Cyclical Co-movement between Output, the Price Level, and Inflation

Joseph H. Haslag†

Yu-Chin Hsu

Department of Economics
University of Missouri at Columbia

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1 Introduction

Over time, there has been a dramatic change in our understanding of the relationship between the price level and output over the business cycle. For several decades, the conventional wisdom maintained that the price level are procyclical. Arguably, the biggest development in our understanding came about because Lucas (1977) offered a transformative elegant definition of the business cycle itself. Armed with the definition that business cycles are deviations in output from trend, researchers applied new econometric techniques to re-consider key business-cycle facts.¹

In this paper, we concentrate on two related sets of business-cycle facts. More specifically, we consider the contemporaneous correlation between the price level and output and between the inflation rate and output. Of course, the relationship between the price level and inflation is tautological; the inflation rate is the time derivative of the log of the price level. The existing evidence indicates a very interesting pair of observations; namely, that the price level is countercyclical and the inflation rate procyclical.²

There is a simple explanation—a kind of folk theorem—that can account for the countercyclical price level and procyclical time derivative of the price level. Consider the price level and output are characterized by two sine waves with a given periodicity. Certainly, there is a phase shift that exists in which the price level movements are negatively correlated with movements in output and the changes in the slope of the sine waves is positively correlated with movements in output. Thus, our first examination is to estimate two correlation coefficients; first, there is the price level and output and second, there is the inflation rate and output. Over the full sample period, we verify that the price level is countercyclical and the inflation rate is procyclical. Next, we use a numerical simulation to verify that a phase shift can account for these correlations.

¹See Lucas (1977, p.9).
²See, for example, Kydland and Prescott (1990), Cooley and Ohanian (1991), and King and Watson (1996).
More concretely, if the price level series and the output series are in phase, the estimated correlation between the price and output is negative and so is the estimated correlation between the inflation rate and output. So, we consider a positive phase shift applied to the price level series. For an appropriate size positive phase shift, the correlation between the simulated price level and simulated output is negative while the correlation between simulated inflation and simulated output is positive. Note that a positive phase shift applied to the price level is consistent with movements in the price level temporally preceding, or Granger causing, output. We test for Granger causality and present evidence consistent that movements in the price level Granger cause

In addition, we examine whether the correlations are stable over the sample period. We present evidence that the correlation between the price level and output changed over time. Based on a rolling set of sample period that are 120 quarters long, the evidence indicates that the price-output correlation is consistent and that countercyclical relationship is becoming stronger when estimated over the most recent 45-year sample than it was when estimated using the 45-year sample beginning in 1947. Meanwhile, the same rolling sample period indicates that the inflation-output correlation is consistent and that the procyclical relationship is weakening over time.

We conduct numerical simulations to study two possible explanations. One is that phase shift is varying over time. Indeed, we demonstrate that with a diminishing positive phase shift in the price level, it is possible to observe a stronger countercyclical price-output correlation and a weaker procyclical inflation-output correlation. Another possible explanation is the wavelength of the price level is different than the wavelength of output. We provide evidence that the wavelength of the price level is slightly longer than the wavelength of output. However, when we test whether the difference between the two wavelengths is significant, we cannot reject the null hypothesis that the two wavelengths are equal.

The outline of the paper is as follows. We provide a brief overview of the developments in the cyclical relationship between the price level and output in
Section 2. The history helps to motivate why the time varying relationship is considered. In Section 3, we present the results of the statistical evidence for the full sample period. We examine the evidence on time-varying correlations in Section 4. We offer a brief summary of our findings in Section 5.

2 Views on the price-output relationship over time

There is a huge literature that reports co-movements of variables at business cycle frequencies. As different techniques emerge, researchers examine different questions. As such, there is an evolution of the stylized facts. In our view, one of the most interesting progressions involves the relationship between output and the price level. In particular, the contemporaneous correlation between the price level and output.

Friedman and Schwartz (1963) provided the rationale for a belief system that nakedly asserted that the price level were procyclical. Friedman and Schwartz’s (1963) central thesis was that monetary disturbances were the most plausible source of cyclical fluctuations. After carefully documenting the relationship between money and output for every business cycle from the Civil War to 1970, Friedman and Schwartz (1963) concluded that monetary disturbances were the primary candidate capable of being the main source of business cycle fluctuations. In part, they search were for sources that were "large enough" to shock a large, industrial economy. Armed with facts presented in the Monetary History and with the Quantity Theory, the monetary surprise literature sought to explain procyclical the price level. In other words, procyclical the price level were listed among the set of business cycle regularities. Accordingly, models were judged on their ability to predict procyclical the price level.3

Kydland and Prescott (1990) overturned this belief system, implementing

3See, for example, Bernanke (1986) and Mankiw (1989). Both raised concerns about the King and Plosser model because it did not explain procyclical prices.
a methodological improvement and using that methodology to report that the price level is countercyclical. The methodological improvement owed to the definition put forward by Lucas (1977). Specifically, Lucas (1977) defined business cycles as the deviations of aggregate output from trend. Kydland and Prescott (1990) adopted this definition, estimated the deviations in trend for both output and the price level (hereafter referred to as the cyclical components), and computed the contemporaneous correlation coefficient. For the sample period 1954 through 1989, they reported the correlation coefficient with the Implicit GNP Deflator was -0.57 and with the Consumer Price Index was -0.57. It is critical to note that the characterization of price-output correlation represents a change in measurement; that is, the definition of cyclical observations changed from characterizing leads and lags at business-cycle turning points to quarterly observations of detrended data.

Given that the price level are countercyclical, the equilibrium models with monetary policy or price surprises had to be modified or simply ignore the price-output correlation. At the heart of the monetary surprise models was a Phillips Curve relationship. Rather than abandon the monetary surprise models, researchers moved to another cyclical relationship. Specifically, equilibrium models sought to explain the relationship between inflation and output. Cooley and Ohanian (1991) provided support for this modified approach, reporting that the relationship between output and inflation is positive during the post-war period.

The purpose of this paper is to develop a simple framework that ties the disparate results between the cyclical component of output and the cyclical component of the price level and inflation. We begin with a characterization of the cyclical relationship between the price level and output and inflation and output using data for the Post World War II period. By examining these results from the perspective of a simulated deterministic relationship, it is possible to account for the observations as the result of a simple phase shift. By taking this approach, the countervailing cyclical relationships are viewed as part of an integrated approach. Once you get an equilibrium model that can account for
the relationship between the price level and output, it becomes much easier to see how the dynamic equilibrium incorporates the relationship between output and inflation. This integration is quite natural since there is a deterministic relationship between the price level and inflation.

In addition, there are important questions about the robustness of the business-cycle relationships over time. For example, Cooley and Ohanian (1991) report the contemporaneous correlation between output and the price level over three different postwar periods: 1948 through 1987, 1954 through 1973, and 1966 through 1987. The evidence indicates that correlation coefficient between output and the price level is algebraically smaller in the 1966-87 sample compared with the one estimated over the 1954-73 sample. In other words, the price level exhibit a stronger countercyclical relationship in more recent data when compared with the relationship in the first two decades after World War II. Cooley and Ohanian (1991) extend their analysis to include annual data from Friedman and Schwartz (1963). After detrending, they report coefficients for four different sample periods: 1870-1900, 1900-1928, 1928-1946, and 1949-1975. They find that the output-price correlation is positive for sample periods before World War and negative for the one sample period using postwar data. Together, these facts provide evidence that is consistent with the notion that the correlation has been changing over time between detrended output and the price level.

The literature has also offered other ways to explain the developments in business cycle facts.

### 3 Full-sample Results

Our first result is to report that the full-sample correlation between detrended the price level and output is significantly negative while the correlation between detrended inflation and output is significantly positive. Using sine functions to create simulated price and output series, we show that positive phase shift of the price level series is consistent with our reported findings. In our case, a left-phase shift of the price level series implies that movements in the cyclical
component of the price level would temporally precede or Granger cause the
cyclical component of output. As a check, we test whether detrended the price
level Granger cause detrended output and vice-versa. The tests are consistent
with unidirectional causation; that is, the price level Granger cause output, but
output does not Granger cause the price level. So, the results are consistent
with the left-phase shift account of the relationship between the price level and
output.

Following Lucas (1977), business cycles are defined as deviations of aggregate
output from trend. We use the Hodrick-Prescott filter to identify the trend
component. For a generic time series, let there exist a sequence of observations
for a random variable $x_t$ for $t = 1, 2, 3, \ldots, T$. At each date $t$, the realization of
can be decomposed into its trend and cyclical components. Formally, let

$$ x_t = x_t^q + x_t^c $$

where $x_t^q$ denotes the trend component and $x_t^c$ represents the cyclical compo-

\footnote{Formally, let $x_t^q$ denote the trend component for $t = 1, 2, \ldots, T$. The Hodrick-Prescott filter minimizes

$$ \sum_{t=1}^{T} (x_t - x_t^q)^2 + \lambda \sum_{t=2}^{T-1} \left[ (x_{t+1}^q - x_t^q) - (x_{t}^q - x_{t-1}^q) \right]^2 $$

Following the literature, we set $\lambda = 1600$ for quarterly observations. See Hodrick and Prescott (1997) for more details.}

\footnote{The price level is defined as the chain-type GDP deflator. All data are obtained from the Federal Reserve Bank of St. Louis FRED database with the Bureau of Economic Analysis as the source data.}

\footnote{4} For both real GDP and the price level measure, we use log transfor-
mations. The data are quarterly observations of real GDP for the period 1947
through 2010.\footnote{5} In addition, we compute the inflation rate for the period 1947:2
through 2010:4, where $\pi_t = p_t - p_{t-1}$ with $p_t$ representing the (log of) the price
level.

There are two ways to compute the cyclical component of the inflation rate.
We could apply the Hodrick-Prescott filter directly to the inflation rate com-
puted from the source data; that is, $\pi_t = \pi_t^q + \pi_t^c$, where $\pi_t^q$ is the result of

\footnote{The price level is defined as the chain-type GDP deflator. All data are obtained from the Federal Reserve Bank of St. Louis FRED database with the Bureau of Economic Analysis as the source data.}
decomposing the inflation time series. Note, however, that equation (1) implies that

\[ \pi_t = p_t^q - p_{t-1}^q + p_t^c - p_{t-1}^c. \] (2)

Thus, based on equation (2), the cyclical component of the price level could be used to compute a "consistent" cyclical component of the inflation rate by setting \( \pi_t^c = p_t^c - p_{t-1}^c \). In small samples, the direct method of computing the cyclical component of the inflation is likely to commingle the trend component of the price level with the cyclical component of the price level. Since there is no guarantee that \( \pi_t^c = p_t^c - p_{t-1}^c \), we perform a check, computing the correlation coefficient for \( p_t^c - p_{t-1}^c \) from the consistent approach and \( \pi_t^c \) from the direct approach. The correlation coefficient is 0.99. Because the correlation coefficient is less than one, though not significantly so, we recognize that the trend component of the price level in equation (2) is commingled into the cyclical component of the inflation rate computed by the direct approach. We confirm that the correlation is close enough that the results reported in this paper when using the direct approach are qualitatively identical to those obtained using the consistent approach. Therefore, throughout this paper we will use the direct approach to computing the cyclical component of the inflation rate.

Table 1 reports two summary statistics for the cyclical components of output, the price level and the inflation rate for the full sample. Figures 1, 2, and 3 plot the cyclical components for output, the price level and the inflation rate, respectively. In Table 1, we report the standard deviation of the cyclical components and the contemporaneous cross-correlation of the cyclical component output to the cyclical component of both the price level and the inflation rate separately.

<table>
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<tr>
<th>Variable</th>
<th>Stand Dev</th>
<th>Corr Coeff w/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.0167</td>
<td>1</td>
</tr>
<tr>
<td>p</td>
<td>0.0095</td>
<td>-0.216</td>
</tr>
<tr>
<td>( \pi )</td>
<td>1.583</td>
<td>0.293</td>
</tr>
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</table>
The standard deviation tells us the average amplitude of the cyclical component. The average amplitude of the cyclical component of output is 1.7 percent while the average amplitude of the cyclical component of the price level is slightly less than one percent.

The correlation coefficient characterizes the average co-movement in the cyclical components. Because business cycles are defined by Lucas (1977) as deviations from trend output, we compute the correlation coefficients relative to real GDP. As Table 1 shows, the negative correlation coefficient between output and the price level indicates that the price level is mildly countercyclical. Interestingly, the evidence indicates that inflation is mildly procyclical. The standard error of the estimate of the correlation coefficient is 0.063. Thus, in both cases, we can reject the null hypothesis that the correlation coefficient is equal to zero at the five-percent confidence level.

3.1 A Phase-shift explanation

The correlation coefficient between the cyclical components of the price level and output has the opposite sign of the correlation coefficient between the cyclical components of inflation and output. One way to account for these two results is to submit that there is a phase shift between the cyclical components of the price level and output. Because the inflation rate is the slope of the price level curve with respect to time, the evidence suggests that the slope of the cyclical component of the price level is, on average, the opposite sign of the slope of the cyclical component of the price level. Moreover, the change of the slope of the cyclical component of the price level—that is, the cyclical component of the inflation rate—on average, is the same sign as the slope of the cyclical component of output.

To illustrate how a phase shift can account for this pair of results, we use a simple numerical example. Consider two sine waves. Let $\hat{y} = \sin(x)$ for $x$.

\[\text{See Ashley, Granger and Schmalensee (1980) for the estimator of the variance of the correlation coefficient.}\]
$\in \mathbb{R}$. Similarly, let $\hat{p} = -\sin(x + \gamma)$ where $2\pi > \gamma > 0$. In comparing $\hat{y}$ and $\hat{p}$ it is straightforward to see that $\hat{p}$ is the inverse of $\hat{y}$ and graphically is phase shifted to the left by $\gamma$. In other words, the graphs of the two variables would correspond to a correlation coefficient of $-1$ for $\gamma = 0$. Let $\rho(\hat{y}, \hat{p})$ denote the correlation coefficients for $\hat{y}$ and $\hat{p}$. Let $\hat{\pi} = \frac{d\hat{p}}{dx}$, or the slope of the sine function characterizing $\hat{p}$. Formally, $\frac{d\hat{y}}{dx} = \cos(x)$ and $\frac{d\hat{p}}{dx} = -\cos(x + \gamma)$. Hence the slope of the simulated price and simulated output are opposite. The change in the slope of the simulated price series is represented by $\frac{d^2\hat{p}}{dx^2} = \sin(x + \gamma)$.

Clearly, $\gamma$ determines the correlation coefficient between simulated output and simulated price level. Indeed, by shifting the price level series "far enough" it is possible to have a positive correlation. Similarly, the correlation coefficient between the simulated output series and the simulated inflation rate depends on the size of the phase shift.\footnote{While this analysis presents the results for a left-phase-shift applied to the price level series, it is obviously equivalent to consider a right-phase shift to the simulated output series. We normalize on the output cycle because business cycles are defined in terms of deviations in output from trend.}

Let $\gamma \in [0, \hat{\gamma}]$ where $\hat{\gamma}$ is defined as the value of $\gamma$ for which $\rho(\hat{y}, \hat{p}) = 0$. There exists values $\gamma \in [\gamma, \hat{\gamma}]$ for which $\rho(\hat{y}, \hat{p})$ is negative and $\rho(\hat{y}, \hat{\pi})$ is positive. In this sense, the cyclical relationship between the price level and output can be characterized as a phase shift. Analytically, the degree of the phase shift is small enough to maintain the negative correlation between $\hat{y}$ and $\hat{p}$. The phase shift parameter essentially moves the slope of the simulated output series toward the change in the slope of the simulated price level series for small enough values of $\gamma$.

In our numerical example, let $\gamma = 0.75$. We plot $\hat{y} = \sin(x)$ and $\hat{p} = -\sin(x + 0.75)$ for $x \in [-2\pi, 2\pi]$ in Figure 4. To develop the intuition further, by shifting the sine curve for the simulated price level to the left of the simulated output sine curve, we draw attention to the trough of the simulated price curve. In the neighborhood of the simulated price curve’s trough, the slope of the curve is increasing. Meanwhile, just prior to the trough in the simulated price curve,
the slope is negative and the slope of the simulated output curve is positive. Turning points are the easiest to illustrate the two correlations, but the sample statistics are in line with the intuition at the turning points. For $\gamma = 0.75, \rho(\hat{y}, \tilde{p}) = -0.732$ and $\rho(\hat{y}, \tilde{\pi}) = 0.26$. Thus, we demonstrate that a left-phase shift in the simulated price curve relative to the simulated output curve can account for the negative correlation between the price level and output and the positive correlation between inflation and output.

### 3.2 Implications for Granger causality

By applying a left-phase shift to the simulated price level series relative to the simulated output series, it is natural to test whether the actual time series exhibit Granger causality. Specifically, Granger causality indicates whether movements in the price level help to predict movements in output. A left-phase shift corresponds to movements in the simulated price level temporally preceding movements in the simulated output series. Here, temporal precedence is consistent with the ability of movements in one variable to help forecast movements in the other variable. Therefore, we can test whether movements in the cyclical component of the price level temporally proceed movements in the cyclical component of output. Such evidence is consistent with a left-phase shift characterizing the relationship between the two time series.

To check whether the temporal precedence is one-way or two-way, we assume that both series are with autoregressive lag length 4 as in [11.2.6] of Hamilton (1994) and estimate the following two regressions by ordinary least squares separately:

\[
p_t = \mu_p + \sum_{i=1}^{4} \alpha_i p_{t-i} + \sum_{i=1}^{4} \beta_i y_{t-i} + \epsilon_{pt}, \tag{3}
\]

\[
y_t = \mu_y + \sum_{i=1}^{4} \delta_i y_{t-i} + \sum_{i=1}^{4} \theta_i p_{t-i} + \epsilon_{yt}. \tag{4}
\]

where $\epsilon_{pt}$’s are error terms that are assumed to be independent over time and so are $\epsilon_{yt}$’s, but $\epsilon_{pt}$ and $\epsilon_{yt}$ might be correlated contemporaneously. Take the price
question, for example. Granger causality occurs if the coefficients on the lagged output levels are significantly different from zero after taking into account the lagged price levels. Therefore, the null hypothesis that output does not Granger cause price is formulated as $H_{p,0} : \beta_i = 0$ for $i = 1, 2, 3, 4$. If $H_0$ is rejected, then we conclude that movements in the cyclical component of output temporally precede, or Granger cause, movements in the cyclical component of the price level. Instead of using an $F$ test to test $H_{p,0}$ because the errors are potentially heteroskedastic, we use a Wald test and we use the heteroskedasticity-robust covariance matrix estimator.\(^8\) Table 2 reports the Wald statistics for the null hypotheses that $H_{p0} : \beta_i = 0$ for $i = 1, 2, 3, 4$, and $H_{p,0} : \theta_i = 0$ for $i = 1, 2, 3, 4$.

<table>
<thead>
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<th>Coefficient</th>
<th>Wald-statistic</th>
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<tbody>
<tr>
<td>$\beta_i = 0$</td>
<td>2.91</td>
</tr>
<tr>
<td>$\theta_i = 0$</td>
<td>15.15</td>
</tr>
</tbody>
</table>

The Wald-statistic indicates that movements in the cyclical component of output do not temporally precede movements in the cyclical component of the price level after accounting for lagged values of the price level. In contrast, the evidence indicates that with 99 percent confidence, one can reject the null hypothesis that movements in the cyclical component of the price level do temporally precede movements in the cyclical component of output.

4 Time varying correlations

Cooley and Ohanian (1991) use annual data to assess the relationship between output and the price level during the 20th Century. They present evidence consistent with the notion that the cyclical components exhibited different properties depending on the subsample. In particular, the contemporaneous correlation between output and the price level was positive during the Interwar

\(^8\)The following test results on Granger causality tests remain the same when we use the covariance matrix estimator under homoskedasticity.
Period—between 1921 and 1940—and negative when using data from the Post World War II period.

In this section, our aim is to assess whether the contemporaneous correlation coefficients change over time. In our assessment, the data are taken at quarterly frequencies. We compute the contemporaneous correlations for two separate estimation windows. First, we consider a fixed-window sample that is 45 years long—120 quarters—in which the starting date begins in 1947:1 and moves forward so that the final 45-year window begins 1966:1. Thus, the initial 120-quarter sample covers the period 1947:1 through 1994:4 and the last 120-quarter sample spans the period 1966:1 through 2010:4. Second, we consider a sample period in which the number of observations increases; that is, the window length starts with a 45-year sample and then adds a quarterly observation to the sample. We refer to this latter approach as an expanding window. In the expanding-window approach, the first observations is computed for the sample period 1947:1 through 1994:1 while the last correlation coefficient is computed for the sample period 1947:1 through 2010:4.

In the fixed-window sample, the results of the contemporaneous correlation coefficient computed for each 120-quarter sample between the price level and output is reported in Figure 5. For inflation, the 120-quarter sample starts in 1947:2 and the correlation coefficient for the cyclical components of the inflation rate and output are reported in Figure 6. Figure 5 shows that the contemporaneous cross correlation exhibits greater countercyclicality as the fixed-window moves forward through time. The correlation coefficients are initially around -0.25, but as the start date of the window moves toward the mid 1950s, the correlation coefficients approach -0.7. As the fixed-window sample spans periods that begin in the mid 1960s, the correlation coefficient increases to about -0.55. Meanwhile, in Figure 6, the evidence suggests that the correlation between output and inflation is weakening as the fixed-window length moves forward in time. For windows starting in 1947, for example, the correlation coefficient is 0.28. The correlation coefficient declines as the sample dates begin in the 1950s, with the correlation coefficient falling to 0.12. As the sample periods move from
the 1950s through to the 1960s, the coefficients increase slightly to around 0.16.

Two points emerge from the fixed-window length analysis. First, the evidence points to substantial variation in the contemporaneous relationship between the cyclical components of output and the price level and between the cyclical components of output and inflation. The correlation coefficients indicate that the relationship between output and the price level is more intensely countercyclical while the relationship between output and the inflation rate is less procyclical as the constant-length sample period omits observations closer to World War II.

In addition, we consider correlation coefficients through time with an expanding window length. Formally, we successively add one quarter to the sample period, computing the correlation coefficients for both output-price relationship and the output-inflation relationship. Figure 7 reports the results from the expanding-window analysis for the correlation between the cyclical components of output and the price level. As Figure 7 shows, the contemporaneous correlation between output and the price level is getting weaker as the window length increases. In the initial 45-year window, the correlation is -0.26 but falls to slightly greater than -0.22 when the full sample is used.

Figure 8 plots the correlation coefficients for the cyclical components of output and inflation. Here, Figure 8 shows that there is very little movement in the correlation coefficient as the sample periods increase in size. The coefficients vary in a narrow range between 0.28 and 0.3.

We conduct Chow test to test if there is a structural change in the correlation coefficient of each pair of series. We assume there is a known break-point at 1973:1 because the oil shocks in the 1970s represented distinguishable identifiable macroeconomic shocks that could have accounted for the differences in the correlation coefficients over time.9 For a pair of time series random variables \( \{(x_t, y_t) : t = 1, \ldots, T\} \), let \( \rho_{xy} \) denote the correlation between \( x_t \) and \( y_t \) which

9See Hamilton (1987) and (2011) for excellent discussions on the evolution of oil prices shocks on the macroeconomy. Hamilton applies regime-switching methods to examine the differences in cyclical fluctuations over time.
is estimated

\[ \hat{\rho}_{xy} = \frac{T^{-1} \sum_{t=1}^{T} x_t y_t - \bar{x}\bar{y}}{\hat{\sigma}_x \hat{\sigma}_y}, \]

where \( \bar{x} \) and \( \bar{y} \) are sample averages of the time series and \( \hat{\sigma}_x \) and \( \hat{\sigma}_y \) are sample standard deviations of the time series. It is true that

\[ \sqrt{T}(\hat{\rho}_{xy} - \rho_{xy}) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \xi_t + o_p(1), \]

\[ \xi_t = \frac{(x_t - \mu_x)(y_t - \mu_y)}{\sigma_x \sigma_y} - \frac{\rho_{xy}}{2} \left( \frac{(x_t - \mu_x)^2}{\sigma_x^2} + \frac{(y_t - \mu_y)^2}{\sigma_y^2} - 2 \right) \] (5)

Therefore, we have

\[ \sqrt{T}(\hat{\rho}_{xy} - \rho_{xy}) \xrightarrow{d} N(0, \Sigma_{xy}), \]

\[ \Sigma_{xy} = \lim_{T \to \infty} \text{Var} \left[ \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \xi_t \right]. \] (6)

We use the heteroskedasticity-autocorrelation-consistent (HAC) estimator by Newey and West (1987) to estimate \( \Sigma_{xy} \). To be more specific, let for \( \tau \) be a positive integer, then \( \Sigma_{xy} \) is estimated by

\[ \hat{\Sigma}_{xy} = \hat{\Gamma}(0) + \sum_{\ell=1}^{\tau-1} 2(1 - \frac{\ell}{\tau})\hat{\Gamma}(\ell), \]

\[ \hat{\Gamma}(\ell) = \frac{1}{T - \ell} \sum_{t=1+\ell}^{T} \hat{\xi}_t \hat{\xi}_{t-\ell} \text{ for } \ell = 0, \ldots, T - 1, \] (7)

where \( \hat{\xi}_t \) is the estimated influence function of \( \xi_t \) by plugging in estimates of all unknown parameters in \( \xi_t \). Let \( \rho_{xy}(1) \) and \( \rho_{xy}(2) \) denote the correlation between \( x_t \) and \( y_t \) before and after the break-point respectively. Define \( \hat{\rho}_{xy}(1) \), \( \hat{\rho}_{xy}(2) \), \( \hat{\Sigma}_{xy}(1) \) and \( \hat{\Sigma}_{xy}(2) \) similarly. Let \( T_1 \) and \( T_2 \) denote the sample sizes before and after the break-point respectively. Then we have

\[ \frac{\hat{\rho}_{xy}(1) - \hat{\rho}_{xy}(2) - \rho_{xy}(1) + \rho_{xy}(2)}{\sqrt{\frac{\hat{\Sigma}_{xy}(1)}{T_1} + \frac{\hat{\Sigma}_{xy}(2)}{T_2}}} \xrightarrow{d} N(0, 1). \]

Therefore, we construct our test based on (??) and we set \( \tau = 4 \) when calculating the HAC variance.\(^{10}\) The test results are presented in Table 3. We conduct a

\(^{10}\)The testing results are not sensitive to the choice of \( \tau \) between 3 to 10.
two-sided t-test. From Table 3, we conclude that there is a structural change in the correlation coefficient between the price and output series at 1973:1, but we accept that there are no structural changes in the correlation coefficients between the price and inflation series, and the output and inflation series.

<table>
<thead>
<tr>
<th>series</th>
<th>t-statistic</th>
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<tbody>
<tr>
<td>price-output</td>
<td>2.979</td>
</tr>
<tr>
<td>price-inflation</td>
<td>0.579</td>
</tr>
<tr>
<td>output-inflation</td>
<td>0.933</td>
</tr>
</tbody>
</table>

Thus, the evidence indicates three things with respect to the time-varying nature of the contemporaneous relationships between output and the price level and output and inflation. First, the evidence suggests that relationship between output and the price level has not been constant over time. In the fixed-window sample, the correlation coefficients indicate a stronger negative correlation as the window becomes more modern. In other words, as the 45-year window excludes the period between World War II and the Federal Reserve-Treasury Accord, the evidence points to a strong countercyclical relationship. That relationship weakened only modestly as the sample period focused on the 45-year period beginning in the 1960s. In particular, the correlation coefficient begins to indicate slightly stronger countercyclicality somewhere after 5 years of data are added to the sample estimate. This is consistent with, though much less pronounced with the evidence produced in the fixed-sample length estimation. As the sample length continues to increase, the correlation coefficient indicates that the countercyclical relationship is weakening. This also is consistent with the smaller correlation coefficients observed in the fixed-window length results that includes more recent observations.

Second, the relationship between output and inflation weakens over time. This result is more pronounced in the fixed-window where the procyclical relationship weakens as the sample period includes more recent observations.

Third, the time-varying nature of the contemporaneous correlation tells us
something about the changes in cycles that can account for the time-varying cross-correlations we observe. A corollary holds; namely, we reported Granger causality results in the previous section. With time-varying correlations, the additional evidence is consistent with the notion that the Granger causality is a small-sample result. In the next section, we present a simple atheoretical depiction of two cycles. Using these cycles, we consider changes between these cycles that are consistent with our presentation of the time-varying changes in the cross correlations. In other words, we begin our investigation by searching over changes in the properties of the time series of the price level and output that can account for time-varying correlations. In particular, we will use simulated cycles to assess whether time-varying phase shifts or differences in the wavelengths can account for the time-varying correlation coefficients. In addition, the evidence is consistent with changes in the correlation coefficients for output and the price level as the sample period lengthens.

4.1 Time-varying phase shifts

With a slight modification to our numerical example, it is possible to demonstrate that a time-varying phase-shift can account for the time-varying correlation coefficients. To formalize this notion, let \( \tilde{p} = -\sin (x + \gamma t) \) so that the phase shift is changing over time. Consider, for example, a simple change to phase-shift parameter. With \( \gamma = 0.35 \), the correlation between simulated the price level and simulated output is \(-0.94\) while the correlation between simulated inflation and simulated output is \(0.15\). (Recall that with \( \gamma = 0.75 \), the correlation coefficients were \( \rho (\tilde{y}, \tilde{p}) = -0.732 \) and \( \rho (\tilde{y}, \tilde{\pi}) = 0.26 \).) Thus, by letting the phase shift parameter decline over time in the simulated series, we see that the correlation between the price level and output becomes more countercyclical while the correlation between inflation and output becomes less procyclical. Hence, this reduction in the phase shift parameter over time is consistent with the observations we presented in the post-war period in the fixed-window analysis.
4.2 Cycles of different wavelengths

In addition to a time-varying phase shift, the evidence is consistent with cycles of different wavelengths. In other words, the cyclical components of output and the price level have different wavelengths. To extend our simulation, consider a case in which output is simulated as \( \tilde{y} = \sin(x) \) and the simulated values for the price level are determined by \( \tilde{p} = -\sin(kx) \). Here, the parameter \( k \) denotes the wavelength so that \( k > 1 \) (or \( < 1 \)) represents a time series with a higher (lower) number of cycles per unit of time. Note that the slope of simulated output is \( \cos(x) \) and the slope of the simulated price series is \( -k \cos(kx) \). Note that as \( k \) deviates more from 1 in either direction, the correlation between the simulated price and output series decays. Not surprisingly, as the wavelength of one series increases relative to the other, the wavelength that is occurring more frequently will look more like noise compared with the other series.

To check whether the wavelengths of the two cycles are identical, we assume that both series follow AR(\( m \)) processes:

\[
\begin{align*}
p_t &= \mu_p + \sum_{j=1}^{m} \rho_{pj} p_{t-j} + \epsilon_{pt}, \\
y_t &= \mu_y + \sum_{j=1}^{m} \rho_{yj} y_{t-j} + \epsilon_{yt},
\end{align*}
\]

where \( \epsilon_{pt} \)'s are error terms that are assumed to be identical and independent over time and so are \( \epsilon_{yt} \)'s. Note that \( \epsilon_{pt} \) and \( \epsilon_{yt} \) are allowed to be correlated contemporaneously and the \( \text{Cov}(\epsilon_{pt}, \epsilon_{yt}) \) is allowed to vary over time so it is compatible with the time varying correlation we observed. If \( p_t \) is generated according to (8), then by [6.1.14] of Hamilton (1994), the spectral density \( s_p(\omega) \) for \( \omega \in [0, \pi] \) is defined as

\[
s_p(\omega) = \frac{\sigma_p^2}{2\pi \left(1 - \sum_{j=1}^{m} \rho_{pj} e^{-ij\omega}\right) \left(1 - \sum_{j=1}^{p} \rho_{pj} e^{ij\omega}\right)},
\]

where \( i = \sqrt{-1} \) and \( \sigma_p^2 = \text{Var}(\epsilon_{pt}) \). To estimate \( s_p(\omega) \), we use the the system ordinary least squares estimator to estimate autoregressive parameters and get
\[ \hat{\rho}_{pj} \text{ for } j = 1, \ldots, m \text{ and } \hat{\sigma}_p^2. \] Hence, for \( \omega \in [0, \pi] \), \( s_p(\omega) \) is estimated by

\[
\hat{s}_p(\omega) = \frac{\hat{\sigma}_p^2}{2\pi \left( 1 - \sum_{j=1}^p \hat{\rho}_{pj} e^{-ij\omega} \right) \left( 1 - \sum_{j=1}^p \hat{\rho}_{pj} e^{ij\omega} \right)}.
\]

We plot the spectral densities of the price and output series in Figure 9 and 10. In Figure 9, we set \( m = 3 \) and in Figure 10, \( m = 4 \). In both figures, we normalize both spectral densities such that \( \int_0^\pi \hat{s}_Y(\omega)d\omega = 1 \) so the two densities are comparable. It is true that \( 2 \int_0^\pi s_p(\omega)d\omega = \sigma_p^2 \), see [6.1.17] of Hamilton (1994). Therefore, the normalization is not harmful.

To test if the normalized spectral densities for price and output series are identical, first note that according to Herglotz’s Theorem, see e.g. Corollary 4.3.2 of Brockwell and Davis (1991), two (non-normalized) spectral densities are identical iff the autocovariance functions of the stationary processes are identical. In our case, two normalized spectral densities are identical in AR processes are identical iff all the autoregressive parameters are the same for both processes. As a result, it is equivalent to \( H_0: \rho_{pj} = \rho_{yj} \text{ for all } j = 1, \ldots, m \).

The Wald-statistics are summarized in Table 4. As we can see, we cannot reject the null hypotheses that the two spectral densities are identical.

<table>
<thead>
<tr>
<th>Model</th>
<th>Wald-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(3)</td>
<td>5.774</td>
</tr>
<tr>
<td>AR(4)</td>
<td>6.715</td>
</tr>
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</table>

Thus, the simulations help to identify potential sources of the time-varying correlations between output and the price level and between output and inflation. For one thing, the simulations indicate that a time-varying positive phase shift can account for the movements in the contemporaneous correlations between output and the price level and output and the inflation rate over time. In other words, as the positive phase shift diminishes over time, the contemporaneous correlation between output and the price level is more countercyclical over time while the correlation between output and inflation is less procyclical.
over time. In effect, the smaller positive shift results in the two simulated series being more in phase.

We also consider the role that different wavelengths in the simulated series could have on the contemporaneous correlations. In the actual series, the evidence indicates that we cannot reject the null hypothesis that the wavelength of the price level is the same as the wavelength of the output. As such, it does not appear that changes differences in the wavelengths can explain why the correlation coefficients are changing over time.

4.3 Other explanations

There are other explanations that can account for the time-varying correlations between the price level and output and inflation and output. Here we consider two. First, there is the correlation between shocks to output and shocks to the price level. More specifically, we modify the simulated series for output and the price level as follows. Let $\tilde{y} = \sin(x) + \epsilon_y$ for $x \in \mathbb{R}$ and let $\tilde{\pi} = -\sin(x + \gamma) + \epsilon_p$. Note that we simulate $\epsilon_y$ such that it is drawn from a pseudo-normal distribution with mean zero and standard deviation equal to 0.167. Consider a case in which $\epsilon_p = \eta \epsilon_Y + u_t$, where $\eta > 0$ and $u$ is a random variable with mean zero and variance $\sigma_u^2 > 0$. For our purposes, we examine a case in which $\rho(\epsilon_y, \epsilon_p) = 0.99$.

The results of these modified simulations indicate that allowing for more correlated shocks can account for the time-varying correlations we observe in the data. With this high degree of correlated shocks, we compute the correlation for the simulated output and price level series for this case, finding $\rho(\tilde{y}, \tilde{\pi}) = -0.94$ and $\rho(\tilde{y}, \tilde{\pi}) = 0.15$. Compared with the baseline correlations of the simulated value [recall that $\rho(\tilde{y}, \tilde{\pi}) = -0.73$ and $\rho(\tilde{y}, \tilde{\pi}) = 0.26$], the results are consistent with correlation between output and the price level becoming more countercyclical over time and that the correlation between output and inflation becoming less procyclical over time.\footnote{We conduct a Chow-test to see if there is a structure change in the correlations between the error terms from (8) and (9) before and after 1973:1 and we fail to reject that there is a structure change.}
5 Summary and Conclusion

In this paper, we examine two sets of business cycle facts. One is the contemporaneous correlation between output and the price level. The other is the contemporaneous correlation between output and inflation. The first is countercyclical and the second is procyclical. Because the inflation rate is the time derivative of the log price level, we are interested in examining properties of the time series that will account for the two facts. The Folk Theorem is that the price level series is a positive phase shift of the output series. We verify that the Folk Theorem holds by simulating a positive phase shift. Because a positive phase shift is consistent with Granger causality, we test for that. Indeed, the price level does Granger cause output in the actual time series.

In addition, we ask whether the contemporaneous correlation are stable over time. Two types of evidence are presented. First, we compute correlation coefficients for the price level-output and the inflation-output over different sample periods. In one the set of rolling 45-year windows is used while the other starts with a 45-year sample and adds next time-series observation to each sample period. The evidence from this approach indicates that the price level-output correlation is becoming smaller algebraically over time; that is, the price level is more countercyclical in the most recent sample when compared with the statistic computed with older data. The evidence also indicates that the inflation-output correlation is smaller, meaning that inflation is less procyclical when estimated over the most recent samples than it was when estimated with older data. We test for a structural break. The test statistic indicates that we reject the null hypothesis of constant correlation coefficients.

The simulations offer some clues as to could possible explain the movements in the correlation coefficients over time. We present simulation results suggesting that a time-varying phase shift could account for the pattern in correlation coefficients that have evolved over time. Another possible explanation is that the phase shift is constant, but that price level and output have different wave-lengths. The power spectrum for the two variables are consistent with the wave-
length for the price level being longer than the wavelength for output. However, we cannot reject the null hypothesis that the two wavelengths are equal.

Lastly, we consider explanations that do not depend on changing the properties of the cycle relative to one another. For example, if there are correlated shocks present, we present simulations consistent with the notion that as the correlation between price level and output shocks increase, the correlation between the price level and output, not surprisingly, becomes more negative. Moreover, the correlation between inflation and output becomes less positive.

In our view, the questions and answers developed in this paper are a starting point. Indeed, a more ambitious project would be to tackle the frictions that would give rise to a positive phase shift between the price level and output.
References


Figure 1:

Cyclical Component of Real GDP
Figure 2:

Cyclical component of the price level
Figure 3:

Cyclical component of the Inflation rate
Figure 4:

Numerical Example of Positive Phase Shift
Figure 5:

Time-Varying Output-Price Correlation: Fixed-Window Case
Figure 6:

Time-Varying Output-Inflation Correlation: Fixed-Window Case
Figure 7:

Time-Varying Output-Price Correlation: Expanding-Window Case
Figure 8:

Time-Varying Output-Inflation Correlation: Expanding-Window Case
Figure 9:

Spectral Densities of the Price and Output Series: AR(3)
Figure 10:

Spectral Densities of the Price and Output Series: AR(4)
Appendix

Table A.1

Parameter Estimates for Output Regression

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Std Err</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>7.78e-06</td>
<td>4.95e-04</td>
</tr>
<tr>
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</tr>
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<td>$\delta_2$</td>
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</tr>
<tr>
<td>$\delta_3$</td>
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</tr>
<tr>
<td>$\delta_4$</td>
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<td>0.063</td>
</tr>
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</tr>
<tr>
<td>$\theta_2$</td>
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<tr>
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<td>0.263</td>
</tr>
<tr>
<td>$\theta_4$</td>
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<td>0.166</td>
</tr>
<tr>
<td>Variable</td>
<td>Parameter Estimate</td>
<td>Std Err</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------</td>
<td>--------</td>
</tr>
<tr>
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<td>$\alpha_1$</td>
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<td>0.035</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>0.011</td>
<td>0.024</td>
</tr>
</tbody>
</table>