Time-varying Long-run Income and Output Elasticities of Electricity Demand∗

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Abstract

It is widely accepted that long-run elasticities of demand for electricity are not stable over time. We model long-run sectoral electricity demand using a time-varying cointegrating vector. Specifically, the coefficient on income (residential sector) or output (commercial and industrial sectors) is allowed to follow a smooth semiparametric function of time, providing a flexible specification that allows more accurate out-of-sample forecasts than either fixed or discretely changing regression coefficients. We fit the model to Korean data over 1995:01-2012:12 for the residential sector and 1985:01-2012:12 for the commercial and industrial sectors. The rapid development of Korea over this period provides a very clear case for allowing the coefficient on income/output to vary over time, but the essential modeling strategy is widely applicable.

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∗We note that the essential application of the methodology employed here to electricity demand was developed by one of the present authors (Chang) for a working paper previously circulated under the title “Electricity Demand Analysis Using Cointegration and Error-Correction Models with Time Varying Parameters: The Mexican Case,” (with Eduardo Martinez-Chombo, 2003). The exposition on methodology in this paper supersedes that in the previous paper.
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1 Introduction

Market demand for electricity is of vital importance to energy producers and policy makers. Forecasts of demand are critical in determining reserve margins and elasticities of demand are essential to determine welfare implications of income and price changes. These are particularly important in electricity markets, since electricity cannot be stored in substantial quantities, generation capacity is constrained, and retail markets tend to be heavily regulated. A vast literature has developed to address estimation of electricity demand functions for these purposes.

One of the earliest applications of cointegration to modeling electricity markets was conducted by Engle et al. (1989). Since then, a class of models including fixed coefficient (FC) cointegrating regressions embedded in error-correction models (ECM’s), along the lines of Bentzen and Engsted (1993) for energy markets in general and Silk and Joutz (1997) and Beenstock et al. (1999) for electricity markets in particular, has become a workhorse of this literature.

A stylized fact not addressed by this class of models is parameter instability. A rather large literature on parameter instability and especially structural breaks in energy markets has developed (e.g., Hughes et al., 2008, for the gasoline market; Miller and Ratti, 2009, for effects of oil prices on stock markets, Blanchard and Gali, 2010, for effects of oil prices on the economy). Several have explicitly estimated time-varying elasticities of electricity or energy demand, including Chang and Hsing (1991), Haas and Schipper (1998), Galli (1998), Judson et al. (1999), Halvorsen and Larsen (2001), Medlock and Soligo (2001), and Chang et al. (2013).

An obvious drawback of typical approaches to modeling structural breaks is that they allow only sudden changes in elasticities at a small number of points in time and cannot be reliably used for forecasting. Moreover, if the elasticities are changing gradually, due to the propagation of technology, etc., a model with discrete breaks is not appropriate. Instead, parameter instability may be modeled as a smooth function of time or other covariates.

We apply a smooth time-varying coefficient (TVC) cointegration approach to modeling electricity demand, with an application to sectoral demand in Korea. Structural change

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1 Asymmetry, which may be viewed as another type of parameter instability, is also prevalent in this literature – e.g., Chen et al. (2005) for price transmission in gasoline markets, Kilian and Vigfusson (2011) for effects of oil prices on the economy, Gately and Huntington (2002) and Adeyemi and Hunt (2007) for aggregate energy elasticities.

2 While many of the studies on electricity demand focus on residential demand (e.g., Halvorsen, 1975, Silk and Joutz, 1997, Maddala et al., 1997), there have also been studies on the channels of electricity usage in the commercial and industrial sectors (e.g., Berndt and Wood, 1975; Halvorsen, 1978; or more recently Bernstein and Madiener, 2010; Pielow, 2012).
in electricity usage may be due to the revolution in computing and portable electronic devices and the proliferation of electricity usage in heating, in electric arc furnaces, and in the petrochemical industry. We may expect such change to impact all market sectors and even in countries less dynamic than Korea. However, Korea’s rapid development over the course of the past few decades presents one of the most compelling cases for a model that allows parameter instability. Moreover, because the Korean electricity market has almost no imports or exports of electricity and is dominated by a near monopolistic distributor, supply is stable and predictable. We can therefore expect that structural changes are driven primarily by (domestic) demand. Although we focus on applying a TVC cointegrating regression to Korean electricity data, our approach is quite general and may be applied much more widely in studies on energy usage.

Estimation of the TVC cointegrating regression proposed here is no more complicated than that of FC cointegrating regressions already employed in the energy literature. Both may be estimated consistently using least squares. However, standard statistical inferential procedures based on least squares are generally invalid for either model. Canonical cointegrating regression (CCR), proposed by Park (1992), and adapted by Park and Hahn (1999) to the class of TVC models considered here, allows asymptotically valid inference and requires only additional long-run variance estimation, which may be accomplished using standard software packages.

We demonstrate, both analytically and empirically, that fixed coefficient models overestimate income/output elasticities, which translates into underestimation of scale economies in electricity in the commercial and industrial sectors. The bias persists even if the FC models are estimated over rolling windows to allow for structural change. In the extreme, the more restrictive FC model estimates diseconomies of scale in the Korean commercial sector, while the less restrictive model estimates the opposite: economies of scale.

In spite of the smaller estimated elasticities using the TVC model, we find that the elasticities have increased dramatically over time – by as much as 100% since 1995 in the residential sector. The change provides clear evidence of a temporal pattern in the relationship between income and electricity usage, which could be the result of a number of influences proxied by time, including the continued rise of electronic devices and electricity usage as a percentage of expenditures in all sectors.

To compare the FC and TVC models, we utilize several in-sample and out-of-sample empirical comparisons, including long- and short-run forecasts. The TVC model outperforms the FC model in every comparison in each sector of the Korean electricity market. By some measures, the improvement is quite dramatic. In particular, once we convert the 2012 short-run forecast errors from the two approaches into a dollar amount, as described
in detail below at the end of Section 4.4, we estimate that the TVC approach saves roughly $18 million, $81 million, and $103 million per year over the FC approach in the residential, commercial, and industrial sectors respectively.

The rest of the paper is organized as follows. In the next section, we lay out the benchmark fixed-coefficient and time-varying coefficient models of long-run electricity demand. We discuss the details of data construction and empirical implementation of the model in Section 3. In that section, we construct a novel measure of seasonality using the entire intra-monthly temperature distribution for each month, which we believe to be a stronger measure of seasonality than monthly statistics typically used in this literature. Section 4 contains the main numerical results, and we conclude with Section 5. A technical appendix discusses asymptotically normal estimation and testing using CCR in this context.

2 Long-run Electricity Demand Models

2.1 Benchmark Models

A prototypical log-log long-run residential (household) demand model of electricity, such as that of Halvorsen (1975) or Silk and Joutz (1997), seeks to explain the log of quantity demanded $y_t$, for $t = 1, \ldots, T$, using the log of the real price $p_t$ of electricity and demand shifters that usually include the log of income $x_t$ and the log of the price of a close substitute good, such as natural gas. Aside from these two, the most commonly employed demand shifter in electricity demand models is a measure $s_t$ of temperature or seasonality. We discuss our methodology for constructing such a measure in detail in Sections 2.4 and 3.3 below.

A typical fixed coefficient (FC) model of residential electricity demand with these covariates, $p_t$, $x_t$, and $s_t$, may be written as

$$y_t = \tau + \alpha x_t + \beta p_t + \gamma s_t + v_t \tag{1}$$

for $t = 1, \ldots, T$, where $\alpha$ is the income elasticity of electricity demand, and $v_t$ is disequilibrium error. Such error should be short-lived, because a long-run demand model should be in a long-run equilibrium. The error is therefore typically assumed to be a general stationary sequence. It does not need to be uncorrelated either serially or with the regressors for

\footnotetext[3]{We also estimated a model identical to that below but with a natural gas price. The empirical results were very similar, except the electricity price coefficient estimate was much smaller and statistically insignificant. Because these are symptoms of collinearity, we subsequently omitted the natural gas price. Halvorsen (1975), Silk and Joutz (1997), and others have included additional demand shifters, such as household size and an interest rate, but most of them have been found to be insignificant or small in magnitude.}
consistent estimation, provided that such an equilibrium (cointegrating vector) exists.

The channel through which firms in the commercial and industrial sectors demand electricity is different from that of households. Electricity may be used directly as a factor of production, as in the case of an electric arc furnace or a computer network. The model in (1) is also useful to model firm demand in these sectors, with one important exception: \( x_t \) is output rather than income, so that \( \alpha \) is the output elasticity of conditional electricity demand, consistent with some of the existing literature.\(^4\) An implicit assumption of this approach is that the production function is multiplicative (Cobb-Douglas). In that case, the inverse of \( \alpha \) can be interpreted as the elasticity of scale (for the production function) and the parameter that determines economies of scales (for the cost function). In other words, if \( \alpha < 1 \), (a) output increases by a higher percentage than the electricity input, suggesting increasing returns to scale, and (b) the marginal cost of electricity is declining in output relative to the average cost of electricity, suggesting economies of scale.\(^5\)

Electricity may also be used by firms indirectly as a component of worker compensation. Workers may demand electricity-intensive amenities, such as air conditioning. As a result, seasonal temperature fluctuations also affect firms’ demand for electricity. Given the caveats above, we will henceforth refer to \( \alpha \) as “the elasticity” when the sector is ambiguous, but income elasticity or output elasticity when the sector is clear. Of course, the price elasticity \( \beta \) is also an elasticity, but we will only refer to it as the price elasticity to avoid confusion.

Our main focus in this analysis is on \( \alpha \).

The model in (1) may be used for long-run forecasts if reliable forecasts of the regressors are available. For short-run forecasts, researchers often build on a cointegrating long-run demand model by embedding it in an ECM. We consider an error-correction model of an increment of de-seasonalized electricity demand, \( \Delta y^o_t = \Delta y_t - \gamma \Delta s_t \). Specifically, the fixed coefficient ECM (FC-ECM) is given by

\[
\Delta y^o_t = \pi_v v_{t-1} + \pi_y(L) \Delta y^o_t + \pi_x(L) \Delta x_t + \pi_p(L) \Delta p_t + \varepsilon^v_t,
\]

where \( v_{t-1} \) is the lagged error-correction term (disequilibrium error) from (1), and \( \pi_y(L) \),

\(^4\)We take a different approach to modeling commercial electricity demand than Halvorsen’s (1978). He modeled commercial demand as a function of both output and income, using substitution to eliminate output with only income remaining. Our approach to modeling the industrial demand is similar to Halvorsen’s approach, which utilized value added in manufacturing and mining. Berndt and Wood (1975) also use output for the industrial sector.

\(^5\)The conditional factor demand of electricity may be derived by minimizing a firm’s cost function subject to the constraint of its production function. Substituting the inverse of a Cobb-Douglas production function with one factor into the cost function and solving for the log conditional factor demand reveals a coefficient on log output \( x_t \) that is equal to the marginal cost of output divided by the average cost of output (Varian, 1992, pp. 54-55).
π_x(L), and π_p(L) are finite-order lag polynomials with strictly positive lags only. In practice, we may fit (2) using \( \Delta \hat{y}^2_t = \Delta y_t - \hat{\gamma} \Delta s_t \) with a consistent estimate \( \hat{\gamma} \) of \( \gamma \) and the lagged fitted residual \( \hat{\nu}_{t-1} \) from the FC model (1).

2.2 Time-Varying Coefficient Models

Building on the benchmark FC model in (1), we allow for the possibility of a time-varying long-run elasticity that is a smooth function of time. A constant elasticity is nested as a special case that may be tested empirically. In this sense, we use time as a proxy for unobserved variables that affect the coefficient on \( x_t \).

We approximate this function semi-parametrically by way of a flexible Fourier functional (an FFF) form, which decomposes the function into a linear combination of a polynomial and pairs of periodic functions.

With a time-varying elasticity \( \alpha_t \), the model in (1) becomes a TVC model given by

\[
y_t = \tau + \alpha_t x_t + \beta p_t + \gamma s_t + u_t, \tag{3}
\]

where \((u_t)\) is a latent disequilibrium error sequence assumed to be weakly dependent. We let \( \alpha_t = \alpha(t/T) \), where \( \alpha \) is a function defined over the unit interval and admits an FFF form. Specifically, we use

\[
\alpha_{pq}(t/T) x_t = \lambda_0 + \sum_{j=1}^{p} \lambda_j t^j + \sum_{j=1}^{q} (\lambda_{p+2j-1}, \lambda_{p+2j}) \phi_j(r),
\]

where \( \phi_j(r) \equiv (\cos 2\pi j r, \sin 2\pi j r)' \) for \( r \in [0,1] \), which approximates an FFF form as \( p \) and \( q \) increase.

By defining \( \lambda_{pq} \equiv (\lambda_0, \ldots, \lambda_{p+2q})' \) and \( \phi_{pq}(r) \equiv (1, r, \ldots, r^p, \phi_j'(r), \ldots, \phi_q'(r))' \), we may write \( \alpha_{pq}(t/T) x_t = \lambda_{pq}' \phi_{pq}(t/T) x_t \) or further as \( \lambda_{pq}' x_{pq} \) with \( x_{pq} \equiv \phi_{pq}(t/T) x_t \). In other words, the nonlinear function may be approximated by a linear function of a new regressor vector \( x_{pq} \). Using this specification, the TVC model we estimate is given by

\[
y_t = \tau + z_{pq}' \theta + u_{pq}, \tag{4}
\]

6If the coefficient is a function of \( x_t \) itself, as in the model of Medlock and Soligo (2001), for example, then the definition of the elasticity as a partial derivative of \( x_t \) necessarily includes an additional term, and the coefficient is no longer the elasticity. See also Chang et al. (2013).

7The FFF form has long been used in semiparametric economic analysis: see Gallant (1981). The FFF form was suggested by Park and Hahn (1999) specifically for time-varying cointegrating vectors, and has been used recently by Park and Zhao (2010) to model gasoline demand and by Kim and Park (2013) to model the changing relationship between stock prices and dividends, for example. Since we estimate long-run elasticities and control for seasonality and higher-frequency cycles in the construction of the data (see Section 3), the purpose of the periodic functions is not to capture such cycles, in contrast to Hinich and Serletis (2006) or Pielow et al. (2012).
where \( z_{pq} \equiv (x'_{pq}, p_t, s_t)^\prime, \theta \equiv (\lambda'_{pq}, \beta, \gamma)^\prime, \) and \( u_{pq} \equiv u_t + (\alpha(t/T) - \alpha_{pq}(t/T))x_t \) includes the original disequilibrium error as well as an approximation error from fixing \( p \) and \( q \). The new regressor vector \( z_{pq} \equiv (x'_{pq}, p_t, s_t)^\prime \) contains the original regressors \( x_t, p_t, \) and \( s_t \), and the elements of \( \phi_{pq}(t/T)x_t \) beyond simply \( x_t \).

Note that the TVC model in (4) nests the FC model in (1) with common coefficients \( \lambda_0 (= \alpha \) in the FC case), \( \beta \), and \( \gamma \). That is, the TVC model reduces to the FC model when \( p, q = 0 \), or equivalently when \( \lambda_1 = \cdots = \lambda_{p+2q} = 0 \) for other values of \( p \) and \( q \). Thus, a fixed coefficient null equates to a joint hypothesis that all of these coefficients are zero, while the TVC alternative is that at least one of them is not zero.

The TVC regression in (4) is no more complicated than the benchmark FC regression in (1), with only the addition of easily constructed covariates that enter the regression linearly. As a result, least squares consistently estimates the coefficients of (4), assuming this regression is not spurious – just as least squares consistently estimates the coefficients of the FC regression, assuming that regression is not spurious.

However, as is well-known, standard inferential procedures based on least squares estimation of either model are invalid unless their respective error terms have ideal properties. Realistic violations of these properties may arise from the omission of demand or supply shifters or other misspecification errors. In order to conduct valid inference on the parameters of either model, we rely on the canonical cointegrating regression (CCR) method of Park (1992), which was shown to be valid for a class of TVC models including (4) by Park and Hahn (1999). We leave a more detailed discussion of the estimation procedure to the appendix, noting here only that, like least squares, CCR is no more complicated for the TVC model than for the FC model.

An ECM analogous to that in (2) but with an error-correction term \( u_{pq} \) created from the TVC model in (4) is given by

\[
\Delta y_t^c = \pi_u u_{pq,t-1} + \pi_y(L)\Delta y_t^c + \pi_x(L)\Delta x_t + \pi_p(L)\Delta p_t + \varepsilon_t^u, \tag{5}
\]

which we refer to as the TVC-ECM. The only difference between the FC-ECM and the TVC-ECM lies in the fixed versus time-varying coefficient in the error-correction term. The models are otherwise identical. Intuitively, the TVC-ECM allows short-run adjustment to a time-varying long-run equilibrium, while the equilibrium relationship in the FC-ECM is fixed.
2.3 Fixed Coefficient Models Estimated Over Rolling Windows

An FC model, being a special case of the TVC model with \( p, q = 0 \), will generally suffer from an omitted variable bias if the true model has a functional coefficient of time. These omitted variables will be functions of \( x_t \), so that the error term includes an unexplained deterministic or stochastic trend that is strongly persistent, and its stationary or weakly dependent component is also explicitly correlated with that regressor. Increasing the order \( p \) and \( q \) of \( \alpha_{pq}(r) \) ameliorates the bias in estimating TVC model. It would be reasonable to speculate that, instead of allowing \( \alpha \) to vary by increasing \( p \) and \( q \), an FC model estimated over rolling windows, so that \( \alpha \) may vary over these windows, might effectively eliminate the bias. In fact, it does not eliminate the bias, as we will see in what follows.

In order to compare the income elasticity estimated from our TVC model with that from an FC model estimated over rolling windows, we let \( T_i = \{t_i - h, \ldots, t_i, \ldots, t_i + h\} \) for \( i = 1, \ldots, m \) be a subset of \( T = \{1, \ldots, T\} \) centered at \( t_i \), where \( T_i \) may or may not overlap. Subsequently, we consider estimating an FC model to obtain an estimate for the fixed coefficient over each subset \( T_i \) of \( T \) for \( i = 1, \ldots, m \).

To focus exclusively on the coefficient \( \alpha \), we let the additional covariates be subsumed by the error term with generic notation \((e_t)\), and rewrite the TVC model in (4) as

\[
y_t = \tau + \alpha_t x_t + e_t,
\]

which we estimate using all observations in the entire set \( T \) of samples. In contrast, we write the FC model

\[
y_t = \tau_i + \alpha_i x_t + e_t,
\]

using the generic error term again, which we fit using observations over each subset \( T_i \) of \( T \) for \( i = 1, \ldots, m \).

If \( h \) is sufficiently small, one may expect to have

\[
\hat{\alpha}_{t_i} \approx \hat{\alpha}_i
\]

for all \( i = 1, \ldots, m \), where \( \hat{\alpha}_{t_i} \) is our TVC estimate evaluated at time \( t_i \) of \( \hat{\alpha}_t \) obtained from regression (6) using all observations in \( T \), and \( \hat{\alpha}_i \) is the usual OLS estimate from regression (7) fitted using only observations in \( T_i \). However, the approximation in (8) does not hold.
To see this, note that by a first-order expansion around $x_t$, we have

$$\alpha_t x_t \approx \alpha_{t_i} x_{t_i} + \left( \alpha_{t_i} + x_{t_i} \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i} \right) (x_t - x_{t_i})$$

$$= -x_{t_i}^2 \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i} + \left( \alpha_{t_i} + x_{t_i} \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i} \right) x_t$$

for all $t$ close to $t_i$. If $h$ is small enough, it holds for all $t$ in $T_i$. It follows that

$$\tau_i \approx \tau - x_{t_i}^2 \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i} \quad \text{and} \quad \alpha_i \approx \alpha_{t_i} + x_{t_i} \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i}.$$ 

and, in general, this second approximation contradicts that in (8).

The reason why (8) fails to hold is clear. If the income elasticity is time-varying, then over any time interval the electricity demand responds to income changes in two different channels: directly through income elasticity, and also indirectly through the changes in income elasticity caused by income changes. The magnitude of the indirect effect is determined by the income level $x_{t_i}$ and the rate of change $\partial \alpha_t / \partial x_t$ of $\alpha_t$ with respect to $x_t$ at time $t_i$, the latter of which is approximately given by a positive constant multiple of the time derivative of $\alpha_t$ because the income level $x_t$ is driven dominantly by its increasing time trend. In most cases, the time derivative of $\alpha_t$, as well as the value of $x_t$, is positive.

Therefore, in such case we have

$$\hat{\alpha}_i \approx \hat{\alpha}_{t_i} + x_{t_i} \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i} > \hat{\alpha}_{t_i},$$

and the FC model is expected to estimate significantly larger elasticities, even if the coefficient is allowed to vary using rolling regressions. We may also deduce that

$$\hat{\tau}_i \approx \hat{\tau} - x_{t_i}^2 \frac{\partial \alpha_t}{\partial x_t} \bigg|_{t=t_i} < \hat{\tau},$$

where $\hat{\tau}$ is the estimate of $\tau$ in regression (6) over $T$, and $\hat{\tau}_i$ is the OLS estimate of $\tau_i$ in regression (7) over $T_i$. The constant term is therefore expected to be underestimated if we use the FC model with rolling windows.

2.4 Modeling Seasonality

In this section, we describe how we specify $s_t$, the seasonal component of electricity demand. It is well known that the seasonal variation in electricity demand is mostly due to the change
in temperature distribution. Therefore, in what follows, we let $s_t$ be given by

$$s_t = \int g(r) f_t(r) dr,$$

(9)

where $f_t(r)$ denotes the temperature density for the month of $t$, $t = 1, \ldots, n$, and $g(r)$ is a function we call the temperature response function (TRF).\(^8\) Throughout the paper, we let the temperature be normalized to the unit interval by adding 20°C and then dividing by 60°C, for expositional and computational convenience.

If we assume that $s_t$ is a linear functional of the temperature density $f_t(r)$, the temperature response function $g(r)$ is well defined and the seasonal component $s_t$ is given as in (9).\(^9\) We may interpret $g(r)$ as the functional coefficient of $f_t(r)$ representing how $s_t$ is affected by $f_t(r)$ and, for every $r$ fixed, $g(r)$ measures the relative magnitude of the effect of likelihood value $f_t(r)$ on $s_t$. Furthermore, if we let $r_t$ be the random variable representing the temperature in month $t$ and having density $f_t(r)$, then we have $s_t = E g(r_t)$. We may therefore also interpret $g(r)$ as the effect of temperature on the electricity demand when $r_t = r$.

We may approximate $g(r)$ by

$$g_{pq}(r) = c_0 + \sum_{j=1}^p c_j r^j + \sum_{j=1}^q (c_{p+2j-1}, c_{p+2j}) \phi_j(r)$$

(10)

with $\phi_j(r) \equiv (\cos 2\pi jr, \sin 2\pi jr)'$ for $r \in [0, 1]$, similarly as for the function $\alpha(r)$ generating the time-varying elasticity $\alpha_t$. Using this approximation, we may write

$$s_t = c_0 + \sum_{j=1}^p c_j \int r^j f_t(r) dr + \sum_{j=1}^q (c_{p+2j-1}, c_{p+2j}) \int \phi_j(r) f_t(r) dr + e_{pqt},$$

(11)

where $e_{pqt}$ represents the approximation error.

In all existing studies of which we are aware, the temperature effect in electricity demand is specified as a function of some statistic such as the average temperature for a given day or month, instead of the entire temperature distribution. Note that the average results from setting $c_1 = 1$ and $c_j = 0$ for $j \neq 1$. We strongly believe that the effect of temperature on electricity demand should be given as a functional of the entire temperature distribution, not as a function of the average temperature.

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8We may allow for an additional random error term in (9). Doing so simply adds innocuous error to the OLS regression (12) discussed in Section 3.3. Since doing so would not change the estimation of the TRF, we suppress it for the sake of simplicity.

9This result follows from the well-known Riesz Representation Theorem.
3 Data and Empirical Implementation

We conduct three separate analyses for Korean residential, commercial, and industrial sectors using sales in megawatt hours (MWh) from Korea Electric Power Corporation (KEPCO), which has a 93% market share in Korea. The three sectors recently comprised 14%, 33%, and 53% of KEPCO’s sales, according to its 2012 annual report. Because the raw data series are observed at mixed and irregular sampling frequencies, we construct monthly series indexed by $t$. For the residential sector, we employ 216 monthly observations running from 1995:01 to 2012:12. For the commercial and industrial sectors, we employ 336 monthly observations running from 1985:01 to 2012:12, since price data in these sectors are available over a longer period.

Procedures to construct the income/output series ($x_t$) and the price series ($p_t$) are straightforward and we address them in Section 3.1 below. Procedures for the remaining series ($y_t$) and ($s_t$) are much more involved and we address them separately in Sections 3.2 and 3.3. Demand for electricity in each sector is widely known to experience pronounced cycles. Because data availability makes a month the most convenient unit for measuring time for estimating the model, we can conceptualize a partition of the frequency domain into high-frequency (intra-monthly) and low-frequency (seasonal) cycles.

High-frequency (Intra-monthly) Cycles. On a typical work day residential electricity usage declines in the mornings as workers leave home and increases in the evenings when they return. The patterns for commercial and industrial usage are the reverse. Similarly, during a typical work week, residential electricity usage declines at the beginning of the work week and increases at the end, and usage is reversed for commercial and industrial customers. Of course, a similar pattern would exist for hypothetical intra-monthly measures of macroeconomic income and output. Were we to observe monthly electricity demand directly, we would expect the aggregation of all series to the monthly frequency to smooth out such high-frequency cycles.

However, we observe electricity sales from KEPCO in 21 overlapping billing cycles. The May bill for a customer in billing cycle 1 reflects temperatures from April 2 to May 1, while that for a customer in billing cycle 15 reflects temperatures from April 16 to May 15. We may aggregate all of the sales data for May to obtain a single datum for that month. However, the May aggregate clearly contains some sales from April, while some of the May sales will not be recorded until June. This issue is discussed in detail by Train et al. (1984).

Moreover, billing cycles ending in the same calendar month contain data from different numbers of workdays, so that an aggregate of data from these cycles does not generally re-
flect the same number of workdays as the calendar month that the aggregate represents. For example, the billing cycles 1 and 15 in May 2012 contained 15 and 16 workdays respectively, while the calendar month May 2012 contained 18 workdays.\textsuperscript{10}

Because income and price are measured by the calendar month, but electricity usage is measured by aggregating billing cycles, we create a measure of workday equivalents, similar to the approach of Moral-Carcedo and Vicéns-Otero (2005), to control for calendar effects in the electricity usage series. Section 3.2 describes this approach and construction of $y_t$ in detail.

**Low-frequency (Seasonal) Cycles.** The literature posits a variety of ways to handle seasonal cycles in monthly electricity demand due to temperature fluctuations. One alternative is to further aggregate the data, or else take a seasonal difference of the data. The loss of information and degrees of freedom may be too high to justify such an approach. Direct approaches to modeling temperature fluctuations include a TRF such as that described already in Section 2.4. Such functions are usually either parametrically specified to be non-linear or else nonparametrically specified, and are estimated using an aggregated monthly measure of temperature, such as monthly average, minimum, or maximum temperatures, or monthly heating and cooling degree days (HDD and CDD).\textsuperscript{11}

While a generic monthly measure of seasonality may capture much of the low-frequency cyclicality in demand, high-frequency cycles still matter in the construction of this measure. Consider two examples. A spring or fall month such as March may feature substantial temperature volatility, requiring both heating and cooling, while maintaining an average that would imply little or no electricity usage for either. A heat wave in a summer month will induce peak demand during the day. If the heat wave happens to fall on a holiday weekend, it will have a larger effect on residential demand than if it happened to fall during the middle of the week. Using monthly average, minimum, or maximum temperatures would fail to adequately capture the demand effects of such events. Heating and cooling degree days better distinguish daily fluctuations within a month, but not intradaily fluctuations.

A solution is offered by the availability of intra-monthly (hourly) temperature observations. Many of the empirical analyses of electricity conducted at the monthly frequency implicitly use intra-monthly data in the form of monthly HDD and CDD, which reflect

\textsuperscript{10} Calculated as Tuesdays, Wednesdays, Thursdays, and Fridays other than May 1 (Labor Day) and April 12 (a National Assembly election day), as noted in Section 3.2 below.

daily temperature variations in a month. Similarly, Engle et al. (1986) create a monthly regressor by aggregating daily temperature data using billing cycle weights.

Our approach to using high-frequency temperature data is based on the recently developed semiparametric approach of Park et al. (2010). We estimate a TRF using the whole distribution of temperatures in a month rather than a summary statistic for the month. We thus obtain a more precise monthly measure $s_t$ of variations in demand based on actual temperatures observed in that month. Section 3.3 explains the construction of $s_t$ in detail.

### 3.1 Income, Output, and Electricity Price

As a proxy for disposable income in the residential sector, we construct a monthly measure of gross domestic product (GDP), created from quarterly GDP and monthly industrial production obtained from the Bank of Korea. We use a technique similar to Friedman’s (1962) interpolation of a low-frequency series (i.e., GDP) using a “related” high-frequency series (i.e., industrial production). Specifically, we compute monthly GDP as the monthly proportion of annual GDP, which is set equal to the monthly proportion of annual industrial production. To measure output in the commercial sector, we use the same monthly GDP measure in this sector as for the residential sector. We use monthly industrial production for output in the industrial sector, similarly to Halvorsen (1978). The series $(x_t)$ represents the natural logs of these respective series.

We obtain five series from the Korean Statistical Information Service (KOSIS) in order to construct the real price series for the three sectors. First, we obtain the nominal electricity price series for each sector. The data spans for each sector noted above are dictated by the availability of these three series. Real electricity price indices are constructed using the consumer price index from KOSIS for the residential and commercial sectors and using the producer price index from KOSIS for the industrial sector. Again, $(p_t)$ denotes the natural logs of these series.

### 3.2 Electricity Demand

In order to control for the calendar effects caused by mismatched intra-monthly cycles discussed above, we construct workday equivalents (“effective days”) in each billing cycle. Effective days are created by weighting “special days” (mostly non-workdays, such as Sundays) by typical usage on that type of day relative to a workday, which receives a unit weight. For this purpose, we use daily demand data from KEPCO collected using automatic meter reading but available over a much shorter period: January 1, 2005 until December 31, 2010.
Table 1: Special Days (Non-effective Days). Election days include presidential elections, National Assembly elections, and local government elections. Summer vacation includes all weekdays (Monday-Friday) in the week that includes August 1.

We first construct daily relative demand as \( RD_i = \frac{D_i}{\bar{D}} \), where \( D_i \) is the electricity demand on day \( i \), and \( \bar{D} \) is the average demand on workdays, Tuesday-Friday,\(^{12}\) up to a week before and after day \( i \). We then regress daily \( RD_i \) on a constant and indicators for specific days on which we expect demand to substantially deviate from a typical work day – primarily on weekends and holidays. Table 1 shows the specific days for which we include indicators and Table 2 shows the results of this regression.

The effective day for a workday is given by the (unit) constant and the effective day for each other type of day is given by the constant plus the respective coefficient. We may interpret that demand for electricity in the industrial sector decreases by 22% on Sundays, for example, from its level on a typical workday. We therefore record each Sunday as 78% of an effective day in the industrial sector. Note that the signs for the specific days are all negative for the industrial and commercial sectors and mostly positive for the residential sector, as workers use electricity more intensively at home while factories and offices shut

---

\(^{12}\)Monday is not considered a workday due to comparatively limited early morning electricity usage. Pardo et al. (2002) found loads on each of Tuesday-Friday to differ significantly from Monday, and in regressions similar to ours in Table 2, Moral-Carcedo and Vicéns-Otero (2005) estimated nearly unit coefficients for these four days.
Table 2: Effective Day Estimates. Results reflect a regression of daily relative demand $RD_j$ on indicators for special days noted in Table 1. An indicator is included for the “big holidays” noted in that table, and separate indicators specify days surrounding these holidays. A single indicator is included for “non-Sunday holidays,” which are election days and all of the remaining holidays on the lunar and solar calendars that do not fall on a Sunday.

down or reduce workloads on those days.

Once we estimate a measure of the effective day for each day in the whole sample, we can then aggregate the number of effective days that actually occurred in each billing cycle for each sector. We then average these billing cycle effective days across each of the 21 billing cycles ending in a single month, using billing cycle weights $w_b$ for $b = 1, \ldots, 21$ (shown in Table 3) created from the proportion of sales billed in each cycle, in order to obtain a measure of effective days in all of the billing cycles ending in a given month. It will be useful to denote the weighted average of the effective days in all of the billing cycles ending in a given month as $ED_t$. We then divide our measure of monthly demand $D_t$ in MWh, given by summing all billing cycle data ending in that month, by the number of effective days $ED_t$ to obtain a measure of demand per effective day in month $t$.

A difficulty from calendar effects remains, because different months may contain different numbers of effective days. The seasonal adjustment of industrial production and GDP should eliminate most of the calendar effects for those variables. In order to adjust demand accordingly, we multiply demand per effective day in month $t$ by the average $\overline{ED}$ of $ED_t$ across each month in the sample (30.733 in the residential sector, 29.098 in the commercial sector, 28.434 in the industrial sector). This creates a monthly measure $\exp(y_t)$ of demand in month $t$, and $(y_t)$ is the natural log of this series.
<table>
<thead>
<tr>
<th>Cycle</th>
<th>Res</th>
<th>Com</th>
<th>Ind</th>
<th>Cycle</th>
<th>Res</th>
<th>Com</th>
<th>Ind</th>
</tr>
</thead>
<tbody>
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<td>2.5%</td>
<td>0.2%</td>
<td>14</td>
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<td>4.6%</td>
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</tr>
<tr>
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<td>3.2%</td>
<td>2.6%</td>
<td>0.2%</td>
<td>15</td>
<td>6.5%</td>
<td>5.6%</td>
<td>2.6%</td>
</tr>
<tr>
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<td>3.6%</td>
<td>2.7%</td>
<td>0.3%</td>
<td>16</td>
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<td>4.7%</td>
<td>2.7%</td>
</tr>
<tr>
<td>4</td>
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<td>5.6%</td>
<td>3.3%</td>
</tr>
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<td>5.2%</td>
<td>3.9%</td>
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<td>3.0%</td>
<td>0.9%</td>
</tr>
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<td>7</td>
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<td>0.6%</td>
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<td>2.5%</td>
<td>0.5%</td>
</tr>
<tr>
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<td>2.7%</td>
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</tr>
<tr>
<td>11</td>
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<td>2.9%</td>
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<td>30</td>
<td>2.5%</td>
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<tr>
<td>13</td>
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<td>3.7%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Billing Cycle Weights \((w_b)\). Weights reflect the proportion of sales billed in each cycle. Billing cycle 30 ends on the last day of each month, so that it reflects the calendar month.

Defining \(E_t = \overline{ED}/ED_t\), multiplication by \(E_t\) converts billing cycle demand \(D_t\) ending in month \(t\) to billing cycle demand in a generic month with an average number of billing cycle effective days. Formally, \(y_t = \ln(D_tE_t)\), and it will be convenient later to use \(D_t = \exp(y_t)/E_t\) in order to forecast demand \(D_t\) directly.

### 3.3 Seasonal Component of Electricity Demand

To estimate our demand models in (1) and (3), we need to obtain the seasonal component \(s_t\) of electricity demand. To do this, we let

\[
y_t^s = s_t + \varepsilon_t,
\]

where \(y_t^s\) is the observed seasonality in \(y_t\), which we define to be the deviation of \(y_t\) from its twelve-month moving averages, and \(\varepsilon_t\) is the error term. Under the specification of \(s_t\) in (9) and its approximation in (11), we have

\[
y_t^s = c_0 + \sum_{j=1}^{p} c_j \int r^j f_t(r) dr + \sum_{j=1}^{q} (c_{p+2j-1}, c_{p+2j}) \int \phi_j(r) f_t(r) dr + \varepsilon_{pq}\]

where \(\varepsilon_{pq} = e_{pq} + \varepsilon_t\). Therefore, once the observed seasonal component \(y_t^s\) and the temperature density \(f_t(r)\) are given for \(t = 1, \ldots, n\), the coefficients \(c_j, j = 0, \ldots, p + 2q\), can be estimated by OLS in regression (12). Clearly, we may use the corresponding \(g_{pq}(r)\) in (10) obtained from those estimates of \(c_j, j = 0, \ldots, 2p + q\), as an estimate of \(g(r)\).

Of course, it is possible to plug our specification of \(s_t\) in (11) directly into regressions.
Provinces | City | Weight Ranges
--- | --- | ---
Seoul, Incheon, Gyunggi-do, Gangwon-do | Seoul | 51.7 – 53.5
Daejin, Chungcheong-do | Daejin | 9.1 – 10.1
Daegu, Ulsan, Gyeongsangbuk-do | Daegu | 10.0 – 10.8
Gwangju, Jeolla-do, Jeju-do | Gwangju | 10.9 – 11.6
Busan, Gyeongsangnam-do | Busan | 15.7 – 16.8

Table 4: **Sales Regions.** City denotes the largest city in each region, from which we sample hourly temperatures. Weight ranges summarize sales percentages across years for each region.

(1) and (3), and estimate the newly defined regressions with the additional TRF terms. In this case, we would not need to estimate $s_t$ separately. Instead, we follow a two-step procedure by first estimating $s_t$ and then using it as a regressor in the main regressions, because we want to compare the FC and TVC models for electricity demand more fairly. If the TVC model is cointegrating, then the FC model is spurious, in which case these TRF terms might pick up some of the omitted nonlinearity in the income elasticity, rather than just the intended seasonality. The two-step procedure avoids this possibility, allowing more accurate comparisons of the two models.

Unfortunately, we do not observe monthly demand directly and the temperature may vary across the entire country. As noted above, we observe demand in billing cycles, and we use the weights $w_b$ for $b = 1, \ldots, 21$, shown in Table 3, for the 21 billing cycles. Because we use the temperature distribution, our weighting procedure differs from that of Train et al. (1984), who aggregate temperature statistics from individual days using billing cycle weights. We can better estimate the distribution of hourly temperature observations over a whole billing cycle and then appropriately weight the cycles ending in the same month, than we could estimate the distribution of hourly observations over a single day and then, as those authors did, apply weights for each day in a given month.

In order to control for some of the spatial distribution of temperatures across Korea, we divide the country into five geographic regions similarly to the approach taken by Moral-Carcedo and Vicéns-Otero (2005) for Spain. Specifically, we use hourly temperatures from the Korea Meteorological Administration for five large and geographically dispersed cities: Seoul, Daejin, Daegu, Gwangju, and Busan. Over each month $t$, we assign weights $w_{at}$ for $a = 1, \ldots, 5$ based on KEPCO sales in the region containing each city. Table 4 shows the provinces in each specific region.\(^\text{13}\)

---
\(^\text{13}\)KEPCO keeps sales percentages in each billing cycle stable by varying the number of customers billed in each cycle. Sales percentages within each region are less stable, so we allow these weights to vary over time.
We estimate the temperature density $f_{ab}(r)$ for each city $a$ in each billing cycle $b$ ending in month $t$, resulting in 105 densities of hourly temperature observations for each month in the sample. Specifically, we use a kernel density estimator with normal kernel and plug-in bandwidth. We then aggregate the densities across cities and billing cycles to obtain a national monthly temperature density, given by $f_t(r) = \sum_{a=1}^{5} \sum_{b=1}^{21} w_{at}w_{ab}f_{ab}(r)$, where $w_{at}$ and $w_{ab}$ are weights assigned to each city and each billing cycle. The values of integrals $\int r^j f_t(r) dr$ and $\int \phi_j(r)f_t(r) dr$ in (12) are obtained numerically as Riemann sums.

In order to estimate the temperature response function $g(r)$, we distinguish three typical responses of electricity demand to temperature. A flat response means that demand does not respond to temperature. We do not expect most industrial uses of electricity, such as for electric arc furnaces for steel or aluminum manufacture, to be responsive to outside temperature fluctuations. Another response is U-shaped, with a minimum typically near 18°C and peaks at the temperature extremes, as electricity is used in both heating and cooling. On the other hand, existing technology drives an asymmetric U, since there are more competing fuels for heating—natural gas and propane, e.g.—than there are for cooling. As we show below, the semiparametric FFF form that we use to estimate $g(r)$ is flexible enough to handle all these three patterns.

The estimates of $c_j$ are shown in Table 5. In order to select $p$ and $q$, we consider $p = 1, 2$ and $q = 0, 1, 2$ and use the cross-validation criterion suggested by Burman et al. (1994) and used by Park (2010) specifically for the FFF form. The criterion suggests $p, q = 1$ for both the residential and commercial sectors and $p = 1, q = 0$ for the industrial sector. There appears to be a strong structural break in the TRF of the commercial sector. In order to determine the date of the break, we calculate values of $R^2$ for a model in which the whole coefficient vector has a break point immediately after dates from 1990.1 and rolling forward to 2008.12. We find that the model with the highest $R^2$ has a break point immediately after

<table>
<thead>
<tr>
<th></th>
<th>Residential Sector</th>
<th></th>
<th>Commercial Sector</th>
<th></th>
<th>Industrial Sector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>coeff.</td>
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<td>est. t-value</td>
<td>est. t-value</td>
<td>est. t-value</td>
<td>est. t-value</td>
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</tr>
<tr>
<td>$c_0$</td>
<td>-0.490 -7.397</td>
<td>-0.636 -9.862</td>
<td>-0.326 -4.049</td>
<td>0.000 -0.061</td>
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<td>$c_1$</td>
<td>1.077 8.815</td>
<td>1.427 11.648</td>
<td>0.911 5.609</td>
<td>0.911 0.046</td>
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<tr>
<td>$c_2$</td>
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<td>0.171 20.378</td>
<td>0.241 30.971</td>
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</tr>
<tr>
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<td>0.324 8.993</td>
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<td></td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.815</td>
<td>0.877</td>
<td>0.935</td>
<td>0.003</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Temperature Response Functions

Residential: 1995.01-2012.12
Commercial: 1985.01-2006.09
Commercial: 2006.10-2012.12
Industrial: 1985.01-2012.12

Figure 1: Temperature Response Functions. Estimated by (10) using least squares coefficient estimates in Table 5 for the residential, commercial, and industrial sectors.

2006.9, so we subsequently use this model in the procedure to obtain $s_t$ for the commercial sector.

Figure 1 shows the TRF’s $g_{pq}(r)$ using (10) with coefficient estimates from Table 5. TRF’s for residential and commercial sectors exhibit the expected asymmetric U, while the TRF for the industrial sector is nearly flat, also as expected. Clearly much more electricity is used for air conditioning at extremely hot temperatures than for heating at extremely cold temperatures in the residential and commercial sectors. The minimums are around the comfortable temperatures of 14°C-18°C. The fact that the commercial sector minimums are a bit less than 18°C for the residential sector is not surprising. After all, the average temperature when commercial buildings are most densely populated (during the day) should exceed that when residential buildings are most densely populated (at night). At a marginal temperature, as workers leave home and arrive at work, energy efficient firms switch on the air conditioning that may have been dormant overnight. Note that the structural break in
Table 6: **ADF Unit Root Tests.** Lag lengths chosen by BIC from a maximum of 8.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Demeaned</th>
<th>Lags</th>
<th>Detrended</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
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<td>$y_t$</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residential</td>
<td>$-1.247$</td>
<td>$7$</td>
<td>$-3.033$</td>
<td>$7$</td>
</tr>
<tr>
<td>Commercial</td>
<td>$-2.378$</td>
<td>$4$</td>
<td>$-1.255$</td>
<td>$4$</td>
</tr>
<tr>
<td>Industrial</td>
<td>$-2.592$</td>
<td>$2$</td>
<td>$-2.241$</td>
<td>$2$</td>
</tr>
<tr>
<td>$x_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Res (GDP, 1995-)</td>
<td>$-1.029$</td>
<td>$2$</td>
<td>$-2.854$</td>
<td>$2$</td>
</tr>
<tr>
<td>Com (GDP, 1985-)</td>
<td>$-3.105$</td>
<td>$2$</td>
<td>$-2.082$</td>
<td>$2$</td>
</tr>
<tr>
<td>Ind (Ind Prod)</td>
<td>$-1.677$</td>
<td>$1$</td>
<td>$-3.424$</td>
<td>$1$</td>
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<tr>
<td>$p_t$</td>
<td></td>
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</tr>
<tr>
<td>Residential</td>
<td>$-0.925$</td>
<td>$1$</td>
<td>$-1.519$</td>
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</tr>
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<td>Industrial</td>
<td>$-1.948$</td>
<td>$1$</td>
<td>$-0.886$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

the commercial sector’s TRF suggests that workers have become slightly less responsive to heat but much more sensitive to cold. This could reflect changes in efficiency or perhaps changes in a temperature elasticity of substitution between alternative fuels.

### 4 Main Empirical Results and Diagnostics

We first test ($y_t$), ($x_t$), and ($p_t$) for unit roots. We then estimate the FC and TVC models. Specification tests show stronger evidence for an authentic (cointegrating) TVC regression than an FC regression. We also compare the TVC model with the FC benchmark in two forecasting exercises.

#### 4.1 Unit Root Pre-testing

The results of augmented Dickey-Fuller (ADF) unit root tests with lag lengths chosen by the Schwarz/Bayesian information criterion (BIC) are shown in Table 6. The intercept $\tau$ in our models above explicitly allows for different means, but a linear trend common to the cointegrated series is implicitly allowed. We therefore present results for both demeaned and detrended series.

It is very clear that ($y_t$) in all sectors have unit roots and almost as clear for ($x_t$). A unit root in the latter is rejected for GDP with size 5% but not as strongly as 1% over the
period since 1985 (used for the commercial sector) when no linear trend is allowed. When such a trend is allowed, we cannot reject a unit root – i.e., an additional (stochastic) trend.

The evidence for \( p_t \) is slightly less conclusive, because the rejection for the commercial sector is stronger with no linear trend. Log asset prices may exhibit a linear trend when an arbitrager may store the asset. Since electricity is not storable in large amounts, we do not expect any linear trend, and the ADF tests that do not allow for a linear trend reject the unit root hypothesis in this sector. Fortunately, our estimation technique is robust to included stationary variables mistakenly believed to have unit roots (Kim and Park, 1998), so our results below should be robust to the order of integration of price, as long as the remaining variables are cointegrated if price is stationary.

4.2 Estimation

Table 7 shows the results from estimating the more restrictive FC model using CCR. The intercepts \( \tau \) reflect the means of any unobservables and discrepancies in units of the various measures included. Differences in estimates of \( \tau \) across sectors are not surprising. However, given that a major unobservable is technology, the negative sign estimate for the commercial sector is unexpected.

It is also not surprising that \( \gamma \) is estimated to be approximately (not significantly different from) unity in the residential and commercial sectors, because the regressor \( s_t \) is created as a prediction of a component (the seasonal component) of the regressand, electricity demand. That the coefficient is not significantly different from zero in the industrial sector is not surprising, either, reflecting the lack of seasonality in industrial demand. Keeping in mind that the TRF in the industrial sector was estimated to be almost flat, the large coefficient estimate with very small \( t \)-value may be attributed to collinearity with the intercept \( \tau \).

The price elasticity \( \beta \) is estimated to be significantly negative as expected in the residential and industrial sectors, but not in the commercial sector, where it is (insignificantly) estimated to be positive. It is quite unlikely that the profit-maximizing commercial sector would have a positive price elasticity.

The income/output elasticities \( \alpha \) are estimated to be positive in all sectors as expected, but the commercial sector appears to have a high output elasticity (>1), suggesting diseconomies of scale in that sector. While diseconomies of scale might be plausible, diseconomies of that magnitude seem implausible and the other coefficient estimates for this sector already cast doubt on the validity of estimates from the FC model.

Table 8 shows the results from estimating the TVC model in (4) using CCR. The intercept estimates from the TVC model are positive in all sectors, as expected. Again, the
<table>
<thead>
<tr>
<th>Coefficients by Sector</th>
<th>Residential</th>
<th>Commercial</th>
<th>Industrial</th>
</tr>
</thead>
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<tr>
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<td>t-value</td>
<td>est.</td>
</tr>
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<td>5.84</td>
<td>-6.22</td>
</tr>
<tr>
<td>γ</td>
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<td>1.16</td>
</tr>
<tr>
<td>β</td>
<td>-0.42</td>
<td>-2.96</td>
<td>0.15</td>
</tr>
<tr>
<td>α</td>
<td>0.90</td>
<td>11.62</td>
<td>1.97</td>
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</table>

<table>
<thead>
<tr>
<th>Long-run Variances of the CCR Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_n^2$</td>
</tr>
</tbody>
</table>

Table 7: **FC Model Estimates.** Coefficient estimates and $t$-values for the FC model in (1) using canonical cointegrating regressions (CCR’s) for each sector. The CCR methodology for the TVC model is described in the appendix, and the FC model is simply the TVC model with $p, q = 0$.

<table>
<thead>
<tr>
<th>Coefficients by Sector</th>
<th>Residential</th>
<th>Commercial</th>
<th>Industrial</th>
</tr>
</thead>
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<tr>
<td>τ</td>
<td>13.88</td>
<td>22.17</td>
<td>8.08</td>
</tr>
<tr>
<td>γ</td>
<td>1.03</td>
<td>30.65</td>
<td>1.04</td>
</tr>
<tr>
<td>β</td>
<td>-0.07</td>
<td>-0.63</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Estimates of the TVC’s : $\alpha_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>est.</td>
</tr>
<tr>
<td>$\lambda_0$</td>
</tr>
<tr>
<td>$\lambda_1 : \frac{t}{T}$</td>
</tr>
<tr>
<td>$\lambda_2 : (\frac{t}{T})^2$</td>
</tr>
<tr>
<td>$\lambda_3 : \cos(2\pi \frac{t}{T})$</td>
</tr>
<tr>
<td>$\lambda_4 : \sin(2\pi \frac{t}{T})$</td>
</tr>
<tr>
<td>$\lambda_5 : \cos(4\pi \frac{t}{T})$</td>
</tr>
<tr>
<td>$\lambda_6 : \sin(4\pi \frac{t}{T})$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long-run Variances of the CCR Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_n^2$</td>
</tr>
</tbody>
</table>

Table 8: **TVC Model Estimates.** Coefficient estimates and $t$-values for the TVC model in (4) using canonical cointegrating regressions (CCR’s) for each sector. The CCR methodology is described in the appendix. BIC is used to select the orders $p$ and $q$ of the FFF form approximation.
estimates of $\gamma$ are close to one in the first two sectors, but insignificant in the industrial sector. The price elasticity $\beta$ is now estimated to have the expected negative sign in all sectors. The magnitudes of the point estimates are larger in the commercial and industrial sectors than in the residential sector, with stronger statistical evidence against zero. Firms are clearly more sensitive to price than households are, which most likely reflects demand charges based on intensity of usage that are levied on large users by KEPCO.

The parameters embedded in $\alpha_t$ that are estimated to have the largest magnitude are $\lambda_0$ and $\lambda_1$ in all sectors, with the addition of $\lambda_2$ in the residential sector. That the first two are positive across all sectors signifies positive and increasing elasticities over time. That the third is negative in the residential sector signifies a decline in the rate of increase, but the coefficient remains positive over the unit interval over which this function is defined. Specifically, the function $0.064 + 0.159r - 0.093r^2$ starts at 0.064, reaches its maximum of about 0.132 at about $r = 0.159/(2 \times 0.093) = 0.85$, and ends at $0.064 + 0.159 - 0.093 = 0.130$ over $r \in [0, 1]$.

The non-zero weights given to the periodic functions allow the function $\alpha(r)$ to deviate from a perfect quadratic function (residential) or linear functions (commercial and industrial). Because the periodic functions are linear combinations of $\cos 2\pi jr$ and $\sin 2\pi jr$ with $j \leq q = 2$ the shortest period that can be modeled by these functions is $2\pi/4\pi = 1/2$ of $r$. In other words, $\cos 4\pi r$ repeats at $r = t/T = 1/2$, which is halfway through the sample – a cycle that is intentionally much too long to account for seasonality.

To better illustrate the variation in the income elasticity $\alpha_t$ in each sector, Figure 2 plots the elasticities over time. The income elasticities of demand in the residential sector are quite small: all elasticities are less than 0.14, suggesting that residential consumers view electricity as a necessity. Nevertheless, the total change in elasticity over the duration of the sample is almost 100%! In other words, an income change in 2012 induced a change in electricity demand double that induced by an income change of equal size in 1995. Such a dramatic increase could be driven by the rise of portable and other electronic devices during this period, which has been particularly dramatic as Korea has rapidly developed.

The commercial and industrial sectors show graphically similar trends in output elasticities of conditional factor demand, with increases of about 25% and 50%, respectively. Again, an increasing reliance on electronic devices, which may also include electric heating systems, could be a possible explanation for such a dramatic increase. It is not surprising that the magnitudes of these elasticities are different from those in the residential sector, because they reflect different channels of demand. Both firm sectors exhibit economies of scale in electricity, but such cost advantages appear to be decreasing over time as output in the digital age requires more electricity.
4.3 Model Comparisons

To compare the results generated by the FC and TVC models, first note the major differences in the magnitudes of elasticities for each sector estimated by the two models. In the residential sector, the TVC model estimates an average income elasticity of about 0.11, while the more restrictive FC model estimates that elasticity to be 0.90. Similarly, the TVC model estimates output elasticities averaging about 0.65 but no more than 0.72 in the commercial sector and about 0.60 but no more than 0.70 in the industrial sector, while the more restrictive FC model estimates these to be 1.97 and 0.89 respectively. The results in all three sectors support the discussion on model comparisons in Section 2.3 above: the more restrictive FC model substantially overestimates the elasticities.

Testing the FC null that $\lambda_1 = \cdots = \lambda_{p+2q} = 0$ against the TVC alternative, with the maintained hypothesis that both FC and TVC regressions are cointegrating, provides a more formal statistical comparison. Using the CCR procedure to obtain asymptotically
Table 9: Tests to Compare the FC and TVC Models and for Cointegration. Wald statistics with $\chi^2$ limiting critical values are shown. The first column shows tests of the null that $\lambda_1 = \cdots = \lambda_{p+2q} = 0$ (the FC model in (1) is valid) against the TVC alternative. The second column shows tests of the null that the TVC model in (4) is cointegrating (authentic) against the null that it is spurious. The third column shows tests of the same null against the same alternative, but for the FC model. See the appendix for more details of the cointegration tests.

<table>
<thead>
<tr>
<th>Sectors</th>
<th>Comparison Test Stat.</th>
<th>5% C.V.</th>
<th>VAT: TVC Test Stat.</th>
<th>5% C.V.</th>
<th>VAT: FC Test Stat.</th>
<th>5% C.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Res</td>
<td>330.11</td>
<td>12.59</td>
<td>12.16</td>
<td>9.49</td>
<td>260.03</td>
<td>9.49</td>
</tr>
<tr>
<td>Com</td>
<td>725.39</td>
<td>11.07</td>
<td>13.24</td>
<td>9.49</td>
<td>642.07</td>
<td>9.49</td>
</tr>
<tr>
<td>Ind</td>
<td>399.83</td>
<td>11.07</td>
<td>9.10</td>
<td>9.49</td>
<td>333.43</td>
<td>9.49</td>
</tr>
</tbody>
</table>

normal parameter estimates allows a Wald statistic with a $\chi^2$ limiting distribution and $p + 2q$ degrees of freedom, which is 6 for the residential sector and 5 for the commercial and industrial sectors. The first column of Table 9 very clearly rejects the FC model in favor of the TVC model in every sector.

Although the above tests maintain that the regressions for each sector are cointegrating under both the null and alternative, there is no reason that this must be so. The null FC model omits nonstationary terms and therefore may be spurious if indeed the true model has a time-varying coefficient. As an indirect comparison of the two models, we test each separately for cointegration using the Wald-type variable addition test introduced by Park (1990) and adapted to the time-varying coefficient setting by Park and Hahn (1999).

The idea underlying this test is that the residuals from a spurious regression will contain nonstationary trends of some sort. Under the null of a cointegrating regression, adding superfluous trends to the model will not substantively effect estimation of the long-run elasticities, and the coefficients on these trends will be correctly estimated to be zero. Under the alternative of a spurious regression, the additional trends may not be superfluous, in the sense that including them may absorb some of the otherwise latent trend in the residual series. The variable addition test is based on the difference between the residual sums of squares of the two models, which is zero only under the null and deviates from zero under a well-specified alternative. The appendix contains a more detailed discussion.

Following Park (1990), we add fourth-order polynomial trends to each of the TVC models estimated above. The second column of Table 9 shows the test statistics for the TVC model. These statistics are compared against a chi-squared critical value with 4 degrees of freedom. We fail to reject cointegration in the industrial sector, but reject cointegration in the residential and commercial sector with 5% size. Nevertheless, we cannot reject cointegration in any sector with 1% size (13.28 critical value). Of course, because we use
polynomials in our test, there is a possibility of over-rejection from choosing the order of the FFF form to be too small. Size distortion may also result from the aggregation and interpolation discussed in the data construction above, even though these operations do not alter the long-run cointegrating relationships.\textsuperscript{14}

If the TVC model is a better approximation of the data generating process, then the error term of the FC model contains omitted variables, which are given by nonstationary \((x_t)\) with linearly or quadratically changing coefficients. A variable addition test with a quadratic trend added may be expected to have very good power to detect such an omission. Indeed, the third column of Table 9 shows that the case for cointegration using the FC specification is virtually nonexistent.\textsuperscript{15} The evidence overwhelmingly rejects the nulls that the FC regressions are cointegrating, clearly suggesting spurious regressions with inconsistent parameter estimates and providing further evidence in favor of the less restrictive TVC model.

As an additional comparison, Figure 3 shows time series plots of the annualized fitted residuals from each model. The TVC residuals are considerably smaller than the FC residuals in nearly every year in each sector.

\section*{4.4 Forecasting Comparisons}

For a further comparison of the FC and TVC models in (1) and (4), we conduct quasi-out-of-sample long-run forecasts. Specifically, we set aside all but the last 12 months of data (2012) to estimate the models, resetting \(T\) to be the subsample size up to December 2011. The forecast targets are \(D_{T+h} = \exp(y_{T+h})/E_{T+h}\), demand for a given sector in all billing cycles ending in month \(T+h\) expressed in MWh. We use only data through December 2011 for estimation and forecast up to 12 steps ahead, so that \(h = 1, \ldots, 12\). In the TVC model, we set \(\alpha(t/(T+12))\) for estimation and use \(\alpha((T+h)/(T+12))\) for the forecasts. This specification prevents \(r\) from exceeding unity so that the signs of neither the periodic functions \(\cos 2\pi jr\) and \(\sin 2\pi jr\) nor their rates of change switch during the forecast horizon.

The forecasts are \(\hat{D}_{T+h} = \exp(\hat{y}_{T+h})/E_{T+h}\), where \(\hat{y}_{T+h}\) is the \(h\)-step-ahead forecast of \(y_{T+h}\), except we use the actual regressors rather than forecast regressors at time \(T+h\). Our long-run forecasts are therefore not out-of-sample. Since long-run forecasts of GDP in particular are widely available, this exercise is not far from what a forecaster of long-run electricity demand might do: rely on third-party forecasts of GDP rather than specify a law

\textsuperscript{14}Ghysels and Miller (2013, 2014) show substantial size distortions in residual-based and likelihood-based cointegration tests resulting from aggregation and interpolation, but they did not consider variable addition cointegration tests.

\textsuperscript{15}Beenstock \textit{et al.} (1999) obtained a qualitatively similar result using residual-based fixed coefficient cointegration tests of the electricity market in Israel.
Figure 3: **Annualized Fitted Residuals.** Annual average of monthly fitted residuals from the FC model in (1) and from the TVC model in (4).
<table>
<thead>
<tr>
<th></th>
<th>Residential Sector</th>
<th></th>
<th>Commercial Sector</th>
<th></th>
<th>Industrial Sector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecast</td>
<td>Forecast</td>
<td>Forecast</td>
<td>Forecast</td>
<td>Forecast</td>
<td>Forecast</td>
</tr>
<tr>
<td></td>
<td>demand</td>
<td>FC</td>
<td>TVC</td>
<td>demand</td>
<td>FC</td>
<td>TVC</td>
</tr>
<tr>
<td>Jan-12</td>
<td>5,665</td>
<td>5,871</td>
<td>5,641</td>
<td>12,792</td>
<td>14,848</td>
<td>13,238</td>
</tr>
<tr>
<td>Feb-12</td>
<td>5,712</td>
<td>5,825</td>
<td>5,538</td>
<td>12,554</td>
<td>14,654</td>
<td>12,939</td>
</tr>
<tr>
<td>Mar-12</td>
<td>5,160</td>
<td>5,259</td>
<td>5,099</td>
<td>11,294</td>
<td>12,040</td>
<td>11,174</td>
</tr>
<tr>
<td>Apr-12</td>
<td>5,246</td>
<td>5,389</td>
<td>5,172</td>
<td>10,596</td>
<td>11,524</td>
<td>10,684</td>
</tr>
<tr>
<td>May-12</td>
<td>4,825</td>
<td>5,027</td>
<td>4,772</td>
<td>9,702</td>
<td>11,030</td>
<td>10,150</td>
</tr>
<tr>
<td>Jun-12</td>
<td>4,877</td>
<td>5,253</td>
<td>5,017</td>
<td>10,225</td>
<td>11,795</td>
<td>10,941</td>
</tr>
<tr>
<td>Jul-12</td>
<td>5,002</td>
<td>5,396</td>
<td>5,082</td>
<td>11,432</td>
<td>13,153</td>
<td>11,858</td>
</tr>
<tr>
<td>Aug-12</td>
<td>6,256</td>
<td>6,009</td>
<td>5,786</td>
<td>12,440</td>
<td>15,159</td>
<td>13,712</td>
</tr>
<tr>
<td>Sep-12</td>
<td>5,407</td>
<td>5,540</td>
<td>5,269</td>
<td>11,431</td>
<td>12,821</td>
<td>11,706</td>
</tr>
<tr>
<td>Oct-12</td>
<td>4,805</td>
<td>4,833</td>
<td>4,647</td>
<td>9,395</td>
<td>10,315</td>
<td>9,804</td>
</tr>
<tr>
<td>Nov-12</td>
<td>5,158</td>
<td>5,210</td>
<td>4,919</td>
<td>10,332</td>
<td>11,589</td>
<td>10,699</td>
</tr>
<tr>
<td>Dec-12</td>
<td>5,423</td>
<td>5,594</td>
<td>5,191</td>
<td>12,197</td>
<td>14,152</td>
<td>12,571</td>
</tr>
<tr>
<td>MAPE</td>
<td>3.44</td>
<td>2.83</td>
<td>13.69</td>
<td>3.95</td>
<td>11.18</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Table 10: Long-run Forecasts from (1) and (4). The forecast target is \( \exp(y_{T+h})/E_{T+h} \) which measures demand in GWh across all billing cycles ending in month \( T+h \) for 12 months in 2012. The forecasts are given by \( \exp(\hat{y}_{T+h})/E_{T+h} \), where \( \hat{y}_{T+h} \) is a series of 12 forecasts from the respective models with increasing forecast horizon. MAPE is the average absolute percentage deviation of the forecast from the target across an increasing horizon \( h \).

We also conduct short-run forecasts using the error-correction models FC-ECM and TVC-ECM in (2) and (5). We forecast during the same period described above, except the forecasts are recursive one-step-ahead forecasts using data up to the period immediately in order to forecast GDP. Since our main point is to compare the deterministically time-varying coefficient with the fixed coefficient, whether or not we use forecasts of the regressors does not seem relevant.
Residential Sector | Commercial Sector | Industrial Sector
--- | --- | ---
Forecast | Forecast | Forecast
demand | FC | TVC | demand | FC | TVC | demand | FC | TVC
Jan-12 | 5,665 | 5,774 | 5,621 | 12,792 | 13,448 | 13,141 | 20,971 | 19,901 | 20,095
Feb-12 | 5,712 | 5,634 | 5,532 | 12,554 | 12,595 | 12,582 | 20,569 | 20,417 | 20,331
Mar-12 | 5,160 | 5,277 | 5,177 | 11,294 | 11,041 | 11,177 | 21,327 | 21,413 | 21,424
Apr-12 | 5,246 | 5,286 | 5,181 | 10,596 | 10,561 | 10,623 | 20,525 | 20,718 | 20,808
May-12 | 4,825 | 4,872 | 4,814 | 9,702 | 10,000 | 9,985 | 20,642 | 20,902 | 21,195
Jun-12 | 4,877 | 5,147 | 5,082 | 10,225 | 10,751 | 10,695 | 20,717 | 20,140 | 20,692
Jul-12 | 5,002 | 5,083 | 5,073 | 11,432 | 11,199 | 11,211 | 21,083 | 20,686 | 21,128
Aug-12 | 6,256 | 5,780 | 5,849 | 12,440 | 13,985 | 13,546 | 20,605 | 20,447 | 21,173
Sep-12 | 5,407 | 5,519 | 5,403 | 11,431 | 10,577 | 10,842 | 20,452 | 19,470 | 19,857
Oct-12 | 4,805 | 4,873 | 4,693 | 9,395 | 9,523 | 9,662 | 19,847 | 20,130 | 20,395
Nov-12 | 5,158 | 5,079 | 5,007 | 10,332 | 10,184 | 10,124 | 20,885 | 20,326 | 20,746
Dec-12 | 5,423 | 5,451 | 5,341 | 12,197 | 12,266 | 11,973 | 21,514 | 20,517 | 20,974
MAPE | 2.30 | 2.05 | 3.46 | 2.87 | 2.28 | 1.81

Table 11: Short-run Forecasts from (2) and (5). The forecast target is \( \exp(\gamma_{T+1}) / E_{T+1} \times 10^{-3} \) which measures demand in GWh across all billing cycles ending in month \( T + 1 \) for 12 months in 2012. The forecasts are given by \( \hat{\exp(\gamma_{T+1}) / E_{T+1}} \times 10^{-3} \), where \( \hat{\gamma}_{T+1} = \hat{\gamma} + \hat{\gamma} \hat{s}_{T+1} \) is a one-step-ahead forecast for each of the 12 months and \( \hat{\gamma} \hat{s}_{T+1} \) is the prediction from the respective ECM. MAPE is the average absolute percentage deviation of the forecast from the target.

Strictly speaking, these forecasts are not out-of-sample, either, because we use the whole sample to estimate \( \gamma \) and to construct \( \hat{s}_{T+1} \), which we take as given in \( \hat{\gamma}_{T+1} \). However, in a comparison of forecasts from the two models, we do not expect using the whole sample to estimate \( \gamma \) to favor either model. As discussed in Section 3, the seasonal covariate \( s_t \) is calculated in a separate estimation step and in the same way for both FC and TVC models.

Taking \( \hat{\gamma} \) as given could be less innocuous, because it is necessarily estimated differently in the FC and TVC models. However, the periods of the trigonometric functions in \( \alpha_{pq}(r) \) are restricted to be long enough that they will not pick up seasonality, as discussed above. Consequently, the coefficient estimate \( \hat{\gamma} \) on the seasonal covariate \( s_t \) should not be biased when those functions are omitted by fixing \( \alpha \) in the FC model. For the residential and commercial sectors, the coefficient estimates \( \hat{\gamma} \) are very similar across the FC and TVC.
models. As noted above, the vast difference in \( \gamma \) between the two models for the industrial sector is more likely due to a variance inflated by collinearity, because the TRF in that sector is nearly flat.

Note that both of the models in (2) and (5) have fixed coefficients. The time-varying coefficient in the TVC-ECM enters only through the error-correction term, which is then taken as given by both ECM’s. Consequently, the main difference in the two forecasting methods is to allow for disequilibrium errors resulting from fundamentally different long-run equilibria.

Table 11 shows the results of the out-of-sample forecasts using first-order lag polynomials in both of the error-correction models. MAPE of the 12 forecasts \( \hat{D}_{T+1} \) from each model for each sector is calculated as above. The improvement from using the TVC approach in the residential, commercial, and industrial sectors is about 11%, 17%, and 20% respectively.

It is not surprising that the improvement is more modest than with the long-run forecasts with increasing horizon. Not only is the forecast horizon fixed to be only one period, but overlapping billing cycles mean that a May demand datum contains a substantial amount of demand in April. The marginal information to be forecast in May is therefore reduced, because the May target is more correlated with the April predictors than would otherwise be the case. As a result, the gain from a better forecasting model is also reduced. However, it is clear that demand in any sector is forecast more accurately when the disequilibrium error is estimated more precisely using the TVC model.

Finally, to put a price on the short-run forecast error, the total absolute error of the short-run FC forecasts in 2012 was 1,505 GWh, 4,786 GWh, and 5,714 GWh for the residential, commercial and industrial sectors. Average prices in the three sectors were 123.2 KRW/kWh, 95.95 KRW/kWh, and 90.41 KRW/kWh respectively. The forecast error in 2012 therefore translates into approximately 185 billion KRW, 459 billion KRW, and 517 billion KRW. At an exchange rate of 1,063 KRW/USD, those forecast errors amount to about $174 million, $432 million, and $486 million.

Using the TVC model generates total absolute error in 2012 of 1,349 GWh, 3,889 GWh, and 4,507 GWh, or a difference of 156 GWh, 897 GWh, and 1,207 GWh in the three sectors. Using the same calculation, we find that the savings from forecasting with the TVC ECM over the FC ECM is 19 billion KRW ($18 million), 86 billion KRW ($81 million), and 109 billion KRW ($103 million) in the respective sectors.
5 Concluding Remarks

The importance of predicting elasticities accurately to inform policy makers and distributors such as KEPCO underscores the need for and innovation of improved models for this purpose. Our time-varying coefficient approach to electricity demand clearly allows us to model the stylize fact of parameter instability in this literature. The smooth semiparametric approach allows both parameter instability and forecasting in a way that would not be feasible in a prototypical model with discrete structural breaks.

Further, we show that allowing parameter instability using a fixed-coefficient model over rolling windows does not allow for parameter instability in a satisfactory way: the elasticities are grossly overestimated if the data-generating process does indeed have a time-varying coefficient. The more flexible models with a time-varying long-run relationship between income/output and electricity demand clearly dominate the fixed-coefficient models in every dimension of comparison that we have considered: an analytical comparison and several empirical in-sample and pseudo-out-of-sample comparisons. Converting the short-run forecast error into a dollar amount, the time-varying approach saves roughly $18 million, $81 million, and $103 million per year in the residential, commercial, and industrial sectors respectively.

Our main empirical findings reveal dramatically increasing income/output elasticities over the sample periods in each sector. Specifically, we find demand for electricity to be very income inelastic – clearly a consumer necessity – but becoming more income elastic over time. Conditional factor demands in the commercial and industrial sectors are more (output) elastic than residential sector demand is (income) elastic, and those elasticities have also increased substantially. This finding suggests that those sectors exhibit economies of scale with respect to electricity in their cost functions, but that the economies of scale are weakening over time.

We speculate that the latent cause of the increase in elasticities is driven by the proliferation of electronic devices. In the residential sector, such proliferation increases the proportion of income spent on devices that require electricity, making demand more sensitive to income changes. In the firm sectors, such sensitivity could be driven by increases in electronic devices in the production of economic output. Over time, output relies more directly on electricity as a factor of production.
Appendix: Estimation and Testing

The CCR estimator discussed here is a straightforward extension of Park and Hahn (1999) to accommodate an additional I(0) regressor \((s_t)\). In order to avoid excessive notation in our exposition on estimation and testing procedures for the TVC regression in (4), we make a simplifying assumption that \(\tau = 0\). We also assume that \(s_t \sim I(0)\) and uncorrelated with the error and that a cointegrating vector of \((y_t, x_t, p_t)'\) exists.

Defining \(w_t = (u_t, \Delta x_t, \Delta p_t)' = (w_{1t}, w'_{2t})'\), we assume that an invariance principle holds for \(w_t\), such that \(T^{-1/2} \sum_{t=1}^{T} w_t \rightarrow_d B(t)\), where \(B = BM(\Omega)\) is Brownian motion with variance \(\Omega\). As is well known, \(\Omega = \sum_{k=-\infty}^{\infty} E w_t w'_{t-k}\), and we define \(\Sigma = E w_t w'_{t}\) and \(\Delta = \sum_{k=0}^{\infty} E w_t w'_{t-k}\), as is typical in this literature. We employ a typical partition of the long-run variance given by

\[
\begin{bmatrix}
\omega_{11} & \omega_{12} \\
\omega_{21} & \Omega_{22}
\end{bmatrix},
\]

so that \(\omega_{11}\) is the long-run variance of \((u_t)\), while \(\Omega_{22}\) is the long-run variance of \((\Delta x_t, \Delta p_t)'\), etc., and \(\Delta\) and \(\Sigma\) are partitioned accordingly.

Estimating a feasible CCR is a two-step procedure. We first estimate the model in (4) using least squares and construct consistent estimates of the variances above. Standard consistent long-run variance estimators are acceptable, and we use a nonparametric estimator with Parzen window and Andrews lag truncation. We use \(\tilde{\theta}, \tilde{\Sigma}, \text{etc.}\) to denote the consistent estimates from this step.

We then construct

\[
z^*_pqt \equiv \begin{bmatrix} \phi pq(t/T) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ p_t \\ s_t \end{bmatrix} - \begin{bmatrix} (\hat{\delta}'_{12}, \hat{\Delta}'_{22}) \tilde{\Sigma}^{-1} w_t' \\
0 \\
0 \\
0 \end{bmatrix},
\]

a \((p+2q+3) \times 1\) canonical regressor vector, and

\[
y^*_t \equiv y_t - w'_{2t} \tilde{\Sigma}^{-1} (\hat{\delta}'_{12}, \hat{\Delta}'_{22})' \begin{bmatrix} \phi pq(t/T) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tilde{\theta} - \tilde{\omega}_{12} \tilde{\Omega}_{22}^{-1} w_{2t},
\]

the canonical regressand. The canonical cointegrating regression,

\[
y^*_t = \tau + z^*_pqt \tilde{\theta} + u^*_pqt,
\]

thus has an error term of \(u^*_pqt \equiv u_{pqt} - \tilde{\omega}_{12} \tilde{\Omega}_{22}^{-1} w_{2t}\). This CCR may be estimated us-
ing least squares, but with standard errors estimated from a variance estimator given by 
\( \tilde{\omega}^2 \left( \sum z_{pq}^{*} z_{pq}^{*\prime} \right)^{-1} \), where \( \tilde{\omega}^2 \) is a consistent estimator of \( \omega^2 = \omega_{11} - \omega_{12} \Omega^{-1}_{22} \omega_{21} \) using the 
variance estimators from the first step regression.

The cointegration tests discussed above are variable addition tests based on that of Park 
(1990). Specifically, these are Wald tests given by 

\[
W_T = \tilde{\omega}^{-2} \left( \sum_t (\hat{\nu}_{pq}^*)^2 - \sum_t (\hat{\nu}_{pq}^{**})^2 \right),
\]

where \( (\hat{\nu}_{pq}^*) \) is a series of fitted residuals from the CCR regression in (13) and \( (\hat{\nu}_{pq}^{**}) \) is 
a series of fitted residuals from a CCR regression based on augmenting that in (13) with 
additional variables that have non-zero coefficients only under the alternative hypothesis 
of no cointegration. We use trend polynomials suggested by Park (1990). The limiting 
distribution is \( \chi^2 \) with degrees of freedom given by the number of variables added.

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