Applied Signaling: Graduate School Admissions and Frequency of STEM Majors

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We offer a closer look at screening by graduate admissions committees in their selection of student applicants, and at applicants’ strategic behavior given screening methods. Essentially, a signaling game takes place between student applicants seeking to signal ability and admissions committees seeking to infer ability from proffered applications. In equilibrium, students’ decisions to adopt a STEM or a non-STEM major reflect both their desire to imply high ability and the importance that admissions committees place on GPA. We find that, relative to GPA, admissions committees placing a higher weight on graduate entrance-exam scores leads to a higher fraction of students selecting STEM majors. Illustrations of the impact on the equilibrium fraction of STEM majors of grade inflation, and of alternative undergraduate grading systems, are also provided.

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1. Introduction

A graduate school application is usefully modeled as a signaling game between a student applicant and a graduate program. Graduate programs use measures to estimate an applicant’s quality, including GPA, an entrance exam score, an undergraduate major, letters of recommendation, perhaps an application essay. Assessing the screening methods of a program, a student will invest in attaining an undergraduate record that maximizes her estimated quality so as to gain admission to a desired graduate program.

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Modeling a student applicant as the Sender and a graduate program admissions committee as the Receiver, in an adaptation of the signaling game introduced by Spence [1973], lends theoretical insight into a prominent issue of higher-education policy. What factors influence college students’ choice between STEM (science, technology, engineering and mathematics) and non-STEM majors?

A shortage of students obtaining bachelor’s degrees in STEM disciplines is widely decried; the National Science Foundation has recently introduced or expanded three programs and funding initiatives designed to encourage more STEM majors. Empirical studies by Kohn, Manski and Mundel [1976], Fuller, Manski and Wise [1982] and Turner and Bowen [1999] find that disciplines deemed more difficult—generally those labeled STEM—promise greater earnings opportunities, and attract students who are aware of their higher ability (measured in these papers by SAT scores), but nonetheless exhibit a lower probability of completing a bachelor’s degree in the discipline. Rumberger and Thomas [1993] provide supporting evidence of higher earnings for STEM-major graduates. Berger [1988] finds empirical evidence consistent with the notion that students’ choices of major may be viewed as a utility-maximizing decision for preferences that place a strong weight on future earnings. Montmarquette [2002] finds lower probabilities of degree completion in STEM than non-STEM disciplines. More potent dynamic tests in Arcidiacono [2004] strengthen these conclusions. Stinebrickner and Stinebrickner [2011] track students through college, finding frequent switches from a STEM to a non-STEM major after low grades in early STEM courses, and almost no balancing switches into STEM majors, suggesting a view that a STEM degree is more costly in time and effort. Arcidiacono and Koedel [2012] document a similar pattern, finding racial minorities disproportionately likely to switch out of STEM majors.

The first five of the papers just mentioned combine to offer a clear pattern of the choices facing an undergraduate who sees a bachelor’s as a terminal degree, or plans several years of employment before contemplating graduate study. An

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1 Cf., e.g., Tables 9 and 11 in Arcidiacono and Koedel [2012].
2 Spence’s model directly shows how education leading to employment may in equilibrium be a signal of the student’s quality or productivity.
increasing fraction of US undergraduates, though, come early in their collegiate studies to contemplate commencing graduate study directly after their bachelor’s degree. DesJardins, Ahlburg, and McCall [2006] point to “a number of studies that model application behavior ... but few if any models of the admission-decision process within institutions.” Arcidiacono and the Stinebrickners suggest that the structure of a theoretical model may be valuable in understanding the behavior of these students, who focus their collegiate choices and purposeful behavior less directly on earnings and more directly on how impressive a record they can provide to graduate program admissions committees.

The signaling game we analyze can provide that structure. Students plan on applying to graduate programs at a portfolio of universities. Those programs’ admission committees evaluate students and make offers of admission and/or fellowship in seeking to end up with the highest possible quality in their next entering class. These committees consider students’ scores on standardized exams, grade-point averages (GPAs), and choices of majors. Our model simplifies to one student seeking, at reasonable cost, to have one program evaluate her application as highly as possible.\(^3\)

The literature cited clearly supports the common opinion that a STEM major, while likely to yield better technical preparation for graduate study (in STEM or some non-STEM fields) and hence higher graduate-entrance-exam scores, also leads to lower GPAs. Our model departs from the seminal Spence model by assuming that the choice of a STEM major, and with it a portfolio of \{higher entrance-exam score, lower GPA\}, might be a signal of ability, should the admissions committee so interpret it. As in all signaling models, the ability variable is subject to differing interpretations in differing settings. Here, ability in rigorous analysis might be the most natural interpretation.

Following setup of the model and characterization of equilibrium, we explore the related impact of a graduate program placing more weight on the student’s major or on entrance-exam score, and then illustrate this impact via consideration of extreme policies, instituting an every-class-ranked, or a pass/fail grading system.

\(^3\) This is surely the student’s goal at least with respect to the most desired match.
2. Program screening criteria

Each year, a graduate program determines the size of its incoming class, choosing some number $n$ of offers to make. Given $n$, the program will select the $n$ best applications it has received, or put differently, will select all applicants who were rated at the $n^{th}$ rank or higher (assuming the program maps all dimensions of an application into a single rating).\(^4\) Of the many dimensions the program observes about an applicant (recommendation letters, personal interviews, reputation of and past experience with the undergraduate school, a personal statement, perhaps others), we consider only the STEM/non-STEM major distinction, GPA, and the score on some required entrance exam.\(^5\) These screening criteria are, by a considerable margin, the most important in application evaluation (Arcidiacono [2004]).

While some admission committee decisions may exhibit less structure, in this model a committee maps an applicant’s GPA, $g$, and normalized entrance-exam score, $x$, into an overall applicant rating $r(g, x, m) = (1 - w)g + wx + \mu \cdot 1_m$, where the weight $w \in [0,1]$ is a program-specific parameter, the indicator $1_m$ takes the value 1 if the major $m = m_{STEM}$, and 0 if $m = m_{NS}$, a non-STEM major, and $\mu \geq 0$ is the premium, if any, placed on an applicant having completed a STEM major.\(^6\) Applicants are then ranked descendingly by ratings $r(g, x, m)$, that is, an applicant with higher rating $r$ is ranked above, and accepted before, one with a lower rating. To fit the usual $[0, 4]$ range for GPA, a raw entrance-exam score $p$, on an exam with

\(^4\) Some admissions committees must return to the Graduate School fellowships which were offered and declined, and presumably do not offer a fellowship to a student estimated to be so strong as sure to receive a fellowship offer from a much stronger and likely preferred program. The resultant strategizing putably exhibits the main characteristics of our model; details are beyond the scope of this paper.

\(^5\) Our limited knowledge of professional-school graduate admission behavior suggests that a larger set of applicant characteristics may affect decisions. If so, this model may fit academic graduate programs better; its characterizations still may well reasonably fit, ceteris paribus, decisions at professional schools.

\(^6\) Dependence of the rating on the perceived caliber of the undergraduate school is submersed. Of course, the assumption that the rating linearly combines aspects of an application is a simplification, unlikely to realistically limit applicability of results. Also, to the extent that a committee pays more attention (or sole attention) to average of the last two years’ grades, or average of grades in the major or in certain courses, we treat this as encoded in $g$. 

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maximum score \( \tau \), is normalized via \( x = \frac{4\rho}{\tau} \). Then \( r \) maps observables \((g, x, m) \in \sum \subseteq [0,4]^2 \times \{m_{STEM}, m_{NS}\} \) onto \([0, 4+\mu] \), and the weight \( w \) is naturally interpreted as the importance of the entrance-exam score relative to the GPA.

3. Adapting the basic signaling model

Signaling becomes relevant when each Sender knows her ability \( a \) but cannot inform the Receiver directly and credibly. The game-theoretic question is whether an indirect signal can be credible. The standard signaling game has a Sender select from a feasible set of signals to send, with higher signals disproportionately more costly to lower-ability Senders. The structure of predicted play is then that a Receiver, upon observing a Sender’s signal, selects a decision that is a best response given the Receiver’s belief as to the relation between signals and Sender abilities; Sender chooses a signal that is a best response, given her ability and various signals’ costliness, given the Sender’s belief as to the relation between signals and Receiver decisions. Mutual best responses generate an equilibrium that justifies both players’ beliefs.

Our incarnation of a signaling game has Sender (an applicant) select a signal \( s = (g, x, m) \in \sum \), where \( \sum \subseteq [0,4]^2 \times \{m_{STEM}, m_{NS}\} \) contains the range of feasible signals, with \( m_{NS} \) indicating choice of a non-STEM major. As usual, the Sender’s strategy is represented as a function \( \sigma: [0,1] \rightarrow \sum \), mapping the Sender’s ability \( a \) (known to the applicant but not credibly observable to the admissions committee) into a three-dimensional signal \( s \).

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7 In many disciplines, the entrance-exam raw score is itself a vector of raw scores. Our assumption that an applicant is assigned a scalar rating implies that this vector is mapped into a scalar raw score \( x \). Our analysis assumes further that this mapping into a single raw score is known to the student (perhaps via an advisor), which seems tenable.

8 The assumption that the privately informed player knows her/his type is nearly universal in analysis of games of incomplete information. Little but complication would be introduced if this assumption were weakened to knowing an unbiased estimate of her/his type. Our results clearly suggest that a student suffers if her/his type estimate is biased upward, and offer the inference that a high rate of switches from STEM to non-STEM majors amidst undergraduate years may result from upwardly biased estimates of ability leading to interim GPAs well below expectations.
The receiver (admissions committee) selects a strategy \( \pi: \Sigma \to [0,1] \), mapping an observed signal \( s \) into a probability \( p \) of accepting (or offering a fellowship, whatever the applicant’s desired outcome) an applicant sending signal \( s \).

4. **Major selection and the tradeoff assumptions**

A student’s objective function is \( u(p, c) \), assumed to be increasing in the acceptance probability \( p \) and decreasing in the cost \( c \); this cost is detailed in section 5. Naturally, a STEM or non-STEM major choice will affect GPA \( (g) \), entrance-exam score \( (x) \), and cost.

Spence [1973] echoes longstanding arguments that a student’s score on an entrance exam can be considered an exogenous index of student ability.\(^9\) To provide structure for empirical analyses, we must go further. The claim that a student of given ability will score higher on the GRE for Chemistry if he majored in Chemistry than if he majored in, say, History, Dance or Spanish, is uncontroversial. We strengthen the argument of Spence and forbearers to a claim that the increasing function \( x(m, a) \) mapping ability \( a \) into an entrance-exam score \( x \) reaches higher values \( x \) at each ability level if the student’s major is STEM rather than non-STEM.

While a student chooses between many possible majors, our model simplifies the choice to a decision between STEM or non-STEM enrollment.\(^{11}\) These majors reflect a typical tradeoff students face across a wide range of majors: as a major increases in its rigor/difficulty, the student will attain a lower GPA (Fuller, 1982), but a higher return to the student’s ability level.

The lower-GPA part of this tradeoff is obvious; the higher-exam-score part is also logical. Ability is modeled herein as yielding both a higher exam score if a STEM

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\(^9\) Viewed broadly, graduate admissions is a perennial game in which faculty serve overlapping, multi-year terms on admissions committees, and build up experience interpreting applicants’ dossiers, while nearly every applicant is a one-time player of the game (though likely having received some advice from faculty accumulating experience over years). A full-fledged analysis of the dynamic game, employing Crémer [1986], would take us far afield; the natural equilibria likely fit the static game we analyze here.

\(^{10}\) The argument stems from the fact that many of these exams target a student’s ability to reason, apply logic and think critically, in a way that captures inherent ability, at least partially.

\(^{11}\) While obvious that more than two majors exist for a student to choose from, modeling more majors adds only notational complexity.
major is picked, and a greater impact of a STEM major on the exam score for higher-ability students: (1): \( x(m_{STEM}, a) - x(m_{NS}, a) \) is a nonnegative, increasing function of \( a \). Ability is correspondingly modeled as yielding both a lower GPA if a STEM major is picked, and a lesser impact of a STEM major on GPA for higher-ability students: (2): the function \( g(m, a) \), mapping major choice and ability into an expected GPA, is increasing in \( a \) for either \( m \), and \( \{g(m_{STEM}, a) - g(m_{NS}, a)\} \) is a nonpositive, decreasing function of \( a \).

5. Differing ability, differing cost

At this point, we may start to analyze different student types, as identified by their exogenous ability \( a \), defining certain students as naturally more productive than others.

We have already assumed that higher ability makes the choice of a STEM major more productive in the sort of learning that enhances entrance-exam scores, and makes the reduction in GPA associated with a STEM major smaller. We now build these assumptions into the cost to the student of the effort in obtaining an undergraduate degree. Cost is treated as a single-dimension summary of pecuniary costs (foregone income, tuition, books, etc.), time costs, effort costs, and all other disutilities. Only two characteristics of the cost \( c \) are salient:

\[
c(m_{STEM}, a) - c(m_{NS}, a) > 0, \tag{3}
\]

\[
\frac{\partial [c(m_{STEM}, a) - c(m_{NS}, a)]}{\partial a} < 0. \tag{4}
\]

The former corresponds directly to the nonnegativity of \( x(m_{STEM}, a) - x(m_{NS}, a) \) and nonpositivity of \( g(m_{STEM}, a) - g(m_{NS}, a) \); the latter to the increasingness of this \( x \) difference and decreasingness of this \( g \) difference.

More directly, equation (3) faces a student of fixed ability \( a \) with higher costs in a STEM than a non-STEM major; equation (4) shows that the higher the ability of a student, the less the cost difference between the two majors and the less troubling the GPA/exam score tradeoff.
6. A separating equilibrium of the signaling game

Spence's seminal signaling model found a separating equilibrium with the following characteristics. A potential employer would offer a job and a wage level that varied with the amount of post-secondary education the worker had attained, with the chosen wage level an optimal response for the employer given his belief as to the extent to which a greater amount of education signaled a higher ability level. A student would select an amount of post-secondary education to attain, as a signal of her ability, with the chosen amount an optimal response to the relationship between education amount and wage that she believes is the basis for the employer’s wage offers.

This incarnation of that model retains the feature that ability is a single variable known to the applicant but not the grad admissions committee. Here, however, the signal of ability sent is a three-dimensional vector \( s = (g, x, m) \). The dimensionality must adjust somewhat the signaling equilibrium we seek: the grad admissions committee believes there is a relationship between \( (g, x, m) \) and the applicant’s ability, and incorporates these beliefs in optimally choosing an acceptance probability. The applicant directly chooses a major \( m \in \{m_{STEM}, m_{NS}\} \), and implicitly a cost of effort, so that the signal \( s \) sent is an optimal response to her beliefs as to the admissions committee’s choice of a relation between signals and acceptance probabilities.

The student’s beliefs are represented simply as a function
\[
p[g(m, a'), x(m, a'), a'] = p[s(m, a')],
\]
giving the believed acceptance probability of a particular quality of grad application (i.e., a particular signal) that would stem from a major \( m \) and a (possibly truthful, possibly misrepresented) ability level \( a' \). Given these beliefs, choosing the major \( m \) is expected to attain a level of utility
\[
u[p[s(m, a)], c(m, a)].
\]
Let
\[
\Delta(a) = \left|u[p[s(m, a)], c(m, a)]\right|_{m = m_{NS}}^{m_{STEM}}.
\]
This \( \Delta(a) \), simply the arithmetic difference between two levels of expected utility given beliefs, is the size of the incentive for an applicant of ability \( a \) to choose a STEM major over a non-STEM major. The first term uses the argument \( a \) and the major choice \( m_{STEM} \) to determine an expected signal \( s \) and via \( s \) an expected
acceptance probability \( p \), using this and the cost of \( m_{\text{STEM}} \) for ability \( a \) to specify an expected utility level for major \( m_{\text{STEM}} \). The second term subtracts the corresponding expected utility at ability \( a \) for major \( m_{\text{NS}} \).

Let us initially dispense with two trivializing possibilities. [i] Were \( \Delta(0) \) to be nonnegative, a student that knew she were of the lowest ability possible, would nonetheless find the costliness of a STEM major so low, its adverse impact on her GPA so tiny, and its beneficial impact on her entrance-exam score so large, that she would select it over a non-STEM major. Given (1)-(4) above, this preference would be even stronger for a student of more-than-minimal ability. The model would then predict that non-STEM departments would have no majors whatsoever.\(^ {12} \) To achieve a more realistic prediction, we set aside this possibility, and assume (5): \( \Delta(0) < 0 \). [ii] Were \( \Delta(1) \) to be nonpositive, a student that knew she were of the highest ability possible, would nonetheless find the costliness of a STEM major so high, its adverse impact on her GPA so large, and its beneficial impact on her entrance-exam score so tiny, that she would select a non-STEM major over a STEM major. Given (1)-(4) above, this preference would be even stronger for a student of less-than-maximal ability. The model would then predict that STEM departments would have no majors whatsoever. To achieve a more realistic prediction, we also set aside this possibility, and assume (6): \( \Delta(1) > 0 \).

As the student’s ability is increased from \( a \) to \( a + \varepsilon \), the incremental increase in ability by (1) increases slightly the marginal benefit of a STEM major on entrance-exam score, by (2) decreases slightly the marginal cost of a STEM major on GPA, by (3) shifts slightly downward the effort cost of a STEM major, and by (4) slightly reduces the marginal effort cost of a STEM major. By (5), a student of ability 0 chooses a non-STEM major, and by (6), a student of ability 1 chooses a STEM major.

It is then a straightforward application of the intermediate value theorem that there exists a threshold ability level \( a^* \) such that a student of ability \( a^* \) would be

\(^{12}\) A sufficiently large value of \( \mu \), the admissions-committee premium for a STEM-major application, could yield this possibility. In a very few graduate disciplines, an applicant whose undergraduate major was not in the discipline is at such a huge disadvantage that admissions committees might routinely set \( \mu \) so high. A graduate nursing program might be an example.
indifferent between a STEM and a non-STEM major. This generates our separating equilibrium: a student of ability \( a < a^* \) optimally chooses a non-STEM major, and a student of ability \( a > a^* \) optimally chooses a STEM major, each responding to their beliefs about admission-committee behavior. That this behavior leads a graduate admissions committee optimally to rate an application \((g, x, m)\) as \(r(g, x, m)\) and then admit the \( n \) highest-rated applicants is straightforward. Hence, the behavior of an admissions committee supports the equilibrium beliefs of applicants, and the major-selection behavior of applicants supports the equilibrium beliefs of admissions committees.\(^{13}\)

7. The comparative static of the entrance-exam weight

Recall that a graduate program established an evaluation system that converts an application \((g, x, m)\) into a rating \(r(g, x, m) = (1 - w)g + wx + \mu \cdot 1_m\), where \(w\) is the relative importance of the entrance-exam score to the GPA. The principal policy-relevant characterization of this signaling model arises from treating the equilibrium threshold ability level \( a^* \) as a function \( a^*(w) \) of the graduate program’s emphasis on entrance-exam scores in evaluating applicants.\(^{14}\)

Consider any given weight \( w_0 \in (0,1) \), a student of ability \( a_0 = a^*(w_0) \) who is thus indifferent between a STEM and a non-STEM major, and an incremental

\(^{13}\) A full characterization of the signaling equilibrium must cover certain other, more technical details (of no interest to most readers). Let \( \Sigma^R \subseteq \Sigma \) be defined by \( \Sigma^R = \{(g', x', m') \in \Sigma | \exists \ a \in [0,1] | \{(g', x', m') = [g(m', a), x(m', a)] \}. \) Here, \( \Sigma \) is the space of all vectors \((g, x, m)\) where \(g\) and \(x\) are real numbers in \([0,4]\) and \(m\) is either \( m_{STEM} \) or \( m_{NS} \). However, only the smaller set \( \Sigma^R \) (a pair of curves in the space \( \Sigma \)) are possible applications that could result from a student choosing one major or the other and doing the best she can, given the costs she faces. For example, \((2.02, 3.998, m_{NS})\) might only be seen if the student somehow arranged for a much-better-prepared expert to take her entrance exam, or \((0.9, 3.9, m_{STEM})\) might only be seen if some malicious hacker had succeeded in falsifying her transcript with lower grades than she earned. Technically, the equilibrium could restrict an applicant’s strategies to elements of \( \Sigma^R \), and have only these strategies the domain of the rating function \( r \). The definitions we have used are less artificial, treating \( \Sigma \) as an applicant’s strategy space. Given that choice, the equilibrium must also specify \([a]\) what admissions committees believe about the applicant’s ability \( a \) if any \( s \in \Sigma \setminus \Sigma^R \) is submitted, and \([b]\) what an applicant believes an admissions committee will do if any \( s \in \Sigma \setminus \Sigma^R \) is submitted. It suffices to assume that an admissions committee in event \([a]\) will assume \( a = 0 \), and an applicant believes event \([b]\) will yield a rejection.

\(^{14}\) Technically, the rating function’s influence should be modeled as \( a^*(w, \mu) \). We submerge this second variable, as the economically significant comparative static is with respect to \( w \), and \( \mu \) is a comparatively blunt instrument.
increase to \( w = w_0 + \varepsilon \). This student finds that the increase in entrance-exam score that accompanies the STEM major now counts more highly, and the decrease in GPA less highly. Hence the STEM major becomes strictly preferable, and thus \( da^*/dw < 0 \): the threshold ability level where major choice is indifferent falls as \( w \) is increased.

The last step is the policy relevance: an increase in the admission committees’ relative weight on entrance-exam scores yields an increase in the fraction of STEM majors among those undergraduate students who contemplate an immediate continuation to graduate school.¹⁵ This may be happening gradually and naturally, across a variety of universities and disciplines, in reaction to widespread undergraduate grade inflation that presumably reduces the value of GPA as a signal of ability. It might also be possible as a loosely coordinated decision of a dozen or a score of top schools, establishing a policy of paying more attention to entrance-exam scores specifically because by so doing they can encourage a higher fraction of applicant to select STEM majors. (If several top schools’ policy adoptions became known, most of the next tier of schools might be expected to follow suit.)

8. Illustrations: alternative grading systems

We have seen that less emphasis on GPA by graduate admissions committees leads to a greater fraction of students selecting STEM majors. Here we illustrate two extremes, by imagining a large consortium of prominent liberal-arts colleges decided to institute an alternative grading system on all their campuses.

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¹⁵ A similar result for a model of undergraduate students planning to seek employment on graduation may be possible. With no standardized measure corresponding to an entrance-exam score, the student’s job application would be modeled as \((g, m)\), a GPA and either a STEM or non-STEM major. A potential employer would have a rating function \( r(g, m) = g + \mu \cdot 1_m \). The functions \( g(m, a) \) and \( c(m, a) \) would still meet assumptions (2)-(4). Though there would be complications associated with one dimension of the application being binary and only one continuous, it would still be the case a strict preference for a non-STEM major at ability 0 and for a STEM major at ability 1 should yield a threshold \( a^{**}(\mu) \) ability level where indifference between majors prevails. It should be possible to show that \( da^{**}/d\mu < 0 \): employers placing a larger premium on a STEM major relative to GPA will lead a larger fraction of these undergraduates to choose STEM majors.
First, suppose they all fought grade inflation by an every-class-ranked system. That is, no letter grades would be given; every instructor would be required to rank all \( N \) students who completed course requirements 1, 2, ..., \( N \) without ties being allowed. To use the same interpretations as before, let \( N \) denote the highest-ranked student. With this grading system, let the GPA variable \( g \) become the mean relative rank (the mean of \( R_{jk}/N_k \), student \( j \)'s rank in course \( k \)).

Maintaining assumptions (1)-(6) above, we would see the same separating equilibrium as before. However, the natural conclusion is that higher informational content in an applicant’s mean relative rank than in GPA would imply that admissions committees place a higher weight in relative rank, thus reducing \( w \), the entrance-exam weight.

For the other extreme, the alternative grading system is pass/fail grading for all courses. While graduate admissions committees could in theory attempt to assess the quality of classes passed and failed, we assume the information such as assessment would require is too expensive or subjective. Hence, the signal a student from one of these colleges can send will still be represented \( s = (g, x, m) \), but \( g \) will be nothing more than the ratio of credits passed to credits attempted. For students who might have a credible graduate school application, this ratio will likely be 1; for the rare applicant who as of the application date has failed one class, it could be as low as 0.972.

Of course, a graduate admissions committee would find this ratio quite uninformative, and thus in essence these students would face a maximal weight on entrance-exam scores.

With a low \( w \), the every-class-ranked grading system would yield in equilibrium a higher threshold ability level \( a^* \), and thus a lower fraction of STEM majors among students intending direct continuation to graduate school. With a maximal \( w \), the pass/fail system would yield the lowest equilibrium threshold ability level \( a^* \), and thus the highest fraction of STEM majors.

There are at least two other dimensions in which these extreme grading systems present contrasts with the usual letter-grade system; each would require a
more elaborate version of our model for definitive conclusions. It seems straightforward, though, that the higher informational content of an applicant’s signal including an every-class-ranked average ranking would yield at least somewhat more precise information to graduate admissions committees, and thus more efficient matching in the market for graduate fellowships. Efficiency in the fellowship market would presumably be lowest under pass/fail grading. So these extremes show a conflict between a desire to increase STEM majors and a desire for fellowship market efficiency.

Perhaps the more important dimension, and also the one that would stretch the model further, is the impact of the grading system on student effort. It is plausible that graduate admissions committees being able to observe both entrance-exam scores and the informationally preferable every-class-ranked average ranking would lead a student of ability $a^*$ or better to put forth more effort in order to make it more difficult for students of slightly lower ability to imitate her application. If so, this joins fellowship market efficiency as two benefits of this grading system that trade off against the lower fraction of STEM majors.

The impact of pass/fail grading on effort is less easily grasped in the equilibrium discussed above. What is clear is that lower effort, and with it lower cost, would likely affect the signal sent to admissions committees in only one dimension, lower entrance-exam score, so long as the effort put forth is enough to pass (this is surely below the effort to obtain an A). For many students, then less will be learned under a pass/fail system, due to decreased effort. The aspect of effort determination that is unambiguous: a student with high enough ability to attain an entrance exam score yielding admission to his first-choice graduate program when the weight $w$ is maximal has a reduced incentive for effort. So the pass/fail system increases the fraction of STEM majors at the costs of reduced fellowship market efficiency and reduced effort and learning for at least some students, definitely including the most able. For those students who would choose a non-STEM major under letter grading but are above the threshold for choosing a STEM major under pass/fail grading, the impact of pass/fail grading on effort incentives is unclear. Those students, if their effort does not decrease significantly,
may well learn more, become more employable, and have a greater chance of getting into some graduate STEM program.

9. **Less efficient equilibria**

In his survey of the signaling games literature, Sobel [2009] states: “The separating equilibrium is a benchmark outcome for signaling games.” Signaling games, however, often have multiple equilibria.

A pooling equilibrium may exist for many signaling games in which Senders of all ability levels “pool” (send the same signal), and the Receiver upon observing the pooled signal chooses the response that maximizes his expected payoff across the ex ante distribution of Senders. For this to be an equilibrium, the Receiver must believe that any signal different from the pooled signal, should it be observed, must have come from the lowest ability type. A pooling equilibrium would be an absurd prediction in the present game: it would have graduate admissions committees receiving identically rated applications from all applicants, responding by selecting the students to be admitted at random, but responding to an unexpectedly strong application by rejecting it. This possibility can safely be dismissed.

Spence also points to the possibility of a lower-level equilibrium trap. In terms of the Job Market model, this trap refers to an inefficient, steady-state hiring equilibrium in which the beliefs of employers systemically cause certain groups to underinvest in education. Such a system is caused by a bias or flaw in employer beliefs, and these beliefs are ironically reaffirmed in the market by employees who act in accordance to the employer’s beliefs. For example, tall men who observe that employers believe all tall men to be of low productivity will recognize this belief, recognize that no amount of education will alter the employer’s belief and choose to not invest in education. They therefore reach an equilibrium in which they are in fact of low productivity. This paradox creates a cycle in which misinformed employer beliefs are continually reaffirmed by the market, but only because they were originally misinformed in the first place.

A lower-level equilibrium trap remains a possibility in the graduate-admissions context. Suppose that an admissions committee observes an additional
demographic variable and believes this variable a sure-fire indication of low ability. It could be that Leland Short Jr. University believes all tall men to be of low productivity, that Browne University believes all blue-eyed females to be of low productivity, or that this is what Irish University believes of anyone whose last name does not start with O’, Mc or Mac. Clearly, the result of such individual beliefs would be that a tall man bases both his choice of major and effort level on plans to apply to universities other than Short, a blue-eyed female omits any consideration of applying to Browne, someone with an Italian or Scandinavian surname discards all thought of applying to Irish. Of course, these universities would then never observe evidence contradicting their beliefs. Those who consider the underlying beliefs mistaken conclude that each of these universities will on average have somewhat lower quality enrollees than in the equilibrium studied above.

A somewhat different low-level equilibrium trap results if a large number of top schools have admissions committees that believe all blue-eyed females are of low ability. Blue-eyed females would respond by removing all thought of getting a graduate degree, and make their STEM/nonSTEM major decision solely on the basis of impact on earnings following a bachelor’s degree. Again, these admissions committees would not, in the resulting equilibrium, see a high-quality blue-eyed female applicant.

10. Conclusion

We have offered both a theoretic approach and a relevant setting in which strategic aspects of preparation of an undergraduate record to compete for graduate admission can be analyzed. Unable to observe directly the ability of an applicant, our signaling game models a graduate admissions committee as attempting to infer ability from the signal that a student sends via the (GPA, exam score, major) summary of her application, by believing that applications rating higher in their scoring process come from higher-ability applicants. Unable directly to demonstrate ability in rigorous analysis, a student seeks to send via her application a signal that a student of lower ability finds too costly to imitate, even though students rationally believe it would lead to a higher probability of an offer from the graduate program. In equilibrium, the beliefs of students and of graduate
admissions committees lead a student above a threshold level of ability to select a STEM major, for the reasons that a student of lesser ability would not attain as significant an increase in exam scores by switching to a STEM major, and would see a STEM major yield a greater reduction in their GPA relative to a non-STEM major.

A greater emphasis on entrance-exam scores relative to GPA is seen above to lead to a higher fraction of grad-school-bound students selecting STEM degrees. To the extent that grade inflation is reducing the value of GPA as an informative signal of student ability, this greater emphasis seems inevitable. Our model further suggests that efforts to make entrance exams for STEM fields more indicative of ability and/or more predictive of post-graduate success will also result in a shift toward more undergraduates choosing STEM majors. Such efforts, if successful, could reduce attrition in graduate programs, if more mismatches could be detected at the admission stage.

Should grade inflation be found an acceptable proxy for graduate admissions committees to place greater reliance on entrance-exam scores, this could allow trends in major selection by students who apply to graduate schools to yield an empirical test of the equilibrium predicted by this signaling model.

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