leq p, \\ \Pi^C(x), & x > p, \end{cases}$$

where $\Pi^A(x)$ is given in (12), $\Pi^B(x)$ in (13), and $\Pi^C(x)$ is the retailer's expected profit when a premium price is charged for pre-orders (i.e., $\Pi^C(x)$ is given in (15)). Combining the results of

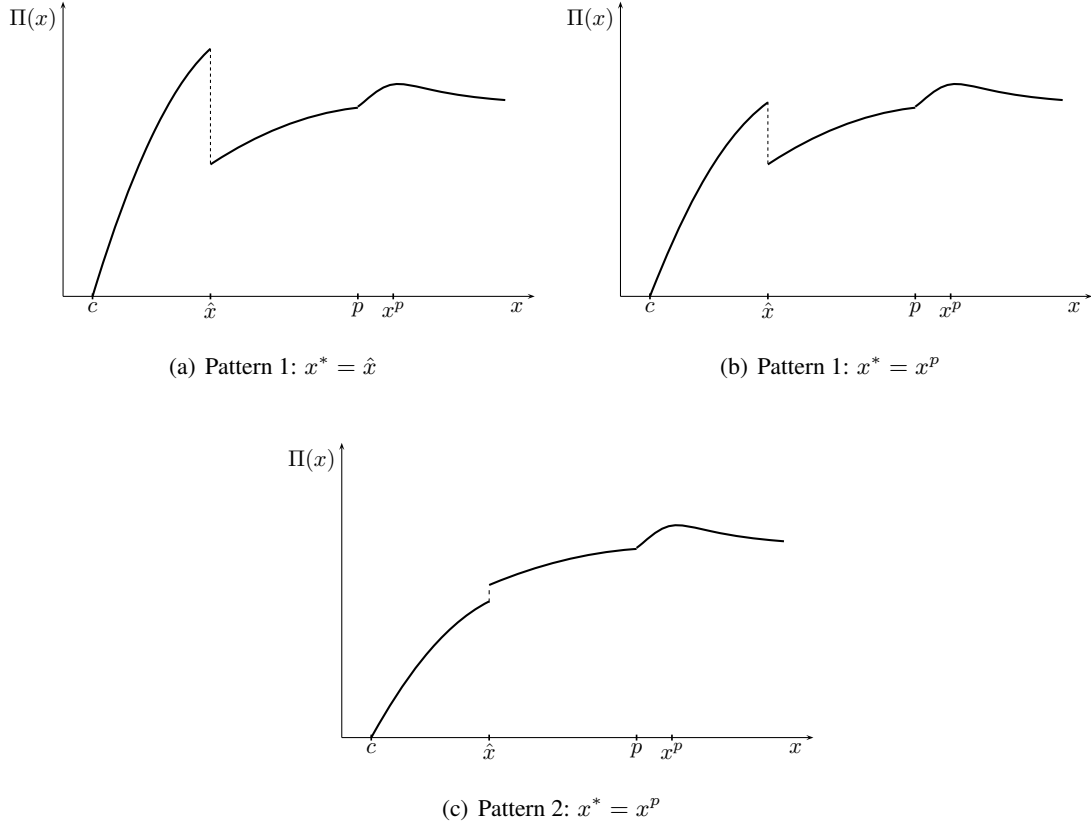


Figure 3: Optimal advance selling price

Propositions 1 and 2, we have the following proposition.

Proposition 3 (Optimal advance selling price). *When both price discounts and premiums for advance selling are considered, the optimal advance selling price is either \hat{x} or x^p .*

Figure 3 illustrates the two possibilities for the optimal advance selling price, denoted by x^* . If pattern 1 in Figure 2 prevails, the optimal advance selling price can be either \hat{x} or x^p . The former is illustrated in Figure 3(a) and the latter in Figure 3(b). If pattern 2 in Figure 2 prevails, the optimal advance selling price is x^p . This is illustrated in Figure 3(c). It follows immediately that $\Pi^A(\hat{x}) > \Pi^B(\hat{x})$ is a necessary condition for $x^* = \hat{x}$ and that $\Pi^A(\hat{x}) \leq \Pi^B(\hat{x})$ is a sufficient condition for $x^* = x^p$.

To provide further understanding of conditions that delineate the choice between $x^* = \hat{x}$ and $x^* = x^p$, we next examine how the values of $\Pi(\hat{x})$ and $\Pi(p)$ are affected by important parameters in the model. Let $m = m_e + m_i$ denote the total expected number of (experienced and inexperienced) consumers, and let α denote the proportion of experienced consumers in the

market. Thus, $m_e = \alpha m$ and $m_i = (1 - \alpha)m$. Rewriting the expressions for $\Pi(\hat{x})$ and $\Pi(p)$ as functions of α yields

$$\Pi(\hat{x}) = \alpha m \bar{F}_v(\hat{x})(\hat{x} - c) + (1 - \alpha)m(\hat{x} - c), \quad (16)$$

$$\Pi(p) = \alpha m \bar{F}_v(p)(p - c) + (p - s)(1 - \Phi(\tau_i - z_\beta))(1 - \alpha)m \bar{F}_v(p). \quad (17)$$

Lemma 5 (Some properties of $\Pi(\hat{x})$ and $\Pi(p)$). *Parameters s , α , σ , ψ , and τ_i have the following effects on $\Pi(\hat{x})$ and $\Pi(p)$:*

- (i) *As s increases, $\Pi(\hat{x})$ decreases and $\Pi(p)$ increases.*
- (ii) *As α increases, $\Pi(\hat{x})$ decreases and $\Pi(p)$ increases.*
- (iii) *As σ or ψ increases, $\Pi(\hat{x})$ decreases and $\Pi(p)$ increases.²³*
- (iv) *As τ_i increases, $\Pi(\hat{x})$ increases and $\Pi(p)$ decreases.*

Since $\Pi(x^p) > \Pi(p)$, this lemma together with Proposition 3 implies the following corollary.

Corollary 1. *Advance selling at the premium price x^p is more likely than at the discount price \hat{x} if the salvage value s is large, the proportion of experienced consumers α is large, the standard deviation of consumer valuations σ or the standard deviation of μ , ψ , is large, or the parameter τ_i in the distribution of the number of inexperienced consumers is small.*

The two tradeoffs discussed following Proposition 1 apply for the choice between \hat{x} and x^p . Intuitively, as the salvage value s increases, the retailer faces smaller overage cost and hence is more willing to charge a premium price for pre-orders. A higher α means the proportion of experienced consumers in the population is greater. Since only experienced consumers with high valuations are willing to pay a premium price, a larger proportion of such consumers in the population gives the retailer more incentives to charge a premium price for pre-orders. In a sense, increasing σ or ψ has a similar effect as increasing α . The distribution of consumers' valuations spreads out, implying that more experienced consumers have sufficiently high evaluations to be willing to pre-order at a premium price. As τ_i decreases, there is less uncertainty about the number of inexperienced consumers, reducing the risk associated with overage and underage.

²³That $\Pi(p)$ increases in σ and ψ holds under the assumption $\mu_0 < p$.

(a) s varies					(b) α varies				
s	\hat{x}	$\Pi(\hat{x})$	x^p	$\Pi(x^p)$	α	\hat{x}	$\Pi(\hat{x})$	x^p	$\Pi(x^p)$
10	145.54	7.29	214.58	6.19	0.1	141.93	8.06	212.70	4.32
20	144.73	7.17	213.57	6.25	0.2	141.93	7.73	212.12	4.87
30	143.87	7.04	212.46	6.32	0.3	141.93	7.41	211.50	5.42
40	142.94	6.91	211.20	6.40	0.4	141.93	7.08	210.72	5.96
50	141.93	6.76	209.84	6.50	0.53	141.93	6.66	209.57	6.66
58.7	140.99	6.62	208.54	6.62	0.6	141.93	6.43	208.82	7.03
70	139.66	6.42	206.59	6.82	0.7	141.93	6.10	207.54	7.54
80	138.40	6.23	204.63	7.08	0.8	141.93	5.78	205.90	8.03
90	137.05	6.02	202.40	7.52	0.9	141.93	5.45	203.76	8.54

(c) σ varies					(d) τ_i varies				
σ	\hat{x}	$\Pi(\hat{x})$	x^p	$\Pi(x^p)$	τ_i	\hat{x}	$\Pi(\hat{x})$	x^p	$\Pi(x^p)$
70	148.88	7.86	208.34	6.31	0.5	139.54	6.40	205.69	7.66
80	146.72	7.52	208.81	6.38	0.6	140.11	6.48	206.67	7.42
90	144.39	7.15	209.30	6.44	0.7	140.62	6.56	207.61	7.18
100	141.93	6.76	209.84	6.50	0.8	141.09	6.63	208.43	6.94
105.5	140.53	6.53	210.13	6.53	0.91	141.57	6.70	209.24	6.70
120	136.68	5.92	210.99	6.62	1.0	141.93	6.76	209.84	6.50
130	133.92	5.48	211.59	6.67	1.1	142.30	6.81	210.45	6.30
140	131.08	5.02	212.21	6.72	1.2	142.64	6.86	210.94	6.11
150	128.19	4.56	212.87	6.77	1.3	142.96	6.91	211.35	5.92

Table 2: Optimal advance selling price

6.2 Numerical Examples

In all of the calculations below we use the following base values: $p = 200$, $c = 100$, $s = 50$, $m = 200000$, $\alpha = 0.5$, $\mu_0 = 180$, $\tau_i = 1$, $\sigma = 100$, and $\psi = 90$. In each sub-table of Table 2 one of the four parameters discussed in Lemma 5 is set to different values to examine how the retailer's optimal choice x^* varies with it. The bold cells correspond to the optimal choices.²⁴

In Table 2(a), for s less than the critical value 58.7 (in a framed box), advance selling at a discount is optimal; for s greater than 58.7, advance selling at a premium is optimal. In Table 2(b), for α less than the critical value 0.53, advance selling at a discount is optimal; for α greater than 0.53, advance selling at a premium is optimal. In Table 2(c), for σ less than the critical value 105.5, advance selling at a discount is optimal; for σ greater than 105.5, advance selling at a premium is optimal. In Table 2(d), for τ_i less than the critical value 0.91, advance selling at a premium is optimal; for τ_i greater than 0.91, advance selling at a discount is optimal. A sub-table similar to Table 2(c) applies to ψ . These results confirm Corollary 1.

²⁴All profits in Table 2 are equal to the reported numbers multiplied by 10^6 .

7 Concluding Remarks

This paper has studied advance selling when the firm faces inexperienced consumers (who do not know their valuations in the advance selling period) as well as a group of experienced consumers (who know their valuations in the advance selling period). We find that it is always in the firm's best interest to adopt advance selling. The optimal pre-order price may be at a discount or a premium relative to the regular selling price.²⁵

A number of issues are worthy of further investigation. One issue concerns the price in the regular selling season. As in many papers in the literature, we have treated this price as exogenously given. A natural extension of the present research is to allow the retailer to choose the regular selling price. If the retailer can credibly commit to a regular selling price in the advance selling season, then the analyses in the present paper should carry through. Without this commitment, one would need to consider options such as dynamic pricing and pre-order price guarantee (e.g., Zhao and Pang, 2011).

Another issue concerns the assumption that the distribution of valuations is the same for experienced and inexperienced consumers. One straightforward generalization of our model is to assume that the distribution of valuations for experienced consumers is a rightward shift of that for inexperienced consumers (i.e., experienced consumers value the product more than inexperienced consumers on average). We expect many of the results in our paper to continue to hold in this extension. An alternative might be to assume a more general level of correlation between the two distribution functions. Much remains to be explored about this case.

A third issue is related to our implicit assumption that there is no second-hand market (or returns). The recent paper by Liu and Schiraldi (2014), which is a study of a durable-goods monopolist launching a new product, points out that results differ whether there is a second-hand market or not. Consumers in our model may change their behavior if there is a second-hand market. In particular, inexperienced consumers are more likely to pre-order in the advance selling season, since they can resell the product after learning a low valuation. As a result, the retailer's optimal pricing strategy will be affected.

Finally, it would be interesting to look at other pricing schemes, such as pre-order price guarantee and dynamic pricing (see, for example, Li and Zhang, 2013, and Zhao and Pang, 2011).

²⁵It is interesting to compare the optimal prices obtained in our model of advance selling with the literature on the durable-goods monopoly problem. This literature suggests a decreasing sequence of prices to discriminate among buyers with different willingness to pay (e.g., Stokey, 1979). In our model the optimal pre-order price does not have to be at a premium to the regular selling price. It may be at a discount to reduce uncertainty and increase overall demand. Many other papers on advance selling suggest an increasing sequence of prices as well. The difference in results from the advance selling literature and the durable-goods monopoly literature is not surprising, as the setups are different.

Appendix

Derivation of (1):

We have:

$$\begin{aligned} f_v(v) &= \int_{-\infty}^{+\infty} f_{v|\mu}(v) f_\mu(\mu) \, d\mu \\ &= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(v-\mu)^2}{2\sigma^2}\right\} \frac{1}{\sqrt{2\pi\psi^2}} \exp\left\{-\frac{(\mu-\mu_0)^2}{2\psi^2}\right\} \, d\mu \\ &= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{\sqrt{2\pi\psi^2}} \exp\left\{-\frac{(\mu-A)^2}{2B^2} - \frac{(v-\mu_0)^2}{2(\sigma^2+\psi^2)}\right\} \, d\mu, \end{aligned}$$

where

$$A = \frac{\mu_0\sigma^2 + v\psi^2}{\sigma^2 + \psi^2}$$

and

$$B^2 = \frac{\sigma^2\psi^2}{\sigma^2 + \psi^2}.$$

Hence,

$$\begin{aligned} f_v(v) &= \frac{B}{\sigma\psi\sqrt{2\pi}} \exp\left\{-\frac{(v-\mu_0)^2}{2(\sigma^2+\psi^2)}\right\} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi B^2}} \exp\left\{-\frac{(\mu-A)^2}{2B^2}\right\} \, d\mu \\ &= \frac{B}{\sigma\psi\sqrt{2\pi}} \exp\left\{-\frac{(v-\mu_0)^2}{2(\sigma^2+\psi^2)}\right\} \\ &= \frac{1}{\sqrt{2\pi(\sigma^2+\psi^2)}} \exp\left\{-\frac{(v-\mu_0)^2}{2(\sigma^2+\psi^2)}\right\}, \end{aligned}$$

which immediately implies $v \sim N(\mu_0, \sigma^2 + \psi^2)$.

Derivation of (2):

Below we show that

$$\int_p^{+\infty} (v-p) f_v(v) \, dv = (\mu_0 - p) \bar{F}_v(p) + (\sigma^2 + \psi^2) f_v(p).$$

The expression (2) will follow immediately. Applying the change of variable $z = (v - \mu_0) / \sqrt{\sigma^2 + \psi^2}$ yields

$$\begin{aligned} \int_p^{+\infty} (v - p) f_v(v) \, dv &= \int_{\frac{p - \mu_0}{\sqrt{\sigma^2 + \psi^2}}}^{+\infty} (\mu_0 + z \sqrt{\sigma^2 + \psi^2} - p) \phi(z) \, dz \\ &= (\mu_0 - p) \left(1 - \Phi \left(\frac{p - \mu_0}{\sqrt{\sigma^2 + \psi^2}} \right) \right) + \int_{\frac{p - \mu_0}{\sqrt{\sigma^2 + \psi^2}}}^{+\infty} \frac{z \sqrt{\sigma^2 + \psi^2}}{\sqrt{2\pi}} \exp \left\{ -\frac{z^2}{2} \right\} \, dz \\ &= (\mu_0 - p) \bar{F}_v(p) + \frac{\sqrt{\sigma^2 + \psi^2}}{\sqrt{2\pi}} \int_{\frac{p - \mu_0}{\sqrt{\sigma^2 + \psi^2}}}^{+\infty} \exp \left\{ -\frac{z^2}{2} \right\} \, d\frac{z^2}{2}. \end{aligned}$$

Applying the change of variable $u = z^2/2$ yields

$$\begin{aligned} \int_p^{+\infty} (v - p) f_v(v) \, dv &= (\mu_0 - p) \bar{F}_v(p) + \frac{\sqrt{\sigma^2 + \psi^2}}{\sqrt{2\pi}} \int_{\frac{(p - \mu_0)^2}{2(\sigma^2 + \psi^2)}}^{+\infty} \exp\{-u\} \, du \\ &= (\mu_0 - p) \bar{F}_v(p) + \frac{\sqrt{\sigma^2 + \psi^2}}{\sqrt{2\pi}} \exp \left\{ -\frac{(p - \mu_0)^2}{2(\sigma^2 + \psi^2)} \right\} \\ &= (\mu_0 - p) \bar{F}_v(p) + (\sigma^2 + \psi^2) f_v(p). \end{aligned}$$

Proof of Lemma 1:

It follows immediately from (2) that \hat{x} increases in η , so we have part (iii) of Lemma 1.

(i) We need to show that $\int_p^{+\infty} (v - p) f_v(v) \, dv$ increases in σ .

$$\begin{aligned} \int_p^{+\infty} (v - p) f_v(v) \, dv &= \int_p^{+\infty} v f_v(v) \, dv - p \int_p^{+\infty} f_v(v) \, dv \\ &= \int_{-\infty}^{+\infty} v f_v(v) \, dv - \int_{-\infty}^p v \, dF_v(v) - p(1 - F_v(p)) \\ &= \mu_0 - \left(v F_v(v) \Big|_{-\infty}^p - \int_{-\infty}^p F_v(v) \, dv \right) - p + p F_v(p) \\ &= \mu_0 + \int_{-\infty}^p F_v(v) \, dv - p. \end{aligned}$$

Hence,

$$\frac{\partial}{\partial \sigma} \left(\int_p^{+\infty} (v - p) f_v(v) \, dv \right) = \frac{\partial}{\partial \sigma} \left(\int_{-\infty}^p F_v(v) \, dv \right).$$

Suppose $p < \mu_0$, then

$$\begin{aligned} \frac{\partial}{\partial \sigma} \left(\int_{-\infty}^p F_v(v) \, dv \right) &= \frac{\partial}{\partial \sigma} \left(\sqrt{\sigma^2 + \psi^2} \int_{-\infty}^{\frac{p-\mu_0}{\sqrt{\sigma^2 + \psi^2}}} \Phi(z) \, dz \right) \\ &= \left(\int_{-\infty}^{\frac{p-\mu_0}{\sqrt{\sigma^2 + \psi^2}}} \Phi(z) \, dz - \frac{p-\mu_0}{\sqrt{\sigma^2 + \psi^2}} \Phi \left(\frac{p-\mu_0}{\sigma} \right) \right) \frac{\sigma}{\sqrt{\sigma^2 + \psi^2}} > 0. \end{aligned} \quad (18)$$

Next, suppose $p > \mu_0$.

$$\int_{-\infty}^p F_v(v) \, dv = \int_{-\infty}^{2\mu_0-p} F_v(v) \, dv + \int_{2\mu_0-p}^{\mu_0} F_v(v) \, dv + \int_{\mu_0}^p F_v(v) \, dv. \quad (19)$$

Applying the change of variable $u = 2\mu_0 - v$ to the second integral yields

$$\int_{\mu_0}^p F_v(2\mu_0 - u) \, du.$$

By symmetry of the normal distribution $F_v(v) + F_v(2\mu_0 - v) = 1$, thus the sum of the last two integrals in (19) equals $p - \mu_0$. Hence,

$$\frac{\partial}{\partial \sigma} \left(\int_{-\infty}^p F_v(v) \, dv \right) = \frac{\partial}{\partial \sigma} \left(\int_{-\infty}^{2\mu_0-p} F_v(v) \, dv \right).$$

The upper limit of the integration on the right-hand side is less than μ_0 , hence, by (18) the above derivative is positive.

(ii) The proof that \hat{x} decreases in ψ follows the same steps as in (i).

Derivations of (5) and (6):²⁶

As shown in Silver et al. (1998), the solution to the Newsvendor Problem (3) is q^* that satisfies

$$\Pr(D_2 \leq q^*) = \beta. \quad (20)$$

Moreover,

$$\pi(q^*) = (p - s) \int_0^{q^*} D_2 g(D_2) \, dD_2.$$

²⁶The solution to the Newsvendor Problem under LN (ν, τ^2) has been provided in lecture notes by Gallego (1995) without a proof.

With $D_2 \sim LN(\nu, \tau^2)$, (20) becomes

$$\Phi\left(\frac{\ln q^* - \nu}{\tau}\right) = \beta,$$

or

$$q^* = \exp\{\nu + \tau z_\beta\}.$$

Next,

$$\pi(q^*) = (p - s) \int_0^{\exp\{\nu + \tau z_\beta\}} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(\ln D_2 - \nu)^2}{2\tau^2}\right\} dD_2.$$

Applying the change of variable $u = \ln D_2$ yields

$$\begin{aligned} \pi(q^*) &= (p - s) \int_{-\infty}^{\nu + \tau z_\beta} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(u - \nu)^2}{2\tau^2} + u\right\} du \\ &= (p - s) \exp\left\{\frac{(\nu + \tau^2)^2 - \nu^2}{2\tau^2}\right\} \int_{-\infty}^{\nu + \tau z_\beta} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(u - (\nu + \tau^2))^2}{2\tau^2}\right\} du \\ &= (p - s) \exp\left\{\nu + \frac{\tau^2}{2}\right\} \Phi(z_\beta - \tau) \\ &= (p - s) (1 - \Phi(\tau - z_\beta)) \exp\left\{\nu + \frac{\tau^2}{2}\right\}. \end{aligned}$$

Derivation of (9):

Let $D_2 \sim LN(\nu, \tau^2)$ and $q^* = \exp\{\nu + \tau z_\beta\}$. Then

$$\begin{aligned} \eta &= \int_{q^*}^{+\infty} \frac{D_2 - q^*}{D_2} g(D_2) dD_2 \\ &= 1 - G(q^*) - \int_{q^*}^{+\infty} \frac{q^*}{D_2} g(D_2) dD_2 \\ &= 1 - G(\exp\{\nu + \tau z_\beta\}) \\ &\quad - \int_{\exp\{\nu + \tau z_\beta\}}^{+\infty} \frac{\exp\{\nu + \tau z_\beta\}}{D_2} \frac{1}{D_2 \sqrt{2\pi\tau^2}} \exp\left\{-\frac{(\ln D_2 - \nu)^2}{2\tau^2}\right\} dD_2. \end{aligned}$$

Applying the change of variable $u = \ln D_2$ yields

$$\begin{aligned}
\eta &= 1 - \beta - \int_{\nu + \tau z_\beta}^{+\infty} \frac{\exp\{\nu + \tau z_\beta\}}{\exp\{u\}} \frac{1}{\exp\{u\} \sqrt{2\pi\tau^2}} \exp\left\{-\frac{(u - \nu)^2}{2\tau^2}\right\} \exp\{u\} \, du \\
&= 1 - \beta - \exp\{\nu + \tau z_\beta\} \int_{\nu + \tau z_\beta}^{+\infty} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(u - \nu)^2}{2\tau^2} - u\right\} \, du \\
&= 1 - \beta - \exp\{\nu + \tau z_\beta\} \exp\left\{-\nu + \frac{\tau^2}{2}\right\} \int_{\nu + \tau z_\beta}^{+\infty} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(u - (\nu - \tau^2))^2}{2\tau^2}\right\} \, du \\
&= 1 - \beta - \exp\left\{\tau z_\beta + \frac{\tau^2}{2}\right\} \left(1 - \Phi\left(\frac{\nu + \tau z_\beta - (\nu - \tau^2)}{\tau}\right)\right) \\
&= 1 - \beta - \exp\left\{\tau z_\beta + \frac{\tau^2}{2}\right\} (1 - \Phi(z_\beta + \tau)).
\end{aligned}$$

Proof of Lemma 3:

The following properties of the standard normal distribution will be used in the proof:

$$\phi(u) \left(\frac{1}{u} - \frac{1}{u^3}\right) < 1 - \Phi(u) < \frac{\phi(u)}{u}, \quad u > 0 \quad (21)$$

and

$$\phi(u) \left(\frac{1}{|u|} - \frac{1}{|u|^3}\right) < \Phi(u) < \frac{\phi(u)}{|u|}, \quad u < 0. \quad (22)$$

(i) First,

$$\eta(\beta, 0) = 1 - \beta - (1 - \Phi(z_\beta)) = 1 - \beta - (1 - \beta) = 0.$$

Next,

$$\lim_{\tau_i \rightarrow +\infty} \eta(\beta, \tau_i) = 1 - \beta - \lim_{\tau_i \rightarrow +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} (1 - \Phi(z_\beta + \tau_i)).$$

It follows from (21) that $1 - \Phi(u) = \frac{\phi(u)}{u}(1 + o(u^{-1}))$. Hence, the above expression can be rewritten as

$$\begin{aligned}
\lim_{\tau_i \rightarrow +\infty} \eta(\beta, \tau_i) &= 1 - \beta - \lim_{\tau_i \rightarrow +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} \frac{\phi(z_\beta + \tau_i)}{z_\beta + \tau_i} \\
&= 1 - \beta - \lim_{\tau_i \rightarrow +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} \frac{\exp\left\{-\frac{(z_\beta + \tau_i)^2}{2}\right\}}{\sqrt{2\pi}(z_\beta + \tau_i)} \\
&= 1 - \beta - \lim_{\tau_i \rightarrow +\infty} \frac{\exp\left\{-\frac{z_\beta^2}{2}\right\}}{\sqrt{2\pi}(z_\beta + \tau_i)} = 1 - \beta.
\end{aligned}$$

Finally,

$$\begin{aligned}
\frac{\partial \eta}{\partial \tau_i} &= \frac{\partial}{\partial \tau_i} \left(-\exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} (1 - \Phi(z_\beta + \tau_i)) \right) \\
&= -(z_\beta + \tau_i) \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} (1 - \Phi(z_\beta + \tau_i)) + \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} \phi(z_\beta + \tau_i) \\
&= (z_\beta + \tau_i) \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} \left[\frac{\phi(z_\beta + \tau_i)}{z_\beta + \tau_i} - (1 - \Phi(z_\beta + \tau_i)) \right].
\end{aligned}$$

By (21) the term in the square brackets is positive, so $\partial \eta / \partial \tau_i > 0$.

(ii) First,

$$\eta(0, \tau_i) = 1 - \lim_{z_\beta \rightarrow -\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} (1 - \Phi(z_\beta + \tau_i)).$$

It follows from (22) that $\Phi(u) = \frac{\phi(u)}{|u|} (1 + o(u^{-1}))$. Hence, the above expression can be rewritten as

$$\begin{aligned}
\eta(0, \tau_i) &= 1 - \lim_{z_\beta \rightarrow -\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} \left(1 - \frac{\phi(z_\beta + \tau_i)}{|z_\beta + \tau_i|} \right) \\
&= 1 - \lim_{z_\beta \rightarrow -\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} \left(1 - \frac{\exp \left\{ -\frac{(z_\beta + \tau_i)^2}{2} \right\}}{\sqrt{2\pi} |z_\beta + \tau_i|} \right) \\
&= 1 - \lim_{z_\beta \rightarrow -\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} + \lim_{z_\beta \rightarrow -\infty} \frac{\exp \left\{ -\frac{z_\beta^2}{2} \right\}}{\sqrt{2\pi} |z_\beta + \tau_i|} = 1.
\end{aligned}$$

Next,

$$\eta(1, \tau_i) = - \lim_{z_\beta \rightarrow +\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} (1 - \Phi(z_\beta + \tau_i)).$$

Since $1 - \Phi(u) = \frac{\phi(u)}{u} (1 + o(u^{-1}))$,

$$\begin{aligned}
\eta(1, \tau_i) &= - \lim_{z_\beta \rightarrow +\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} \frac{\phi(z_\beta + \tau_i)}{z_\beta + \tau_i} \\
&= - \lim_{z_\beta \rightarrow +\infty} \exp \left\{ \tau_i z_\beta + \frac{\tau_i^2}{2} \right\} \frac{\exp \left\{ -\frac{(z_\beta + \tau_i)^2}{2} \right\}}{\sqrt{2\pi} (z_\beta + \tau_i)} \\
&= - \lim_{z_\beta \rightarrow +\infty} \frac{\exp \left\{ -\frac{z_\beta^2}{2} \right\}}{\sqrt{2\pi} (z_\beta + \tau_i)} = 0.
\end{aligned}$$

Finally, we can write η as a function of z_β and τ_i ,

$$\eta(z_\beta, \tau_i) = 1 - \Phi(z_\beta) - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} (1 - \Phi(z_\beta + \tau_i)).$$

Differentiating the above expression with respect to z_β yields

$$\frac{\partial \eta}{\partial z_\beta} = -\phi(z_\beta) - \tau_i \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} (1 - \Phi(z_\beta + \tau_i)) + \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} \phi(z_\beta + \tau_i).$$

The first term and the third term cancel out each other, as

$$\exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} \frac{\exp\left\{-\frac{(z_\beta + \tau_i)^2}{2}\right\}}{\sqrt{2\pi}} = \frac{\exp\left\{-\frac{z_\beta^2}{2}\right\}}{\sqrt{2\pi}}.$$

Thus,

$$\frac{\partial \eta}{\partial z_\beta} = -\tau_i \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} (1 - \Phi(z_\beta + \tau_i)) < 0.$$

Since z_β is increasing in β , it follows immediately that $\partial \eta / \partial \beta$ is also negative.

Derivation of (10):

We have:

$$\begin{aligned} E_\mu [\bar{F}_{v|\mu}(x)] &= \int_{-\infty}^{+\infty} \bar{F}_{v|\mu}(x) f_\mu(\mu) \, d\mu = \int_{-\infty}^{+\infty} \left(\int_x^{+\infty} f_{v|\mu}(v) \, dv \right) f_\mu(\mu) \, d\mu \\ &= \int_{-\infty}^{+\infty} \int_x^{+\infty} f_{v|\mu}(v) f_\mu(\mu) \, dv \, d\mu = \int_x^{+\infty} \int_{-\infty}^{+\infty} f_{v|\mu}(v) f_\mu(\mu) \, d\mu \, dv. \end{aligned}$$

The integral

$$\int_{-\infty}^{+\infty} f_{v|\mu}(v) f_\mu(\mu) \, d\mu$$

is $f_v(v)$, hence

$$E_\mu [\bar{F}_{v|\mu}(x)] = \int_x^{+\infty} f_v(v) \, dv = \bar{F}_v(x).$$

Derivation of (14):

We have:

$$\begin{aligned} f_{\mu|v}(\mu) &= \frac{f_{v|\mu}(v)f_{\mu}(\mu)}{f_v(v)} \\ &= \frac{1}{2\sqrt{\frac{\pi\sigma^2\psi^2}{\sigma^2+\psi^2}}} \exp \left\{ -\frac{(v-\mu)^2}{2\sigma^2} - \frac{(\mu-\mu_0)^2}{2\psi^2} + \frac{(v-\mu_0)^2}{2(\sigma^2+\psi^2)} \right\}. \end{aligned}$$

Regrouping the terms in the exponent yields

$$f_{\mu|v}(\mu) = \frac{1}{2\sqrt{\frac{\pi\sigma^2\psi^2}{\sigma^2+\psi^2}}} \exp \left\{ -\frac{\left(\mu - \frac{\sigma^2\mu_0 + \psi^2v}{\sigma^2 + \psi^2}\right)^2}{\frac{2\sigma^2\psi^2}{\sigma^2 + \psi^2}} \right\},$$

which immediately implies (14).

Proof of Lemma 5:

- (i) Suppose s increases. Then $\beta = (p - c)/(p - s)$ increases. By Lemma 3(ii) η decreases, leading to a decrease in \hat{x} . It follows immediately that $\Pi(\hat{x})$ in (16) decreases in s . Next, differentiating $\Pi(p)$ in (17) with respect to s yields

$$\frac{\partial \Pi(p)}{\partial s} = \frac{\partial \pi}{\partial s} \Big|_{q=q^*} = \mathbb{E} [(q^* - D_2)^+] > 0,$$

where the second equality follows from applying the Envelope Theorem to (3). Hence, $\Pi(p)$ increases in s .

- (ii) First, we differentiate $\Pi(\hat{x})$ in (16) with respect to α :

$$\frac{\partial \Pi(\hat{x})}{\partial \alpha} = m (\bar{F}_v(\hat{x}) - 1) (\hat{x} - c) < 0.$$

Next, we differentiate $\Pi(p)$ in (17) with respect to α :

$$\begin{aligned} \frac{\partial \Pi(p)}{\partial \alpha} &= m \bar{F}_v(p) ((p - c) - (p - s) (1 - \Phi(\tau_i - z_\beta))) \\ &= m \bar{F}_v(p) ((p - s) \Phi(\tau_i - z_\beta) - (c - s)) \\ &> m \bar{F}_v(p) ((p - s) \Phi(-z_\beta) - (c - s)). \end{aligned}$$

Substituting $\Phi(-z_\beta) = 1 - \Phi(z_\beta) = 1 - \beta = (c - s)/(p - s)$ into the above inequality

yields

$$\frac{\partial \Pi(p)}{\partial \alpha} > m \bar{F}_v((c-s) - (c-s)) = 0.$$

(iii) The expected profit $\Pi(\hat{x})$ in (16) can be written as

$$\Pi(\hat{x}) = \alpha m \left(1 - \Phi \left(\frac{\hat{x} - \mu_0}{\sqrt{\sigma^2 + \psi^2}} \right) \right) (\hat{x} - c) + (1 - \alpha) m (\hat{x} - c).$$

Note that

$$\frac{\partial}{\partial \sigma} \left(1 - \Phi \left(\frac{\hat{x} - \mu_0}{\sqrt{\sigma^2 + \psi^2}} \right) \right) = \frac{1}{\sqrt{2\pi(\sigma^2 + \psi^2)}} \exp \left\{ -\frac{(\hat{x} - \mu_0)^2}{2(\sigma^2 + \psi^2)} \right\} \frac{\sigma(\hat{x} - \mu_0)}{(\sigma^2 + \psi^2)^{\frac{3}{2}}} < 0.$$

Hence, the direct effect of an increase in σ on $\Pi(\hat{x})$ is negative. The indirect effect (through \hat{x}) is negative as well. Indeed, $\Pi(\hat{x})$ is an increasing function of \hat{x} and \hat{x} decreases in σ by Lemma 1(i). Next, the expected profit $\Pi(p)$ in (17) can be written as

$$\Pi(p) = (\alpha m(p - c) + (1 - \alpha)m(p - s)(1 - \Phi(\tau_i - z_\beta))) \left(1 - \Phi \left(\frac{p - \mu_0}{\sigma} \right) \right).$$

Since

$$\frac{\partial}{\partial \sigma} \left(1 - \Phi \left(\frac{p - \mu_0}{\sqrt{\sigma^2 + \psi^2}} \right) \right) = \frac{1}{\sqrt{2\pi(\sigma^2 + \psi^2)}} \exp \left\{ -\frac{(p - \mu_0)^2}{2(\sigma^2 + \psi^2)} \right\} \frac{\sigma(p - \mu_0)}{(\sigma^2 + \psi^2)^{\frac{3}{2}}},$$

$\Pi(p)$ is increasing in σ if $\mu_0 < p$, decreasing if $\mu_0 > p$. The proof for ψ is similar and skipped.

(iv) Suppose τ_i increases. By Lemma 3(i) η increases, leading to an increase in \hat{x} . It follows immediately that $\Pi(\hat{x})$ in (16) increases in τ_i . Next, (17) implies $\Pi(p)$ decreases in τ_i . Hence, $\Pi(p)$ decreases in τ_i .

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