CONSERVATIVE CENTRAL BANKS, AND NOMINAL GROWTH,
EXCHANGE RATE AND INFLATION TARGETS

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Abstract

A framework is developed in which inflation biases with different target variables are compared. A nominal growth target measured in consumer prices may yield less stabilization bias than a nominal income growth target. Exchange rate and inflation targets result in less stabilization bias than an income growth target the more important terms of trade stabilization. Persistence in output causes excessive stabilization of productivity shocks and of shocks to the terms of trade under discretion. An inflation-weight conservative central bank is more likely under an inflation target than under an exchange rate target and less likely under a nominal income growth target.

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1. INTRODUCTION

The issue to be addressed in this paper concerns the relative usefulness of nominal growth, exchange rate, and inflation targets as institutional arrangements for the conduct of monetary policy, given that an exchange rate stabilization objective may appear in the loss function for government along with domestic inflation and output stabilization objectives.\(^1\) The idea that an exchange rate term should appear in the loss function is motivated by the report by Fry et al. (2000) that 84 out of 94 central banks responding to a survey by the Bank of England placed positive weight on an exchange rate objective during 1997-1998, the finding by Calvo and Reinhart (2002) that many developing countries reporting floating regimes actually peg the exchange rate of their currencies, and the evidence in Reinhart and Rogoff (2004) that in modern history the most common exchange rate regimes have been pegged exchange rates followed by crawling pegged exchange rates, and that over half of the regimes officially identified by the IMF as managed floats have in fact been regimes in which the exchange rate is tied in some way to another currency.\(^2\)

That an exchange rate term should appear in the loss function is also supported by much of the recent literature relating welfare to the open economy loss function. Corsetti and Pesenti (2001; 2005), Gali and Monacelli (2005), Sutherland (2005a, 2006), Monacelli (2005), Benigno and Benigno (2006), and De Paoli (2006), and others, show that in general and under a number of circumstances, movement in the terms of trade or the real exchange rate has an effect on welfare. This recent work draws on initial contributions by Rotemberg and Woodford (1997) and Woodford (2002) for the closed economy connecting welfare with a quadratic loss function over output and inflation gaps, and demonstrates that the open economy government loss function is not simply that obtained for the closed economy. In line with this work the loss function in this paper is assumed to be a quadratic function of producer price inflation, output gap, and change in terms of trade variable.
It is assumed that a target regime is implemented by delegating the conduct of monetary policy to a central bank via an explicit loss function designed to bring discretionary behavior closer to (if possible to replicate) the optimal equilibrium under commitment. Delegation may be realized by inflation, exchange rate, nominal growth, or other targets. Persistence in output is assumed so as to enrich discussion of dynamic aspects of policy. A framework is developed in which inflation biases with different target variables are compared. It is shown that a target for nominal income growth may be dominated by a target in nominal income based on consumer prices, and that the greater the weight attached to real exchange rate stabilization the more likely it is that an exchange rate or an inflation target may dominate a nominal income growth target.

Stabilization of shocks to the terms of trade is not great enough under a nominal income target and is too great under an exchange rate or inflation target. Exchange rate and inflation targets result in less stabilization bias in absolute value than a nominal income growth target the more weight placed by government on exchange rate stabilization relative to output stabilization and the more sensitive the terms of trade to changes in supply of output. An inflation-weight conservative central bank is very likely under an inflation target, is always more likely under an inflation target than under an exchange rate target, and is always more likely under an exchange rate target than under a nominal income growth target.

The biases associated with discretionary policy are damped down the more sensitive output is to unexpected changes in the terms of trade and the greater the share of imports in output, because these factors flatten out the Phillips curve and reduce output gains from surprise inflation. The optimal stabilization of productivity shocks is smaller, in absolute terms, the more sensitive the supply of output to the terms of trade, and the larger the increase in the price of imported goods induced by output. It is shown that a target of a weighted sum of change in terms of trade and nominal income growth can replicate the optimal equilibrium without resort to a state-contingent rule when output is persistent. A flatter Phillips curve reduces the penalty for not stabilizing the intermediate policy target and makes it more likely that the optimal central banker is an inflation-weight conservative.
The model and the optimal policy under commitment are discussed in the next section. A policy rule that can replicate the optimal equilibrium is provided in Section 3. Nominal income growth, nominal growth based on consumer prices, exchange rate, and inflation targets are compared in terms of stabilization bias in Section 4. In Section 5 the relationship between conservative central banks and targets is considered. Section 6 concludes.

2. THE MODEL

In the literature on dynamic inconsistency in monetary policy, the period loss function is usually assumed to be a quadratic function of an output gap and of inflation for a closed economy. A quadratic form of the period loss function is standard in the literature since the contribution by Barro and Gordon (1983). Rotemberg and Woodford (1997) and Woodford (2002) demonstrate that subject to certain restrictions, welfare of private agents can be expressed by Taylor series approximation as such a quadratic function. A potential welfare enhancing role for monetary policy being provided by the presence of price stickiness associated with Calvo (1983) staggered price-setting. In extensions to the open economy, Clarida et al. (2001; 2002), Benigno and Benigno (2003), and Gali and Monacelli (2005) show that the policy makers’ stabilization objective can in some situations (when terms of trade is proportional to an output gap) continue to be expressed as a quadratic function of output and producer price inflation gaps. However, in these papers and in others it is made clear that a wide range of circumstances can arise in the context of the open economy under which monetary policy should take account of exchange rate volatility.

With incomplete pass-through of exchange rate changes to local prices Corsetti and Pesenti (2001), Devereux and Engel (2003), Sutherland (2005a), and Monacelli (2005) show that exchange rate volatility has an impact on consumer welfare. Corsetti and Pesenti (2001), Gali and Monacelli (2005) and Sunderland (2005b; 2006), and others, demonstrate that with a non-unit elasticity of substitution between domestic goods and foreign goods, even in the presence of complete pass-through from exchange rate shocks to domestic price changes, volatility of the terms of trade is an important consideration for optimal policy. McCallum and Nelson (2000) show that in the presence of trade in intermediate goods and
stickiness of final goods prices, inefficiencies arise that lead to consumer price inflation being related to welfare. Corsetti and Pesenti (2005) show that if firms’ markups are influenced by the exchange rate, then less exchange rate volatility is desirable. Kirsanova et al. (2006) find that international risk sharing of shocks provides a role for the terms of trade or real exchange rate in the welfare function.

In recent work with a more general framework, Benigno and Woodford (2006) establish, under certain regularity conditions, that it is possible to approximate an optimal nonlinear policy problem with linear constraints and a quadratic objective. Benigno and Woodford (2005) show that application of this method to the problem of optimal monetary policy in a model with nominal rigidities and monopolistic competition results in a quadratic stabilization objective in output and inflation. De Paoli (2006) shows that extension of the method of Sutherland (2002) and Benigno and Woodford (2005) to that of a small open economy, results in welfare being expressed as a quadratic loss function in producer price inflation, output gap, and the real exchange rate or terms of trade. In an application to cooperation between two central banks, Benigno and Benigno (2006) derive welfare based objectives quadratic in terms of trade and other variables.

In general, movement in the real exchange rate has an effect on welfare through (disproportionate) effects on the marginal utility of consumption and disutility of work. The findings of this research relating welfare to the open economy loss function are consistent with the evidence by Fry et al. (2000), Calvo and Reinhart (2002), and Reinhart and Rogoff (2004) concerning the importance attached to exchange rate stabilization by developing countries. Thus, the loss function for the government in this paper is assumed to be a quadratic function of producer price inflation, output gap, and terms of trade variable. The government’s (and society’s) preferences are represented by \[ E_0 \sum_{t=1}^{\infty} \beta^{t-1} L_t, \]
where \( E \) is the expectations operator, \( 0 < \beta < 1 \) is a discount factor, and the period loss function is assumed to be:

\[
(1) \quad L_t = \frac{1}{2}((\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 + \kappa (\omega_t - \pi_t + \pi^*)^2).
\]
Here, $\pi_t$ is the rate of inflation in domestic prices, $y_t$ is the log of output, $\omega_t$ represents the change in the log of the price of a unit of foreign currency, and $\pi^f$ is the foreign rate of inflation (assumed to be constant). The last term on the right hand side of equation (1) represents change in the terms of trade and its inclusion in the government loss function captures the importance of exchange rate stabilization to developing countries. If $\theta_t$ is the rate of inflation in consumer prices and $\mu$ is defined as the fraction of imports in consumption, the quadratic term in equation (1) in the terms of trade can be re-written as $$(\kappa/(1-\mu^2))(\omega_t - \theta_t + \theta^f)^2$$, a quadratic term in the real exchange rate. $\lambda$ and $\kappa$ are the relative weights placed on output and terms of trade stabilization versus inflation stabilization. $\kappa$ will be larger the more open the economy. $\pi^*$ and $y^*$ > 0 are the socially optimal levels of the inflation rate and of output. It is assumed that the socially optimal level of output exceeds the long-run natural output level because of distortions in the labor market. The long-run natural output level $\lim_{s \to \infty} E_{t-1}[y_s] = 0$ is normalized to zero.

The rate of consumer price inflation is defined as:

(2) $\theta_t = (1-\mu)(\pi_t + \mu(\omega_t + \pi^f)), \ 0 \leq \mu < 1.$

Fry et al. (2000) and Svensson (2000) note that in practice countries express inflation targets in terms of consumer prices, sometimes exclusive of certain components such as some commodity prices or mortgage interest rates, but that no country targets only domestic inflation. Consumer price inflation is not directly in the society objective function, but may appear in the loss function of the central bank through delegation. Government is assumed to delegate the conduct of monetary policy to an instrument independent central bank by assigning a consumer price inflation target and other targets. This subject is taken up in Section 3.

We specify a standard open economy Phillips curve in which there is persistence and in which unexpected changes in the prices of both domestic and foreign goods affect output:
where $E_{t-1}x$ is the expected value of $x$ given information available at time $t-1$, and $\xi_t$ is an independently and identically distributed random variable with zero mean and finite variance. A positive value for $\xi_t$ indicates a favorable productivity shock and less cost push inflation.

The first term on the right hand side of equation (3) captures persistence in output. Lockwood et al. (1998), Svensson (1997), Lockwood and Philippopoulos (1994), and others, have shown that the assumption of persistence in output enriches the study of commitment versus discretion in the conduct of monetary policy by bringing consideration of dynamic aspects of policy to the fore. In equation (3), an unexpected rise in domestic prices raises output, and an unexpected rise in the price (in domestic currency) of foreign goods raises cost and reduces output. If the latter effect is absent $\alpha_2 = 0$.

It is assumed that domestic and foreign goods are imperfect substitutes and that a lower price of domestically produced goods relative to the price of foreign goods will imply a higher level of domestic output relative to foreign output. Following the formulation in Romer (1993), change in the terms of trade is proportional to change in domestic output relative to foreign output:

$$\omega_t - \pi_t + \pi^f = \gamma((y_t - y_{t-1}) - (y_t^f - y_{t-1}^f)) + \nu_t, \quad 1 > \gamma > 0,$$

where $y_t^f$ is log of foreign output, and $\nu_t$, a shock to the terms of trade. A negative value for $\nu_t$ would reflect a positive shock to demand for domestic output relative to foreign output and result in a smaller depreciation of the currency and/or a higher rate of inflation. Romer (1993) assumes that consumers have CES consumption functions and interprets $\gamma$ as the inverse of the elasticity of substitution between goods. If it is assumed that growth in foreign output is an exogenous stationary stochastic process, $\zeta_t$, equation (4) can be re-written as:
where $\zeta_i \equiv \nu_i + \zeta_i$ will for convenience be referred to as shock to the terms of trade and is assumed to be an independently and identically distributed random variable with zero mean and finite variance.

Equations (2), (3), and (4') can now be solved to provide a reduced form relationship between output and unexpected consumer price inflation as follows:

$$y_t = \rho y_{t-1} + \frac{\alpha_1 - \alpha_2}{(1 + \gamma(\alpha_1 \mu + (1 - \mu) \alpha_2))} (\theta_t - \theta^*_t) + \frac{1}{(1 + \gamma(\alpha_1 \mu + (1 - \mu) \alpha_2))} \xi_t - \frac{(\alpha_1 \mu + (1 - \mu) \alpha_2)}{(1 + \gamma(\alpha_1 \mu + (1 - \mu) \alpha_2))} \zeta_i.$$

The greater the share of imports, $\mu$, and the greater the sensitivity of domestic production to the (domestic) price of imported goods, $\alpha_2$, the flatter the Phillips curve. Also, the greater is $\mu$ or $\alpha_2$ the greater (smaller) in absolute value is the impact of terms of trade (domestic productivity) shock on output.

In decision making the sequence of events is the following. The public’s expectation of inflation is formed (i.e., wage contracts are signed), the productivity and terms of trade shocks are observed by the central bank, and the producer price inflation rate is chosen.

2.1. The Commitment Equilibrium

Under optimal commitment the government internalizes the effects of its decision rule on expectations. The optimal rule is obtained from the Bellman equation:

$$Z(y_{t-1}) = E_{t-1} \min_{\pi_t, \pi^*_t} \{ \frac{1}{2} \left( \pi_t - \pi^*_t \right)^2 + \lambda (y_t - y^*_t)^2 + \kappa (\omega_t - \pi_t + \pi^*_t)^2 + \beta Z(y_t) \},$$

subject to the constraint $\pi^*_t = E_{t-1} \pi_t$ and to equations (2), (3) and (4). Following procedures outlined in Svensson (1997), the socially optimal inflation rule can be obtained as:

$$\pi_t = \pi^*_t - s(\xi_t - x\zeta_t),$$
\[
s = \frac{(\alpha_1 - \alpha_2)(\lambda + \kappa \gamma^2[(1 - \beta \rho^2) + \beta(1 - \rho^2)])}{(1 - \beta \rho^2)(1 + \alpha_2 \gamma^2 + (\alpha_1 - \alpha_2)^2(\lambda + \kappa \gamma^2[(1 - \beta \rho^2) + \beta(1 - \rho^2)])} > 0,
\]

and

\[
x = \alpha_2 - \frac{\kappa \gamma(1 - \beta \rho^2)(1 + \alpha_2 \gamma)}{(\lambda + \kappa \gamma^2[1 - \beta \rho^2 + \beta(1 - \rho^2)])}.
\]

The solution in equation (6) for producer price inflation differs from the commitment solution in Svensson (1997) for inflation in the closed economy by the inclusion of a shock to the terms of trade term and the coefficient indicating stabilization of productivity shocks. The degree of stabilization in response to productivity shocks (or shocks to the terms of trade) is increasing in \( \kappa \) since an increase in the relative importance of stabilizing the terms of trade requires that change in output is stabilized and this is achieved by allowing greater response in inflation. Through effects on supply of output and inflation, the optimal solution under commitment depends on shocks to the terms of trade even if \( \kappa = 0 \). The optimal stabilization of productivity shocks is smaller, in absolute terms, the more sensitive the supply of output to the terms of trade, and the larger the increase in the price of imported goods induced by output. Under commitment, inflation on average matches the socially optimal rate. Stabilization is greater the more shocks persist (since a change in output lingers) and the greater the value attached to these future effects.

2.2. The Discretion Equilibrium

Svensson (1997) establishes that persistence in output introduces into the solution under discretion state-contingent inflation bias and stabilization bias in the productivity shock in addition to the average inflation bias. Policy under discretion is obtained by minimizing expected social loss in equation (1) with respect to \( \pi_t \) taking expectations about inflation as given. The decision rule under discretion is derived from the Bellman equation:

\[
Z^d(y_{t-1}) = E_{t-1} \min_{\pi_t} \{ \frac{1}{2}((\pi_t - \pi^*_t)^2 + \lambda(y_t - y^*_t)^2 + \kappa(\omega_t - \pi_t + \pi^f_t)^2) + \beta Z^d(y_t) \}.
\]
The minimization is subject to equations (2), (3) and (4). The linear-quadratic form of the model implies that the value function is quadratic. Taking \( Z(y_i) = \psi_0 + \psi_1 y_i + (1/2)\psi_2 y_i^2 \), the first-order condition is given by:

\[
(1 + \alpha_2 \gamma)(\pi_i - \pi^*) + (\alpha_1 - \alpha_2)(\lambda + \beta \psi_2) y_i \\
+ (\alpha_1 - \alpha_2)(\beta \psi_1 - \lambda y_i^*) + \kappa \gamma(\alpha_i - \alpha_2)(\omega_i - \pi_i + \pi^*) = 0.
\]

Applying rational expectations and solving for the parameters of the value function yields the inflation rate under discretion:

\[
\pi_{\text{disc}} = a - c(\xi^*_t - f_{\xi^*_t}) - dy_{-1},
\]

where

\[
\alpha_1 \equiv (\alpha_1 - \alpha_2)/(1 + \alpha_2 \gamma),
\]

\[
a = \pi^* + \frac{\alpha_1 \lambda y^*}{(1 - \beta \rho) + \alpha_1 \beta \rho d},
\]

\[
d = \frac{1}{2\alpha_1 \beta \rho} \left( (1 - \beta \rho^2) - \sqrt{(1 - \beta \rho^2)^2 - 4\alpha_1^2 \beta \rho (\lambda \rho - (1 - \rho)(1 - \beta \rho)\kappa \gamma^2)} \right),
\]

\[
\beta \neq 0, \rho \neq 0, (1 - \beta \rho^2)^2 \geq 4\alpha_1^2 \beta \rho (\lambda \rho - (1 - \rho)(1 - \beta \rho)\kappa \gamma^2),
\]

\[
c = \frac{\alpha_1 (\lambda + \beta d^2 + \kappa \gamma^2 [1 - \beta \rho^2 + \beta (1 - \rho)^2])}{(1 + \alpha_2 \gamma) [1 - \beta \rho^2 + \alpha_1^2 (\lambda + \beta d^2 + \kappa \gamma^2 [1 - \beta \rho^2 + \beta (1 - \rho)^2])]},
\]

and

\[
f = \alpha_2 - \frac{(1 - \beta \rho^2) \kappa \gamma^2 (1 + \alpha_2 \gamma)}{(\lambda + \beta d^2 + \alpha_1 \kappa \gamma^2 [1 - \beta \rho^2 + \beta (1 - \rho)^2])}.
\]

In the model, as in the model in Svensson (1997), the discretion equilibrium in equation (9) involves three kinds of bias in comparison with the commitment equilibrium in equation (6). Under discretion average inflation bias arises in that expected inflation is greater than \( \pi^* \), stabilization bias arises since terms of trade and productivity shocks are not stabilized optimally, and a state-contingent bias arising from the appearance of lagged output in the solution for inflation.
In the model, unlike the model in Svensson (1997), when \( \rho = 0 \), a state-contingent bias remains (it can be shown that \( d = -\hat{\alpha}_1 \kappa \gamma^2 \)). In addition, absence of persistence for the closed economy will not eliminate stabilization bias from either productivity or terms of trade shocks. The source of state-contingent bias and stabilization biases in productivity and terms of trade shocks when \( \rho = 0 \) is the presence of a terms of trade variable in the loss function. With \( \kappa > 0 \) there are benefits from avoiding volatility in the terms of trade. Greater stability in the terms of trade can be achieved by reducing change in output and in effect making output persist under discretion even when \( \rho = 0 \). The discretion equilibrium solution for output includes the state-contingent term \( \hat{\alpha}_1^2 \kappa \gamma^2 y_{t-1} \) which is positive as long as a term of trade variable appears in the loss function.

Persistence in output extends the benefits of higher output with the result that there is excessive stabilization of productivity shocks (\( c > s \)) and of shocks to the terms of trade (\( cf > sx \)). This excessive stabilization of shocks applies even with \( \rho = 0 \) if the loss function contains a terms of trade variable. Persistence also implies an effective state-contingent short run target for output that necessitates a state-contingent component to stabilization under discretion. It can be shown that stabilization bias in productivity shocks, state-contingent bias, and average inflation bias are all damped down the more sensitive output is to unexpected changes in the price of foreign goods, since this flattens out the Phillips curve and reduces output gains from surprise inflation.

3. REPETITION OF COMMITMENT EQUILIBRIUM

Government is assumed to delegate the conduct of monetary policy to an instrument independent central bank.\(^1\)\(^1\) It is shown below that it is possible to replicate the equilibrium under commitment by a combination of exchange rate and income target.\(^1\)\(^2\) We assume delegation is implemented by the period loss function assigned to the central bank given by:

\[
\tilde{L}_t = \frac{1}{2}[(\pi_t - \pi^*)^2 + (1 + \chi)(\hat{\lambda}(y_t - y^*)^2 + \kappa(\omega_t - \pi_t + \pi'^*)^2) + \Phi(h_t - h^*)^2],
\]
where \( h_t = (\tau(\omega_t - \pi_t + \pi^f) + (\pi_t + y_t - y_{t-1})) \), \( h^* \) is a designated target for \( h_t \), \( \Phi \) is the weight assigned to stabilization of \( h_t \) relative to \( h^* \), \( \chi \) is a given adjustment to the weight on output and terms of trade objectives relative to the inflation objective, and \( \tau \) is a parameter specifying the emphasis to be placed on change in the terms of trade \( (\omega_t - \pi_t + \pi^f) \) relative to that on growth in nominal income \( (\pi_t + y_t - y_{t-1}) \) in the composite variable \( h_t \). The optimal \( \tau \) will be a function of the parameters in the model.

A nominal income growth target is given by \( \tau = 0 \), and a value of \( \tau = \mu \) implies \( h_t = \theta_t + y_t - y_{t-1} \), a nominal growth target based on consumer prices. Nominal exchange rate and inflation targets will be related to the model in the following sections. We derive below the \( \tau, h^*, \Phi, \) and \( \chi \) that will result in replication of the optimal equilibrium under commitment.

With the period loss function equation (10), the Bellman equation for the central bank is given by:

\[
(11) \quad Z^b(y_{t-1}) = E_{t-1} \min_{\pi_t} \left\{ \frac{1}{2}((\pi_t - \pi^*)^2 + (1 + \chi)(\lambda(y_t - y^*)^2 + \kappa(\omega_t - \pi_t + \pi^f)) + \Phi(h_t - h^*)^2 + \beta Z^b(y_t) \right\}.
\]

The first order condition is given by (the value function is expressed as \( Z^b(y_t) = \varphi_0 + \varphi_1 y_t + (1/2)\varphi_2 y_t^2 \)):

\[
(12) \quad (\pi_t - \pi^*) + \hat{\alpha}_t((1 + \chi)(\lambda + \beta \varphi_2))y_t - \hat{\alpha}_t((1 + \chi)\lambda y^* - \beta \varphi_1) \\
+ \hat{\alpha}_t(1 + \chi)\kappa \gamma (\omega_t - \pi_t + \pi^f) + \Phi(1 + \hat{\alpha}_t(1 + \gamma \tau))(h_t - h^*) = 0,
\]

where \( \hat{\tau} = (1 + \gamma \tau) \). The application of rational expectations to (12) combined with the constraints in (2), (3) and (4) yields the solution for the path of inflation:

\[
(13) \quad \pi_t = \tilde{\alpha} - \tilde{c}(\xi_t - \tilde{f}_{\xi_t}) - \tilde{d}y_{t-1},
\]
where
\[ \tilde{a} = \pi^* - \hat{\alpha}_1 (\beta \phi_1 - (1 + \chi) \hat{\lambda} y^*) + (1 + \hat{\alpha}_1 \hat{\tau}) \Phi h^* \]
\[ \tilde{d} = \frac{\hat{\alpha}_1 \rho (\beta \phi_2 + (1 + \chi) \hat{\lambda}) - (1 - \rho) (\hat{\alpha}_1 (1 + \chi) \kappa \gamma^2 + \hat{\tau} (1 + \hat{\alpha}_1 \hat{\tau}) \Phi)}{(1 + (1 + \hat{\alpha}_1 \hat{\tau}) \Phi)} \]
\[ \tilde{c} = \frac{\hat{\alpha}_1 [\beta \phi_2 + (1 + \chi) (\lambda + \kappa \gamma^2)] + \hat{\tau} (1 + \hat{\alpha}_1 \hat{\tau}) \Phi}{(1 + \alpha_2 \gamma) \{1 + \hat{\alpha}_1^2 [\beta \phi_2 + (1 + \chi) (\lambda + \kappa \gamma^2)] + (1 + \hat{\alpha}_1 \hat{\tau})^2 \Phi\}} \]
and
\[ \tilde{f} = \alpha_2 - \frac{(1 + \alpha_2 \gamma) \{\hat{\alpha}_1 (1 + \chi) \kappa \gamma + \hat{\tau} (1 + \hat{\alpha}_1 \hat{\tau}) \Phi\}}{\{\hat{\alpha}_1 [\beta \phi_2 + (1 + \chi) (\lambda + \kappa \gamma^2)] + \hat{\tau} (1 + \hat{\alpha}_1 \hat{\tau}) \Phi\}} \]

The coefficients $\tilde{a}, \tilde{c}, \tilde{d}$ and $\tilde{f}$ are functions of $\phi_1$ and $\phi_2$, the parameters in equations (2), (3) and (4), the parameters in the delegated period loss function (10), and of the discount factor. $\phi_1$ and $\phi_2$ are identified in the Appendix. It is shown in the Appendix that the values for $h^*$ and $\Phi$ that will eliminate average and state contingent bias and set expected inflation equal to the value under optimal commitment (i.e. set $\tilde{a} = \pi^*$ and $\tilde{d} = 0$) in equation (13) are given by:

\[ h^* = \pi^* - \frac{\hat{\alpha}_1 (1 + \chi) \hat{\lambda} y^*}{[(1 - \beta \rho) + \hat{\alpha}_1 \hat{\tau} (1 - \beta)] \Phi}, \] (14)

and

\[ \Phi = \frac{\hat{\alpha}_1 (1 + \chi) [\rho \lambda - (1 - \rho)(1 - \beta \rho) \kappa \gamma^2]}{(1 - \rho) \hat{\tau} [(1 - \beta \rho) + \hat{\alpha}_1 (1 - \beta \rho) \hat{\tau}]} , \rho \neq 1, \hat{\tau} \neq 0. \] (15)

The weight on the stabilization of terms of trade relative to nominal income growth, $\tau$, is chosen to equate $\tilde{f}$ in equation (13) to $x$ in equation (6) and thus to eliminate stabilization bias in shocks to the terms of trade relative to stabilization bias in productivity shocks. It is shown in the Appendix that values of $\hat{\tau}$ that set $\tilde{f} = x$ are potentially the roots of the quadratic function $J(\hat{\tau})$ set equal to zero:

\[ J(\hat{\tau}) \equiv \hat{\tau}^2 \hat{\alpha}_1 \lambda + \hat{\tau} \{\lambda + m \kappa \gamma^2\} (1 - \hat{\alpha}_1) - \kappa \gamma^2 (1 - \beta \rho^2) \} - (\lambda + m \kappa \gamma^2) = 0, \] (16)
where \( m \equiv (1 - \beta \rho^2 + \beta(1 - \rho)^2) \), provided \( \hat{\tau} \neq 0 \), \( \hat{\tau} \neq -(\lambda + \beta(1 - \rho)\gamma^2)/(\hat{\alpha}_1 \lambda) \). Since \( \lambda + m k\gamma^2 > 0 \) the roots of (16) are real and of opposite sign. The positive and negative roots are given by:

\[
\hat{\tau}^+, \hat{\tau}^-= \frac{1}{2\hat{\alpha}_1 \lambda} \left( -\Omega \pm \sqrt{\Omega^2 + 4\hat{\alpha}_1 \lambda (\lambda + m k\gamma^2)} \right),
\]

where \( \Omega \equiv [\lambda + m k\gamma^2](1 - \hat{\alpha}_1) - \kappa \gamma^2 (1 - \beta \rho^2)] \). It is shown in the Appendix that the negative root, \( \hat{\tau}^- \), is inadmissible. It will be noted that \( \hat{\tau}^+ \) does not depend on \( \chi \).

An increase in the elasticity of output with respect to change in the domestic price of foreign goods reduces \( \hat{\tau}^+ \):

\[
\frac{\partial \hat{\tau}^+}{\partial \alpha_2} = \frac{(1 + \alpha_2 \gamma)(\lambda \hat{\tau}^+ - (\lambda + m k\gamma^2))\hat{\tau}^+}{(1 + \alpha_2 \gamma)^2 J'(\hat{\tau}^+)} < 0.
\]

Since \( J(\hat{\tau}) \) is strictly convex, we have that \( J'(\hat{\tau}^+) = \partial J(\hat{\tau})/\partial \hat{\tau} \bigg|_{\hat{\tau} = \hat{\tau}^+} > 0 \). The result in equation (18) follows since it can be shown that \( J((\lambda + m k\gamma^2)/\lambda) > 0 \) and that this implies \( \hat{\tau}^+ < (\lambda + m k\gamma^2)/\lambda \). The intuition for the result is that a rise in \( \alpha_2 \) increases the impact of a shock to the terms of trade on output, with the result that the relative weight attached to stabilizing income growth is increased relative to the weight attached to stabilizing the terms of trade. It can also be shown, not surprisingly, that an increase in the weight, \( \kappa \), attached to stabilization of the terms of trade in government’s period loss function, causes the weight to be placed on stabilizing the terms of trade, \( \hat{\tau}^+ \), in the intermediate policy target variable to increase.

The value of \( \chi \), adjustment to the weight on the output and terms of trade terms relative to the weight on the inflation term in the loss function, required to eliminate stabilization bias in productivity shocks and in terms of trade shocks is obtained by equating \( \tilde{c} = s \). From equations (6) and (13) \( \tilde{c} = s \) requires that:
\[(19) \quad 1 + \chi = \frac{(1 - \rho)\hat{\varepsilon}(\lambda + \kappa \gamma^2 m)\[1 - \beta \rho^2 + \hat{\alpha}(1 - \beta \rho)]}{\left[\hat{\alpha}(1 - \beta \rho^2) - \hat{\alpha}(1 + \hat{\alpha}(\lambda + \kappa \gamma^2 m)(\rho \lambda - \kappa \gamma^2 (1 - \rho)(1 - \beta \rho)))\right]},\]

where \( u \equiv [\lambda(1 + \hat{\alpha}) + \kappa \gamma^2 \beta(1 - \rho)^2]. \)

The value of \( \hat{\varepsilon}^+ \) in equation (17) together with \( \chi, \Phi, \) and \( h^* \) obtained from equations (19), (15) and (14), respectively, will result in delegation to a central bank of the loss function in equation (10) that replicates the optimal equilibrium under commitment. The optimal solution for \( h^* \) (from equation (14) after eliminating \( (1 + \chi)/\Phi \)):

\[(20) \quad h^* = \pi^* - \frac{(1 - \rho)\hat{\varepsilon}^+[1 - \beta \rho^2 + \hat{\alpha}(1 - \beta \rho)\hat{\varepsilon}^+]}{[1 - \beta \rho + \hat{\alpha}(1 - \beta)\hat{\varepsilon}^+]v} \lambda y^* < \pi^*,\]

where \( v = \rho \lambda - \kappa \gamma^2 (1 - \rho)(1 - \beta \rho). \) To eliminate average inflation bias, \( h^* \) should be set below \( \pi^* \) (provided \( v > 0; \) this ensures that the penalty weight \( \Phi > 0 \))\(^{13}\). An increase in the elasticity of output with respect to change in the terms of trade reduces the reduction in \( h^* \) below \( \pi^* \), or the less conservative the target \( h^* \), i.e. that:

\[(21) \quad \frac{\partial(\pi^* - h^*)}{\partial \alpha_2} = \frac{(1 - \rho)\lambda y^*}{v} \left\{ \frac{-(1 + \alpha)(\hat{\varepsilon}^+)^2 \beta(1 - \rho)^2}{(1 + \alpha \gamma^2 (n')^2) + \left(\frac{n}{n'} + \hat{\alpha}(1 - \beta \rho)^2\right)} \right\} < 0,\]

where \( n \equiv [1 - \beta \rho + \hat{\alpha}(1 - \beta \rho)] \) and \( n' \equiv [1 - \beta \rho + \hat{\alpha}(1 - \beta)] \) by the result \( \partial \hat{\varepsilon}^+ / \partial \alpha_2 < 0 \) in equation (18). An increase in \( \alpha_2 \) reduces the required degree of conservativeness of \( h^* \) since inflation bias is less the more open the economy to exchange rate movements. It can also be shown that the greater the weight placed on terms of trade stabilization in the government loss function (that is \( \kappa \) is greater), the larger will be the reduction in \( h^* \) below \( \pi^* \), or the more conservative the target \( h^* \). This latter result follows since more
weight is now being placed on ensuring that the inflation rate comes in at a level that keeps change in the terms of trade small.

Examination of equation (19) reveals that the adjustment, $\chi$, necessary to eliminate stabilization bias may be positive or negative. A weight-conservative is implied by $\chi < 0$ and a weight-populist is implied by $\chi > 0$. The effect of an increase in $\alpha_2$ on $\chi$ results in expressions in which most terms are consistent with a decline in $\chi$. This is illustrated by considering the special case of a myopic government and central bank.

**Myopic Central Bank**

For myopic government and central bank, $\beta = 0$, equation (16) implies optimal value $\hat{\tau}^+ = ((\lambda + \kappa \gamma^2) / \lambda)$ and $\tau = ((\kappa \gamma) / \lambda)$. More weight is now placed on stabilizing the terms of trade relative to stabilizing income growth the greater is $\kappa$.

The effects of $\alpha_2$ on the penalty weight $\Phi$ and on $\chi$ when $\beta = 0$ are obtained by substituting $\hat{\tau}^+ = ((\lambda + \kappa \gamma^2) / \lambda)$ into equations (15) and (19) and differentiating with respect to $\alpha_2$ (and noting that for $\beta = 0$, $m = 1$, $u = \lambda n = \lambda(1 + \hat{\alpha}_i \hat{\tau}^+)$). The effects are:

$$\frac{\partial \Phi}{\partial \alpha_2} \bigg|_{\beta=0} = -\frac{(1 + \hat{\alpha}_i \gamma)(1 + \hat{\alpha}_i(1 + (\kappa \gamma^2 / \lambda)\nu))}{(1 + \alpha_2 \gamma)^2[1 + \hat{\alpha}_i(1 + (\kappa \gamma^2 / \lambda))]^2(1 - \hat{\alpha}_i \nu)^2} < 0,$$

and

$$\frac{\partial \chi}{\partial \alpha_2} \bigg|_{\beta=0} = -\frac{(1 - \rho)(1 + \hat{\alpha}_i \gamma)\nu(\lambda + \kappa \gamma^2)}{(1 + \alpha_2 \gamma)^2 \lambda(1 - \hat{\alpha}_i \nu)^2} < 0.$$

An increase in the elasticity of output supply with respect to change in the domestic price of foreign goods or an increase in the share of imports reduces the penalty $\Phi$ for not stabilizing the intermediate policy target and reduces $\chi$. A lower value for $\chi$ makes it more likely that the optimal central banker is an inflation-weight conservative. There is a reduction in penalty because the $\Phi$ needed
to eliminate state-contingent bias is less the more open the economy (the smaller is $\hat{\alpha}$). A smaller value for $\Phi$ implies that stabilization bias introduced by stabilizing the intermediate target is less and the stabilization bias is also less for the more open economy.

4. NOMINAL GROWTH, EXCHANGE RATE, AND INFLATION TARGETS

We will now consider nominal growth, exchange rate, and inflation targets as intermediate monetary policy variables by considering the values that they imply for $\tau$ and $\hat{\tau}$. In the previous section the parameters $h^*, \Phi, \chi$ and $\hat{\tau} = 1 + \gamma \tau$ have been selected optimally so as to eliminate average inflation bias, state-contingent bias, and stabilization biases in shocks to productivity and to terms of trade. The intermediate policy target variable has the form $h_t = (\tau(\omega_t - \pi_t^f) + (\pi_t^f + y_t - y_{t-1}))$. Selection of income, exchange rate, or inflation targets amounts to imposing a value for $\tau$ that will in general not be the optimal value, $\hat{\tau}^*$. The ranking of alternative target regimes is now investigated in terms of stabilization bias in shocks to the terms of trade.

4.1. Nominal Income Growth and Nominal Growth based on Consumer Prices Targets

A nominal income growth target implies that $\tau = 0$ (and $\hat{\tau} = 1$) since this yields $h_t = (\pi_t + y_t - y_{t-1})$. A regime of target growth in income based on consumer prices is obtained if $\tau = \mu$ (and $\hat{\tau} = 1 + \gamma \mu$) since this implies that $h_t = (\mu(\omega_t - \pi_t^f) + (\pi_t^f + y_t - y_{t-1})) = \theta_t + y_t - y_{t-1}$. For non-zero values of parameters, examination of $J(\hat{\tau})$ in equation (16) reveals that $J(1) < 0$ and $J(1 + (m \kappa \gamma^2 / \lambda)) > 0$, with the implication that since $J'(\hat{\tau}^*) = \hat{\partial}J(\hat{\tau})/\hat{\partial}J|_{\hat{\tau}=\hat{\tau}^*} > 0$ the feasible range for $\hat{\tau}^*$ is enclosed within $(1, 1 + (m \kappa \gamma^2 / \lambda))$, and that a nominal income growth target cannot replicate the equilibrium under commitment. It is possible that a regime of target growth in income based on consumer prices (with $\hat{\tau} = 1 + \gamma \mu$) is superior to a nominal income growth target.
From equations (13) and (6) stabilization bias in shocks to the terms of trade relative to shocks to productivity can be expressed as:

\[
\tilde{f} - x = \frac{(1 + \alpha \gamma)wJ(\hat{\tau})}{\tilde{\gamma}(\lambda + m\gamma^2)(1 + \hat{\alpha}_1 \hat{\tau} + \beta(1 - \rho)^2 \gamma^2)} \hat{\tau} \neq 0, \hat{\tau} \neq -\frac{(\lambda + \beta(1 - \rho)^2 \gamma^2)}{\hat{\alpha}_1 \lambda},
\]

where \( J(\hat{\tau}) \) is the quadratic function defined in equation (16).

For a nominal income growth target, \( \hat{\tau} = 1 \), the right hand side of equation (24) is positive (for \( \nu > 0 \)) and stabilization of shocks to the terms of trade is not great enough under a nominal income target regime.\(^{15} \) Since it can be shown that \( \partial(\tilde{f} - x)/\partial \hat{\tau} < 0 \) for \( \hat{\tau} > 0 \), a regime of target growth in income based on consumer prices is superior, in the sense of generating a smaller absolute value in stabilization bias of shocks to the terms of trade, to a nominal income growth target if \( \hat{\tau}^* > 1 + \gamma \mu \) (this is a sufficient but not necessary condition).

If an exchange rate stabilization term does not appear in the government loss function (\( \kappa = 0 \)), equation (24) indicates that stabilization bias is eliminated under a nominal income growth target, \( (\tilde{f} - x)_{\hat{\tau} = 1, \kappa = 0} = 0 \) and the equilibrium under commitment is replicated. A nominal income growth target is more likely to be a better prospect for the economy if the government need not be concerned with real exchange rate variability.\(^{16} \)

If a real exchange rate stabilization term does appear in the government loss function (\( \kappa > 0 \)), comparison between nominal growth targets and others is not quite as straightforward. We consider a myopic central bank so as to clarify the discussion. When \( \beta = 0 \), stabilization bias, \( (\tilde{f} - x)_{\beta = 0} \), is illustrated in Figure 1 for different \( \hat{\tau} \) values. We note that \( (\tilde{f} - x)_{\beta = 0} = 0 \) at \( \hat{\tau}^* = 1 + (\kappa \gamma^2 / \lambda) \). In Figure 1, a nominal income growth target is indicated by NI and a nominal growth based on consumer prices
target is indicated by \( NG(CP) \). As drawn, stabilization bias under \( NI \) is larger than that under \( NG(CP) \), since it is implicitly being assumed that \( \gamma\mu \) is somewhat smaller than \( (\kappa\gamma^2 / \lambda) \).

4.2. Exchange Rate and Inflation Targets

A nominal exchange rate target can be implemented by setting \( \tau = 1 - 1 / \gamma \) together with an adjustment to the variable \( h^* \). Since \( \tau = 1 - 1 / \gamma \) yields \( h_i = \omega_t + \pi^f - (\zeta_t / \gamma) \), the last term in the delegated loss function of the central bank given in equation (10) must be re-written as:

\[
\Phi(h_i - h^*_i)^2 = \Phi(\tau(\omega_t - \pi^f) + (\pi_t + y_t - y_{t-1}) - h^*_i)^2,
\]

where \( h^*_i \equiv h^* - (\zeta_t / \gamma) \), so as to compensate for the appearance of \( - (\zeta_t / \gamma) \) in \( h_i \). For \( \tau = 1 - 1 / \gamma \) equation (25) implies \( \Phi(h_i - h^*_i)^2 = \Phi(\omega_t - (h^* - \pi^f))^2 \) and a pegged exchange rate is implemented with a targeted rate of depreciation given by \( h^* - \pi^f \).

An inflation target in consumer prices is obtained by setting \( \tau = \mu - (1 / \gamma) \) since this yields \( h_i = \theta_t - (\zeta_t / \gamma) \). An inflation target in domestic prices is obtained by setting \( \tau = -1 / \gamma \) since this yields \( h_i = \pi_t - (\zeta_t / \gamma) \). Here we will work with an inflation target in consumer prices. If the target is for inflation in domestic prices, the solution will also be sub-optimal in that a state-contingent bias remains.

Exchange rate and inflation targets are special cases of delegation implemented by the period loss function assigned to the central bank given by:

\[
\tilde{L}_t^Y = \frac{1}{2}[(\pi_t - \pi^*)^2 + (1 + \chi)\lambda(y_t - y^*)^2 + \kappa(\omega_t - \pi^f)^2 + \Phi(h_i - h^*_i)^2].
\]

The solution will be given by:

\[
\pi_t = \tilde{a} - \tilde{c}(\tilde{\xi}_t - \tilde{f}_t\tilde{\zeta}_t) - \tilde{d}y_{t-1},
\]
where \( \bar{a} , \bar{c} , \) and \( \bar{d} \) are defined in equation (13), but \( f' \) differs slightly from \( f \) and is given by:

\[
(28) \quad f' = \alpha_2 - \frac{(1 + \alpha_2 \gamma) \{ \hat{\alpha}_1 (1 + \chi) \kappa \gamma + (\tau + (1/\gamma)) (1 + \hat{\alpha}_1 \hat{\tau}) \Phi \}}{\{ \hat{\alpha}_1 [\beta \phi_2 + (1 + \chi)(\lambda + \kappa \gamma^2)] + \hat{\tau} (1 + \hat{\alpha}_1 \hat{\tau}) \Phi \}}.
\]

Noting that \( \tau + (1/\gamma) = \hat{\tau} / \gamma \), elimination of \( \Phi / (1 + \chi) \) and \( \phi_2 / (1 + \chi) \) from equation (28) yields stabilization of shocks to the terms of trade given by

\[
(29) \quad \tilde{f} = \frac{\alpha_2 \gamma (1 - \rho) - \rho}{\gamma},
\]

which is independent of \( \hat{\tau} \). Thus, given that average, state-contingent, and stabilization bias in productivity shocks have been eliminated, the bias remaining in shocks to the terms of trade is the same under an exchange rate as under an inflation target and is given by (subtracting \( x \) in equation (6) from \( f' \) in equation (29)):

\[
(30) \quad \tilde{f} - x = -\frac{(1 + \alpha_2 \gamma) \nu}{\gamma \{ \lambda + \kappa \gamma^2 (1 - \beta \rho^2 + \beta (1 - \rho)^2) \}} < 0.
\]

The negative sign of \( \tilde{f} - x \) indicates excessive stabilization of shocks to the terms of trade under exchange rate and inflation targets. Note that although stabilization bias under exchange rate and inflation targets is identical, \( \Phi, \kappa, \) and \( h^* \) differ between regimes since under an exchange rate target \( \hat{\tau} = \gamma \) and under a consumer price inflation target \( \hat{\tau} = \gamma \mu \).
4.3. **Comparison of stabilization biases**

We consider a myopic central bank so as to clarify the discussion. If $\beta = 0$ equations (24) and (30) imply that stabilization biases under a nominal income growth target and under an exchange rate target (and under an inflation target) are given by:

\[
(\tilde{f} - x)\big|_{\beta=0} = \frac{(1 + \alpha_2 \gamma) \kappa \gamma v}{\lambda (\lambda + \kappa \gamma^2)},
\]

and

\[
(\tilde{f}' - x)\big|_{\beta=0} = -\frac{(1 + \alpha_2 \gamma) v}{\gamma (\lambda + \kappa \gamma^2)} = -\eta,
\]

respectively.

The expressions in equations (31) and (32) indicate that an exchange rate target or an inflation target will do relatively less well in terms of absolute value of smaller stabilization bias than a nominal income growth target if $\kappa \gamma^2 < \lambda$. This situation is illustrated in Figure 1, where stabilization bias under an exchange rate target or an inflation target, $-\eta$, is greater in absolute value than that under a nominal income growth target.

If a real exchange rate stabilization term does not appear in the government loss function, a nominal income growth target can replicate the optimal equilibrium under commitment, and the other target variables cannot. If real exchange rate volatility does affect government loss, the greater the weight on this term in the government loss function, the more likely it is that an exchange rate or an inflation target results in less stabilization bias than a nominal income growth target. If stabilization of the real exchange rate matters, a target for nominal growth rate based on consumer prices may also dominate a nominal income growth target. In response to a positive (negative) shock to the terms of trade, import prices rise (fall) by more relative to domestic prices under the nominal income target than under the exchange rate target (crawling peg).

5. CONSERVATIVE CENTRAL BANKS
The increased relative weight placed on the output objective in the loss function delegated to the central bank is given by $\chi$ in $\tilde{L}_b^b$ or $\tilde{L}_b$ in equations (10) or (26). A Rogoff (1985b) inflation-weight conservative is then taken to be implied by $\chi < 0$. An inflation-weight populist is implied by $\chi > 0$. To obtain insight into the factors influencing the sign of $\chi$ under alternative targets we consider results for a myopic central bank. In the Appendix it is shown that if $\beta = 0$,

$$\chi|_{\beta=0} = \frac{[\hat{\alpha}_t(\lambda + \kappa \gamma^2) - \hat{\tau}]v}{[\lambda \hat{\tau} - \hat{\alpha}_t(\lambda + \kappa \gamma^2)v]}, \quad \hat{\tau} \neq 0, \hat{\tau} \neq -\frac{1}{\hat{\alpha}_t}, \hat{\tau} \neq \hat{\tau}_1 = \frac{\hat{\alpha}_t(\lambda + \kappa \gamma^2)v}{\lambda}.$$  

Equation (33) gives $\chi$ as a function of $\hat{\tau}$ given that average inflation bias, state-contingent inflation bias, and stabilization bias in productivity shocks have been eliminated. The effect of $\hat{\tau}$ on $\chi$ can be directly compared across targets and is illustrated in Figure 2. At $\hat{\tau}^0 = \alpha_t(\lambda + \kappa \gamma^2), \chi = 0$, and $\chi$ is discontinuous at $\hat{\tau}_1 = (\nu / \lambda)\hat{\tau}^0 < \hat{\tau}^0$. An inflation-weight conservative central bank is more likely under an inflation target than under an exchange rate target since $\mu \gamma < \gamma$, and is more likely under both these rules than under a nominal income growth target since $\gamma < 1$. Figure 2 illustrates a situation in which all targets imply a conservative central bank, consistent with the implicit assumption that $1 + \mu \gamma < \hat{\tau}_1$.

For $\beta = 0$, equation (16) yields $\hat{\tau}^+ = ((\lambda + \kappa \gamma^2) / \lambda)$. This is the value for $\hat{\tau}$ that replicates the equilibrium under optimal commitment. From Figure 2 it can be seen that a conservative central bank will replicate the commitment equilibrium provided either $\hat{\tau}^+ > \hat{\tau}^0$ or $\hat{\tau}^+ < \hat{\tau}_1$. In Figure 2 a conservative central bank under optimal commitment is illustrated under the implicit assumption $\hat{\tau}^+ < \hat{\tau}_1$. In this case, greater elasticity of substitution between domestic and foreign goods (a smaller $\gamma$) and greater persistence makes it more likely that a conservative central bank is chosen to replicate the equilibrium under commitment.
6. CONCLUSION

This paper focuses on the usefulness of exchange rate, inflation, and nominal income growth targets as institutional arrangements for the conduct of monetary policy, given that there is persistence in output and that the foreign exchange rate is influenced by the supply of output relative to foreign output. A framework is developed in which inflation biases with different target variables are compared. The presence of terms of trade variable in the loss function is shown to imply state-contingent bias and stabilization biases in productivity and terms of trade shocks in the discretionary solution even in the absence of persistence for the closed economy. Greater stability in the terms of trade can be achieved by reducing change in output and in effect making output persist under discretion. Persistence in output extends the benefits of higher output with the result that there is excessive stabilization of productivity shocks and of shocks to the terms of trade under discretion.

It is found that a target in nominal growth based on consumer prices may result in less stabilization bias in shocks to terms of trade than a nominal income growth target, given that both targets eliminate average, state-contingent, and stabilization bias from productivity shocks. The greater the weight attached to real exchange rate stabilization the more likely it is that an exchange rate or an inflation target may dominate a nominal income growth target. An inflation-weight conservative central bank is very likely under an inflation target, and is always more likely under an inflation target than under an exchange rate target or a nominal income growth target. It is shown that a target of a weighted sum of change in terms of trade and nominal income growth can replicate the optimal equilibrium without resort to a state-contingent rule when output is persistent. The usual qualifications apply to the results obtained, in that they are derived for given structural relations and information structure.
APPENDIX

Replication of the Optimal Equilibrium without State-Dependent Target

The objective is to show that the optimal rule under commitment can be replicated by target
\[ h_t = (\tau (\omega_t - \pi_t + \pi_i') + (\pi_t + y_t - y_{t-1})), \]
a weighted sum of nominal income growth and change in the
terms of trade, without a state-contingent target. The parameter \( \tau \) is to be chosen so as to be consistent
with replication of the optimal equilibrium. Eliminating \( y_t, \pi_t, \) and \( \omega_t \) from the first order condition in
equation (12), applying rational expectations, and taking the parameters of the value function as given
yields the inflation rate given in equation (13).

To complete the solution for inflation as a function of the parameters in the model, the Bellman
equation (11) is examined to identify \( \phi_1 \) and \( \phi_2 \). Substituting (13) into (11) yields a Riccati equation in
\( \phi_2 \):

(A.1) \[ \phi_2 = \tilde{d}^2 + (1 + \chi)(\lambda \rho^2 + \kappa \gamma^2 (1 - \rho)^2) + \Phi(\tilde{d} + (1 - \rho)\tilde{\tau}^2) + \beta \phi_2 \rho^2. \]

Eliminating \( \phi_2 \) from (A.1) using the definition of \( \tilde{d} \) in (13) yields a quadratic equation in \( \tilde{d} \):

(A.2) \[ 0 = \tilde{d}^2 (1 + \Phi)\tilde{h}_t \beta \rho - \tilde{d}[(1 - \beta \rho^2)(1 + \Phi(1 + \tilde{h}_t \tilde{\tau}) - 2\tilde{h}_t \beta \rho(1 - \rho)\tilde{\tau} \Phi)] \]
\[ + \tilde{h}_t \beta \rho(1 + \chi)(1 - \rho)(1 + \chi)(1 - \beta \rho)\kappa \gamma^2 - \Phi(1 - \rho)\tilde{\tau}[(1 - \beta \rho^2) + \tilde{h}_t (1 - \beta \rho)\tilde{\tau}] . \]

If (A.2) is re-written as \( 0 = \tilde{d}^2 A - \tilde{d} B + X \), real solutions for \( \tilde{d} \) exist if \( B^2 - 4AX \geq 0 \). Examination of
(A.2) indicates \( \tilde{d} = 0 \) if

(A.3) \[ \Phi = \tilde{h}_t (1 + \chi)[\rho \lambda^2(1 - \rho)(1 - \beta \rho)\kappa \gamma^2] \]
\[ (1 - \rho)\tilde{\tau}[(1 - \beta \rho^2) + \tilde{h}_t (1 - \beta \rho)\tilde{\tau}] , \quad \rho \neq 1, \tilde{\tau} \neq 0, \tilde{\tau} \neq -(\lambda + \beta (1 - \rho)^2 \kappa \gamma^2) \tilde{h}_t \tilde{\tau} . \]

Substituting (13) into (11) and equating the coefficients of \( y_{t-1} \) on the left-hand and right-hand
sides of equation (11) yields a Riccati equation in \( \phi_1 \):

(A.4) \[ \phi_1 = -(\tilde{a} - \theta^*)\tilde{d} - (1 + \chi) \rho \lambda \gamma^* - \Phi(\tilde{d} + (1 - \rho)\tilde{\tau})(\tilde{a} - \gamma^*) + \beta \phi_1 \rho . \]
Equations (13) and (A.4) can be used to solve for \( \tilde{a} \) and \( \varphi \) in terms of the parameters of the model. The value for \( h^* \), that sets \( \tilde{a} = \theta^* \) given that \( \Phi \) has been selected to set \( \tilde{d} = 0 \), is given by:

\[
(A.5) \quad h^* = \theta^* - \frac{\alpha (1 + \chi) \lambda y^*}{[(1 - \beta \rho \gamma) + \hat{\alpha}_i \hat{\tau}(1 - \beta)] \Phi}, \quad \rho \lambda - (1 - \rho)(1 - \beta \rho) \nu \gamma^2 \neq 0.
\]

The restriction \( \rho \lambda - (1 - \rho)(1 - \beta \rho) \nu \gamma^2 \neq 0 \) ensures that \( \Phi \neq 0 \). If \( \rho \lambda - (1 - \rho)(1 - \beta \rho) \nu \gamma^2 > 0 \), then \( \Phi > 0 \) and \( h^* - \theta^* < 0 \).

The solutions for \( \varphi_1 \) and \( \varphi_2 \) are given by:

\[
(A.6) \quad \varphi_1 = -\frac{(1 + \chi) \lambda y^* (\rho + \hat{\alpha}_i \hat{\tau})}{[(1 - \beta \rho \gamma) + \hat{\alpha}_i \hat{\tau}(1 - \beta)]}, \quad \varphi_2 = \frac{(1 + \chi) \nu \gamma \lambda \Phi}{[(1 - \beta \rho) + \hat{\alpha}_i \hat{\tau}(1 - \beta)]} > 0.
\]

The weight on the stabilization of terms of trade relative to nominal income growth, \( \tau \), is chosen to equate \( \tilde{f} \) in equation (13) to \( x \) in equation (6). Substituting for \( \varphi \) and \( \Phi \) from equations (A.3) and (A.6) into (13) yields the following condition for the optimal value of \( \hat{\tau} \):

\[
(A.7) \quad \tilde{f} - x = -\frac{1 + (\alpha_1 + \alpha_2) \gamma}{\hat{\tau} \gamma (\lambda + m \nu \gamma^2)} v f(\hat{\tau}) = 0, \quad \hat{\tau} \neq 0, \hat{\tau} \neq -\frac{(1 + \chi) \omega \gamma \lambda \Phi}{\hat{\alpha}_i \lambda}, \quad J(\hat{\tau}) \text{ is a quadratic function of } \hat{\tau} \text{ given by:}
\]

\[
(A.8) \quad J(\hat{\tau}) = \hat{\tau}^2 \hat{\alpha}_i \lambda + \hat{\tau} \{(\lambda + m \nu \gamma^2) (1 - \hat{\alpha}_i) - \nu \gamma^2 (1 - \beta \rho \gamma^2) \} - (\lambda + m \nu \gamma^2),
\]

where \( m \equiv (1 - \beta \rho \gamma^2 + \beta (1 - \rho) \gamma^2) \). \( \tilde{f} - x = 0 \) if \( J(\hat{\tau}) = 0 \) provided \( \hat{\tau} \neq 0, \hat{\tau} \neq -\frac{(1 + \chi) \omega \gamma \lambda \Phi}{\hat{\alpha}_i \lambda} \).

Since \( \lambda + m \nu \gamma^2 > 0 \) the roots of \( J(\hat{\tau}) = 0 \) are real and of opposite sign and are given by:

\[
(A.9) \quad \hat{\tau}^+, \hat{\tau}^- = \frac{1}{2 \hat{\alpha}_i \lambda} \left( -\Omega \pm \sqrt{\Omega^2 + 4 \hat{\alpha}_i \lambda (\lambda + m \nu \gamma^2)} \right),
\]

where \( \Omega = [(\lambda + m \nu \gamma^2) (1 - \hat{\alpha}_i) - \nu \gamma^2 (1 - \beta \rho \gamma^2)] \). It will be noted that \( \hat{\tau}^+ \) and \( \hat{\tau}^- \) do not depend on \( \chi \).

Note that the negative root, \( \hat{\tau}^- \), is frequently inadmissible. For example, for \( \beta = 0 \), \( \hat{\tau}^- \) becomes \(-1/\hat{\alpha}_i \), a value for \( \hat{\tau} \) precluded in the solution for \( \Phi \) and other parameters. Selection of \( \hat{\tau}^+ \) results in optimal stabilization of shocks to terms of trade given optimal \( \chi \).
It remains to choose the value of $\chi$ that will adjust stabilization of productivity shocks to the optimal value. Substituting for $\Phi$ and $\varphi_2$ from equations (A.3) and (A.6), the coefficient $\tilde{c}$ in the inflation rate $\theta_i = \theta^* - \tilde{c}(\xi_i - f\xi_i)$ in (13) becomes:

\begin{equation}
\tilde{c} = \frac{(1 + \chi)\hat{\alpha}_i\hat{\nu}}{(1 + \alpha_2\gamma)[(1 - \rho)\hat{\nu} + (1 + \chi)\hat{\alpha}_i,((\rho + \hat{\alpha}_i,\hat{\nu})u - \kappa\gamma^2(1 - \rho)n)]},
\end{equation}

where $n = [1 - \beta\rho^2 + \hat{\alpha}_i,\hat{\nu}(1 - \beta\rho)]$ and $u = [\lambda(1 + \hat{\alpha}_i,\hat{\nu}) + \kappa\gamma^2\beta(1 - \rho)^2]$. Equating $\tilde{c} = s$ in equation (6) yields the optimal adjustment to the relative weight on the output and terms of trade objectives:

\begin{equation}
1 + \chi = \frac{(1 - \rho)\hat{\nu}(\lambda + \kappa\gamma^2m)}{[\hat{\nu}(1 - \beta\rho^2) - \hat{\alpha}_i,((\rho + \hat{\alpha}_i,\hat{\nu})u - \kappa\gamma^2(1 - \rho)n)]},
\end{equation}

where $\hat{\nu} = \rho\lambda - (1 - \rho)(1 - \beta\rho)\kappa\gamma^2$.

Substituting optimal $\hat{\nu} = \hat{\nu}^*$ from equation (A.9) into (A.11) yields the optimal value for the adjustment to weight on output and exchange rate stabilization relative to inflation stabilization, $1 + \chi$. Substituting $\hat{\nu} = \hat{\nu}^*$ from equation (A.9) and $1 + \chi$ from equation (A.11) into Equations (A.5) and (A.3) yields optimal intermediate target and optimal penalty weight:

\begin{equation}
h^* = \theta^* - \frac{(1 - \rho)\lambda^* n}{[1 - \beta(1 - \beta\rho^2) + \hat{\alpha}_i,\hat{\nu}(1 - \beta)]v} \text{ and } \Phi = \frac{\hat{\alpha}_i,\nu}{(1 - \rho)\hat{\nu}^* n}(1 + \chi).
\end{equation}

These values for the parameters introduced into the period loss function, $F_{\eta}^b$, will result in replication of the commitment equilibrium in equation (6).

**Myopic Government and Central Bank**

For the optimal weight on the output and terms of trade terms relative to the weight on the inflation term in the delegated loss function with myopia take $\beta = 0$, $m = 1$, $u = \lambda n = \lambda(1 + \hat{\alpha}_i,\hat{\nu})$, and $\hat{\nu} = ((\lambda + \kappa\gamma^2)/\lambda)$, in equation (A.11) to yield:
Nominal Growth, Exchange Rate, and Inflation Targets

Income growth, growth based on consumer prices, exchange rate, and consumer price inflation targets imply \( \tau = 0 \) (\( \hat{\tau} = 1 \)), \( \tau = \mu \) (\( \hat{\tau} = 1 + \gamma \mu \)), \( \tau = 1 - 1/\gamma \) (\( \hat{\tau} = \gamma \)), and \( \tau = \mu - 1/\gamma \) (\( \hat{\tau} = \gamma \mu \)), respectively. Substituting \( \hat{\tau} = 1 \) into equation (A.7) yields stabilization bias in shocks to terms of trade, \( \tilde{f} - \tilde{x} \bigg|_{\kappa=0} \) and \( \tilde{f} - \tilde{x} \bigg|_{\beta=0} \), under nominal income growth target. Stabilization bias in shocks to terms of trade under an exchange rate or inflation target is expressed in equation (30) and is independent of \( \hat{\tau} \).

Measure of inflation-weight conservativeness (\( -\chi \)) under income growth, growth based on consumer prices, exchange rate, and consumer price inflation targets is obtained by substituting for \( \hat{\tau} = 1 \), \( \hat{\tau} = 1 + \gamma \mu \), \( \hat{\tau} = \gamma \), and \( \hat{\tau} = \gamma \mu \), in equation (A.11), respectively.
REFERENCES


Figure 1: Stabilization Bias in shocks to term of trade when $\beta = 0$ (other inflation biases eliminated).

Note: NI indicates income growth target and NG(CP) indicates nominal growth based on consumer prices target. Stabilization Bias in shocks to term of trade under inflation and exchange rate targets equals $-\eta$. 
Figure 2: Inflation-Weight Conservativeness ($-\chi$)

Note: NI indicates income growth target, NG(CP) indicates nominal growth based on consumer prices target, INF indicates inflation target, and NX indicates exchange rate target.
FOOTNOTE

1 The literature on inflation and nominal income growth targeting is vast. See for example recent contributions by
McCallum and Nelson (1999), Walsh (2003), Guender and Tam (2004), and Kim and Henderson (2005) and the
references within.
2 Levy-Yeyati and Sturzenegger (2005) also find that countries that claim to float act from time to time to stabilize
their exchange rates. In an investigation of whether central banks respond to exchange rate movements, Lubik and
Schorfheide (2006) report that the central banks of Canada and the U.K. include a nominal exchange rate in their
policy rules but that the central banks of Australia and New Zealand do not.
3 In the dynamic inconsistency in monetary policy literature loss functions for the central bank have incorporated
Rogoff (1985b) weight-conservativeness, Persson and Tabellini (1993) and Walsh (1995) performance contracts, and
Svensson (1997) conservative inflation targets.
4 Lockwood (1997) and Svensson (1997) establish that a state-contingent linear inflation contract can replicate the
commitment equilibrium when there is persistence in employment. Beetsma and Jensen (1999) and Roisland (2001)
show that a nominal income growth target can mimic the optimal equilibrium without explicit state-contingency.
5 Empirical results justify assumptions underlying a non-identical quadratic approximation for stabilization functions
in the open and closed economy cases. Anderson and van Wincoop (2004) report findings of quite elastic estimates
of the elasticity of substitution in consumption between domestic and imported goods. Choudhri and Hakura (2006)
present evidence that incomplete pass through is empirically important for emerging economies.
6 The loss function in equation (1) is similar to loss functions that have appeared in earlier literature, such as in
Persson and Tabellini (1996) and Giavazzi and Pagano (1988). Governments may wish to avoid excessive volatility
in the terms of trade to ensure competitiveness of exports and ensure stability of the banking and financial sector.
Persson and Tabellini (1996) observe that an inflation target over consumer prices tends to stabilize the exchange
rate and that an inflation target over domestic prices tends to make output less variable. Krugman and Obstfeld
(2000) note that governments in developing countries are concerned about excessive exchange rate volatility
influencing stability in other markets.
7 Rogoff (1985a) specifies consumer inflation in both domestic and foreign central bank objective functions.
8 The papers cited introduce dynamics by assuming that the employment target of the wage setter depends on the
employment of insiders (or recent employees). Alogoskoufis and Manning (1988) point out that persistence in
employment could also arise if firms face costs in adjusting employment. Researchers have also been concerned with
explaining persistence in the real exchange rate. Benigno (2004) does so in terms of central bank inertia and
nominal rigidity.
9 Open economy models by Clarida et al. (2001) and Gali and Monacelli (2005) (based on Calvo (1983)) obtain a
relationship with terms of trade proportional to an output gap.
10 Razin and Yuen (2002), in an extension of Woodford’s (2003) model of a closed economy, show that the more the
open the economy the flatter the Phillips curve.
11 Under delegation considered in this paper it is assumed that the objective function of the central bank is
announced. Mishkin (2004) argues that transparency regarding a complicated objective function might hinder
conduct of policy with a focus on long-run objectives.
12 Alternative means of implementation of the optimal equilibrium have been considered by Svensson (2003),
Woodford (2003, Chapter 7), Giannoni and Woodford (2003a; 2003b), and Svensson and Woodford (2005) under
which the central bank is charged with implementing some targeting criteria, such as an inflation-forecast targeting
rule. This is part of an attempt to design institutional arrangements that might more readily encourage
implementation of the commitment equilibrium.
13 The assumption $v > 0$ amounts to assuming that for given (positive) level of persistence the relative weight for
government on stabilizing output is greater than that on stabilizing change in the terms of trade.
14 For the economy being open enough an inflation-weight conservative is optimal. For $\beta = 0$, equation (19) implies
that $\chi = -(1 - \dot{\alpha}_1)\nu/[\lambda(1 - \dot{\alpha}_1\nu)].$ For $1 + \chi > 0$, it can be seen that $\chi < 0$ if $\dot{\alpha}_1 < 1/\lambda$.
15 Under a nominal income target regime, too little stabilization of shocks to the terms of trade will imply that output
and inflation are too variable in response to shocks to the terms of trade compared to the commitment equilibrium.
16 This result holds because of an equivalence between closed and open economy models when $\kappa = 0$. Thus, when
$\kappa = 0$ the problem is formally identical to that considered by Beetsma and Jensen (1999) and Roisland (2001) and a
target in nominal income growth results in replication of the optimal equilibrium under commitment.