The Cyclical Dynamics of Illiquid Housing, Debt, and Foreclosures

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Abstract

This paper quantitatively accounts for the cyclical dynamics of key macroeconomic housing and mortgage market variables using a tractable, search-theoretic model of housing with equilibrium mortgage default. To explain these dynamics, the model highlights the importance of liquidity spirals which arise from the interaction of search frictions and endogenous credit constraints. During housing busts, longer selling times spill over into higher foreclosure risk, thereby magnifying the response of credit constraints to the depressed housing market. This contraction in credit then deepens the downturn. During booms, the reverse occurs. Based on these insights, I consider a foreclosure reform that makes all mortgages full recourse, and I show that implementing such a reform would reduce foreclosures and dampen housing dynamics.

Keywords: Housing; Liquidity; Search Theory; Credit Constraints; Household Debt; Foreclosure

JEL Classification Numbers: D31, D83, E21, E22, G11, G12, G21, R21, R31

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1 Introduction

This paper seeks to explain several stylized facts of housing, debt, and foreclosure dynamics. First, house prices, sales, residential investment, mortgage debt, foreclosures, and months supply—a proxy for selling time on the market\(^1\)—are all highly volatile compared to movements in output. Second, house prices tend to move together with sales, investment, and mortgage debt, while months supply and foreclosures move strongly in the opposite direction. For example, an increase in months supply from four to over \textit{eleven} months accompanied the 30%+ drop in U.S. real house prices from 2006 – 2010, reminiscent of the housing backlogs and double digit values of months supply in the early-1980s housing bust. Third, booms and busts are protracted and display asymmetric dynamics, with busts generally slower and shallower.\(^2\)

I account for these facts using a quantitative macroeconomic model with directed search in the housing market, long-term defaultable mortgage debt, idiosyncratic and aggregate uncertainty, and incomplete markets. In the model, standard productivity shocks in the non-housing sector spill over to produce large cyclical variation in housing and mortgage markets, which in turn affects the non-housing sector. I show that important propagation and amplification mechanisms emerge from the interaction between frictional housing and mortgage markets, underlining the importance of a joint analysis.

The first major contribution of this paper is theoretical. I establish \textit{block recursivity} in the housing market, which allows me to use the time path of one sufficient statistic—the shadow housing price—to calculate the dynamics of the entire distribution of house prices and trading probabilities when buyers

\(^1\)Months supply equals the ratio of unsold inventories to the sales rate.
\(^2\)See Burnside, Eichenbaum and Rebelo (2013) and Case and Quigley (2008) for a detailed discussion of the empirical evidence comparing booms and busts.
and sellers exhibit rich heterogeneity. To obtain this result, I introduce one-
period-lived real estate companies that intermediate trades between buyers and
sellers as passive market makers. Real estate companies dispatch real estate
agents to match with buyers and sellers, where agents buy housing from sellers
and sell housing to buyers. Directed matching occurs in submarkets indexed by
the house size and transaction price, and real estate companies ensure an equal
flow of housing from matched sellers to matched buyers. Free entry of real
estate agents establishes a type of no-arbitrage condition that links submarkets
and facilitates the computation of equilibrium dynamics using conventional
methods. This modeling innovation makes it possible to integrate housing
and financial markets while capturing heterogeneity in household portfolios.

Quantitatively, the model generates housing, debt, and foreclosure dynam-
ics in line with the stylized facts above. In particular, model house prices, ex-
sting sales, and months supply display volatilities and co-movements almost
identical to those of U.S. data from 1975 – 2010. Furthermore, the model gen-
erates protracted, asymmetric booms and busts. Most importantly, the model
sheds light on the mechanisms that can cause ordinary economic fluctuations
to produce the unique cyclical behavior observed in housing markets.

To explain these dynamics, I isolate the effects of search and uncover some
important channels that both amplify and prolong housing booms and busts.
First, the interaction of search frictions with endogenous credit constraints
generates liquidity spirals á la Brunnermeier and Pedersen (2009). As first
identified by Hedlund (2013), search frictions create substantial risk for home-
owners. During housing downturns, homeowners must lower their selling price
to avoid long selling delays. However, homeowners with large mortgages are
debt-constrained and forced to set a high price. As a result, heavily indebted
homeowners may fail to quickly sell their house in the event of financial ne-
cessity, causing many of them to end up in foreclosure. Banks anticipate this risk and price higher default premia into mortgages during times of low house prices and liquidity. This chain of events cascades into a vicious cycle of decreasing prices, reduced selling probabilities, higher foreclosures, and tighter credit. The reverse happens in booms. I quantify the impact of these liquidity spirals and conclude that search frictions contribute an additional 20% volatility in house prices and 27% volatility in residential investment.

Search frictions also help explain the strong co-movement of prices and sales and the prolonged nature of housing cycles. By delaying trades, search frictions spread out the impact of economic shocks on housing. As a result, search reduces the volatility of existing sales by over 45% and increases the correlation of sales and prices from 0.27 to 0.59. Therefore, housing booms tend to evolve gradually and exhibit price momentum, as discussed in Case and Shiller (1989) and recently in Head, Lloyd-Ellis and Sun (2014). However, the evolution of housing busts depends largely on their severity. During mild downturns, downward price stickiness results from homeowners who resist lowering their selling price because they expect prices to rebound and because they took out long-term mortgages during more favorable conditions. However, after a boom and large productivity drop, a spike in foreclosures and distressed sales contributes to a precipitous drop in house prices, followed by a prolonged decline drawn out by debt overhang. These scenarios reflect the shallow U.S. housing bust in the early 1990s and the recent sharp downturn, respectively.

Lastly, I consider the effects of a foreclosure reform that makes all mortgages legally as well as effectively full recourse. In particular, I allow banks to costlessly initiate deficiency judgments and seize up to 90% of the assets of foreclosed borrowers whose houses do not cover the full balance of their mortgage. I show that such a reform dramatically alters housing and foreclosure
dynamics, with house price and residential investment volatilities dropping by 12% and 17%, respectively, and existing sales volatility *increasing* by over 38%. Furthermore, fluctuations in months supply drop by over 85% and foreclosures essentially disappear. Less cyclical movement in credit constraints and fewer high leverage borrowers prevent liquidity spirals from emerging, which explains much of the change in dynamics. However, even without liquidity spirals, the economy with recourse mortgages still generates protracted booms and busts.

### 1.1 Related Literature

This paper makes substantial theoretical and quantitative contributions to the modeling and understanding of housing market movements. In doing so, I build upon multiple areas of related research. One strand of the literature, including seminal papers by Stein (1995) and Ortalo-Magné and Rady (2006), establishes how credit constraints magnify income shocks and amplify house price movements. Even so, the literature has struggled to develop housing models that produce sufficient house price volatility. Davis and Heathcote (2005) make one of the earliest attempts and successfully generate sufficient volatility in residential investment, but not in house prices.

Several recent papers model housing in an incomplete markets setting, such as Iacoviello and Pavan (2013), Kiyotaki, Michaelides and Nikolov (2011), Ríos-Rull and Sánchez-Marcos (2008), Chu (2013), and Favilukis, Ludvigson and Van Nieuwerburgh (2013). The latter two, along with Kahn (2009), make progress in generating volatile house prices and highlight the importance of inelastic construction, time-varying risk premia, and inelastic substitution between housing and consumption, respectively. However, none of the previous papers addresses all of the stylized facts described in the introduction, includ-
ing notably the strong countercyclicality of months supply and foreclosures.

Another strand of the literature deviates from the Walrasian framework by
developing search models of housing, as in early papers by Wheaton (1990)
and Krainer (2001). Most related to my work here are recent contributions
by Novy-Marx (2009), Burnside et al. (2013), Caplin and Leahy (2011), Díaz
and Jerez (2013), and Head et al. (2014). Novy-Marx (2009) and Díaz
and Jerez (2013) both show how search frictions magnify shocks to fundamentals,
with Díaz and Jerez (2013) emphasizing the importance of directed search,
rather than random search, in housing markets. Burnside et al. (2013) in-
troduce learning and social dynamics to generate housing booms which are
only sometimes followed by busts. Head et al. (2014) generates house price
momentum in a city-level model of housing with free entry of buyers. I add to
this literature by integrating a frictional, decentralized housing market into a
fully closed production economy with imperfect credit markets and substantial
household heterogeneity, which allows me to simultaneously address all of the
major stylized facts on housing, debt, and foreclosure dynamics.

My paper also fits into the literature on mortgage default. Mitman (2012),
Hintermaier and Koeniger (2011), and Jeske, Krueger and Mitman (2012)
study foreclosures in an environment with one-period mortgages, which forces
homeowners to refinance each period. Chatterjee and Eyigungor (2011), Cor-
bae and Quintin (2014), and Garriga and Schlagenhauf (2009) analyze fore-
closures in steady state and transition with long-term mortgage contracts. I
extend this work by studying foreclosure dynamics with long-term mortgages
and aggregate uncertainty.

Lastly, my paper complements Menzio and Shi (2010) and Hedlund (2013)
by utilizing block recursivity to develop a directed search model of housing with
two-sided heterogeneity and computationally tractable aggregate dynamics.
2 The Model

2.1 Households

Households inelastically supply one unit of time to the labor market and are paid wage \( w \) per unit of stochastic labor efficiency \( e \cdot s \), where \( s \in S \) follows a finite Markov chain with transitions \( \pi_s(s'|s) \) and \( e \) is drawn from the cumulative distribution function \( F(e) \) with compact support \( E \subset \mathbb{R}_+ \). Households initially draw \( s \) from the invariant distribution \( \Pi_s(s) \).

Households derive utility from composite consumption \( c \) and housing services \( c_h \). Homeowners with house size \( h \in H = \{h_1, h_2, h_3\} \) receive a dividend \( c_h = h \) of housing services each period, while renters purchase housing services \( c_h \in [0, h] \) from a competitive spot market at price \( r_h \) (relative to the numeraire consumption good). All homeowners are owner-occupiers and can only own one house at a time.

Households save by purchasing one-period bonds with price \( q_b \in (0, 1) \) from financial intermediaries. Homeowners also have the option to borrow against their house with mortgage debt. I detail the structure of mortgage contracts in the financial intermediaries section.

2.2 Consumption Good Sector

Consumption good firms operate a constant returns to scale production function using capital \( K_c \) and labor \( N_c \) to produce composite consumption,

\[
Y_c = z_c F_c(K_c, N_c).
\]

Total factor productivity \( z_c \) follows a finite state Markov chain with tran-
position probabilities $\pi_z(z'_c|z_c)$. Firms rent capital from financial intermediaries at rental rate $r$ and pay wage $w$ per unit of labor efficiency. Output can be consumed, added to the capital stock, or used to build new housing.

### 2.2.1 Housing Services for Renters

Landlords convert the consumption good into housing services at the rate $A_h$ using a linear, reversible technology and sell these housing services competitively at price $r_h$.

### 2.3 Construction Sector

Construction firms operate a constant returns to scale production function using land/permits $L$, structures $S_h$, and labor $N_h$ to produce new housing,

$$Y_h = F_h(L, S_h, N_h).$$

Firms purchase new land/permits from the government at price $p_l$, pay wage $w$ per unit of labor efficiency, and purchase structures $S_h$ from the consumption good sector. The government supplies a fixed amount $\bar{L} > 0$ of new land/permits each period and all revenues go to wasteful government spending. Construction firms sell new houses directly to real estate firms at price $p_h$ and do not experience any building delays.

Houses come in discrete sizes $h \in H$ and face a small probability $\delta_h$ of complete depreciation. Therefore, the aggregate housing stock depreciates deterministically at the rate $\delta_h$ and evolves according to

$$H' = (1 - \delta_h)H + Y'_h.$$
2.4 Real Estate Sector

The real estate sector is populated by a continuum of real estate firms that facilitate housing trades between buyers and sellers. In the absence of a centralized market, buyers and sellers match bilaterally with real estate agents in an environment with search frictions. First, sellers attempt to match with real estate agents to sell their house. Next, buyers attempt to match with real estate agents to purchase a house recently sold by a seller. Real estate firms simply act as conduits to transfer houses from sellers to buyers but greatly improve the tractability of the model, as I discuss later.

2.4.1 Decentralized House Selling

Sellers direct their search to real estate agents by choosing a selling price $x_s \geq 0$ for their house $h \in H$. Formally, sellers choose $x_s$ to enter submarket $(x_s, h) \in \mathbb{R}_+ \times H$. Sellers commit to the selling price, conditional on successfully matching with a real estate agent, and pay utility cost $\xi$ if they fail to match. Real estate firms hire a continuum $\Omega_s(x_s, h)$ of real estate agents to enter each submarket at cost $\kappa_s h$. The ratio of real estate agents to sellers in submarket $(x_s, h)$, or market tightness, is $\theta_s(x_s, h) \geq 0$, and is determined in equilibrium. A seller in submarket $(x_s, h)$ successfully matches with a real estate agent with probability $p_s(\theta_s(x_s, h))$, while a real estate agent in submarket $(x_s, h)$ successfully matches with a seller with probability $\alpha_s(\theta_s(x_s, h)) = \frac{p_s(\theta_s(x_s, h))}{\theta_s(x_s, h)}$. The function $p_s: \mathbb{R}_+ \to [0, 1]$ is continuous and strictly increasing with $p_s(0) = 0$, and $\alpha_s$ is strictly decreasing. Real estate

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3The utility cost discourages homeowners nearly indifferent about selling from posting a selling price that causes their house to take extremely long to sell.

4In unvisited submarkets, $\theta_s(x_s, h)$ is an out-of-equilibrium belief that helps determine equilibrium behavior.
agents may match with multiple sellers if $\alpha_s > 1$, but sellers always match with at most one real estate agent. By the law of large numbers, real estate firms know exactly how many matches agents will have with sellers, and to ensure that real estate firms are passive market participants, I do not allow them to hold housing inventories. Agents and sellers take $\theta_s(x_s, h)$ parametrically.

### 2.4.2 Decentralized House Buying

Buyers direct their search to real estate agents by choosing a submarket $(x_b, h)$ with purchase price $x_b \geq 0$ and house size $h \in H$. Buyers match with a real estate agent with probability $p_b(\theta_b(x_b, h))$ and agents match with a buyer with probability $\alpha_b(\theta_b(x_b, h)) = p_b(\theta_b(x_b, h)) / \theta_b(x_b, h)$, where $\theta_b(x_b, h)$ is the market tightness. The functions $p_b$ and $\alpha_b$ have the same properties as $p_s$ and $\alpha_s$, respectively. Successful buyers immediately move into their house, while unsuccessful buyers remain as renters until the next period. Real estate firms hire a continuum $\Omega_b(x_b, h)$ of real estate agents to enter each submarket at cost $\kappa_b h$ per agent. Real estate agents and buyers take $\theta_b(x_b, h)$ parametrically.

### 2.5 Financial Sector

Intermediaries trade bonds $b' \in B > 0$ and mortgages $m' \in M > 0$ with households, accumulate capital to rent to firms, and manage their stock of repossessed foreclosure housing. Capital evolves according to

$$K' = (1 - \delta_c)K + I_c.$$

Intermediaries have access to international bond financing at interest rate $i$, although I focus on equilibria with a zero net supply of such bonds.
2.5.1 Mortgages

Borrowers who take out a mortgage of size $m'$ receive $q_{m}^0 m'$ at origination, where $q_{m}^0 \in (0, 1)$ is the mortgage price. Perfect competition partitions the mortgage market by loan size and borrower characteristics and causes intermediaries to earn zero expected profits loan-by-loan.\(^5\) Therefore, mortgage prices $q_{m}^0$ depend on the initial balance $m'$, the borrower’s house size $h$, the aggregate state of the economy $\Gamma$, and the borrower’s initial savings $b'$ and persistent labor efficiency component $s$.

In subsequent periods, borrowers repay their loan according to a flexible repayment schedule. Specifically, borrowers choose how quickly to pay down principal, while the remaining balance accrues interest at rate $r_m$.\(^6\) Borrowers can only increase their mortgage debt by paying off their existing balance and taking out a new loan.

Intermediaries incur proportional origination costs $\zeta$ and servicing costs $\phi$ over the life of each mortgage. Additionally, intermediaries face two sources of non-repayment risk. First, if a borrower’s house stochastically depreciates, the intermediary absorbs the entire mortgage loss without any penalty for the borrower. Secondly, a borrower may choose to default, which causes the intermediary to initiate foreclosure proceedings.

Intermediaries price the origination cost and default risk into $q_{m}^0$, while the servicing cost and depreciation risk affect the interest rate $r_m$. By front-\(^5\)The government distributes all ex-post profits/losses to households through a proportional wealth tax/subsidy $\tau$. This arrangement bypasses the need to explicitly assign ownership of intermediaries. Instead, intermediaries in the model are risk-neutral entities that discount the future at the international bond rate $i$.
\(^6\)The infinite-duration structure of mortgage contracts proxies for other mortgage instruments that I do not include in the model, such as second mortgages and home equity lines of credit, which operate to provide borrowers flexibility in paying down their total mortgage debt.
loading all default risk into the initial mortgage price, intermediaries exhibit one-sided commitment and do not subject households to mortgage re-pricing risk. This modeling approach for mortgages greatly simplifies computation by reducing the mortgage state to only the loan balance.

### 2.6 Legal Environment

I model a single legal environment for mortgage default, although actual laws vary by state. Mitman (2012) explores this legal variation more in-depth.

In the model, mortgage default causes the following chain of events:

1. The borrower’s mortgage debt is erased, and a foreclosure filing is placed on the borrower’s credit record \( f = 1 \). The borrower’s other financial assets are left intact, i.e. mortgages are non-recourse.

2. The intermediary repossesses the borrower’s house as Real Estate Owned (REO) property and tries to sell it in the decentralized selling market.
   
   (a) The intermediary has reduced search efficiency \( \lambda \in (0, 1) \) and, upon successful sale, loses a fraction \( \chi \) of the sale price.\(^7\)

   (b) The intermediary absorbs all mortgage losses but must pass along any potential profits from the foreclosure sale to the borrower.

3. Households with \( f = 1 \) lose access to the mortgage market\(^8\), and the foreclosure flag stays on their record at the beginning of the next period with probability \( \gamma_f \in (0, 1) \).\(^9\)

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\(^7\)This proportional loss accounts for various foreclosure costs and foreclosure property degradation.

\(^8\)Fannie Mae and Freddie Mac do not purchase mortgages issued to borrowers with recent foreclosure filings, making it much less appealing to lend to these borrowers.

\(^9\)Foreclosure filings stay on a borrower’s credit record for a finite number of years.
2.7 Decision Problems

2.7.1 Households

Each period consists of three subperiods. Homeowners have cash at hand \( y \in \mathbb{R}_+ \), mortgage balance \( m \in M \), house size \( h \in H \), labor efficiency shock \( s \in S \), and foreclosure flag \( f \in \{0, 1\} \). Renters have individual state \((y, s, f)\).

The aggregate state \( \Gamma = (z_c, \Phi, K, \{H_{REO}(h)\}_h) \) consists of the productivity shock \( z_c \), the distribution \( \Phi \) of homeowners and renters over individual states, the capital stock \( K \), and the REO housing stock \( \{H_{REO}(h)\}_h \). The aggregate law of motion is \( \Gamma' = G(\Gamma, z'_c) \). I describe household decisions below.\(^{10}\)

Selling and Default Decisions:

At the beginning of the period, households learn their shocks, receive cash at hand \( y = (1-\tau)(wes + b) \), and sellers choose a price \( x_s \) sufficiently high to ensure mortgage repayment upon sale, \( y + x_s \geq m \). Afterward, non-selling homeowners make default decisions. The option value of selling is \( R_{sell}(y, m, h, s, f, \Gamma) \).

\[
R_{sell}(y, m, h, s, f, \Gamma) = \max \left\{ 0, \max_{y + x_s \geq m} \left[ \max \left( V_{rent} + R_{buy}(y + x_s - m, s, 0, \Gamma) - W_{own}(y, m, h, s, 0, \Gamma), 1 - p_s(\theta_s(x_s, h; \Gamma)) \right) \right] \right\}
\]

\[
W_{own}(y, m, h, s, 0, \Gamma) = \max \left\{ (V_{rent} + R_{buy})(y + \max\{0, J_{REO}(h, \Gamma) - m\}, s, 1, \Gamma), V_{own}(y, m, h, s, 0, \Gamma) \right\}
\]

\(^{10}\)The supplementary appendix gives the complete list of household value functions.
**Buying Decisions:**

In subperiod 2, buyers choose submarket \((x_b, h)\). Buyers with access to credit in subperiod 3 can afford \(x_b \leq y - y\), where \(y < 0\), while buyers with bad credit can only afford \(x_b \leq y\). The option value of buying is \(R_{buy}(y, s, f, \Gamma)\).

\[
R_{buy}(y, s, 0, \Gamma) = \max \{0, \max_{h \in H, x_b \leq y - y} p_b(\theta_b(x_b, h; \Gamma))[V_{own}(y-x_b, 0, h, s, 0, \Gamma) - V_{rent}(y, s, 0, \Gamma)]\}
\]

(3)

**Consumption and Portfolio Decisions:**

In subperiod 3, renters choose consumption \(c\), housing services \(c_h \in [0, h]\), and bond holdings \(b'\). Homeowners choose consumption, bond holdings, and for those with access to mortgage credit, a new mortgage balance \(m'\).

\[
V_{own}(y, m, h, s, 0, \Gamma) = \max_{m' \in M, b' \in B, c \geq 0} u(c, h) + \beta \mathbb{E} \left[ (1 - \delta_h)(W_{own} + R_{sell})(y', m', h, s', 0, \Gamma') + \delta_h(V_{rent} + R_{buy})(y', s', 0, \Gamma') \right]
\]

subject to

\[
c + q_b(\Gamma)b' + [m - q_m(m', b', h, s, \Gamma)m'] \leq y
\]

\[
q_m(m', b', h, s, \Gamma) = \begin{cases} 
q_m^0(m', b', h, s, \Gamma) & \text{if } m' > m \\
\frac{1}{1 + r_m(\Gamma)} & \text{if } m' \leq m
\end{cases}
\]

\[
y' = (1 - \tau(\Gamma'))(w(\Gamma')e's' + b')
\]

\[
\Gamma' = G(\Gamma, z'_c)
\]

(4)

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11 Choosing \(m' \leq m\) entails a payment of \(m - \frac{1}{1 + r_m(\Gamma)}m'\) while choosing \(m' > m\) means paying off the existing mortgage and taking out a new loan of size \(m'\).
\[ V_{\text{rent}}(y, s, 0, \Gamma) = \max_{\forall \theta' \in B, c \geq 0, \ c_h \in [0, h]} \ u(c, c_h) + \beta \mathbb{E} \left[ (V_{\text{rent}} + R_{\text{buy}})(y', s', 0, \Gamma') \right] \]

subject to
\[ c + r_h c_h + q_b(\Gamma)b' \leq y \]
\[ y' = (1 - \tau(\Gamma'))(w(\Gamma')e's' + b') \]
\[ \Gamma' = G(\Gamma, z'_c) \]

2.7.2 Production Firms

Consumption good firms, construction firms, and landlords solve standard static profit maximization problems, resulting in the following conditions:

**Consumption Good Sector**:

\[ r(\Gamma) = z_c \frac{\partial F_c(K_c(\Gamma), N_c(\Gamma))}{\partial K_c} \] (6)
\[ w(\Gamma) = z_c \frac{\partial F_c(K_c(\Gamma), N_c(\Gamma))}{\partial N_c} . \] (7)

**Housing Services for Renters**:

\[ r_h = \frac{1}{A_h} \] (8)

**Construction Sector**:

\[ p_l(\Gamma) = p_h(\Gamma) \frac{\partial F_h(L(\Gamma), S_h(\Gamma), N_h(\Gamma))}{\partial L} \] (9)
\[ 1 = p_h(\Gamma) \frac{\partial F_h(L(\Gamma), S_h(\Gamma), N_h(\Gamma))}{\partial S_h} \] (10)
\[ w(\Gamma) = p_h(\Gamma) \frac{\partial F_h(L(\Gamma), S_h(\Gamma), N_h(\Gamma))}{\partial N_h} . \] (11)
2.7.3 Real Estate Firms

Real estate firms purchase new housing \( Y_h \) and hire agents \( \Omega_s \) and \( \Omega_b \) to intermediate trades between buyers and sellers, solving

\[
\max_{Y_h \geq 0, \Omega_s(x_s,h) \geq 0, \Omega_b(x_b,h) \geq 0} \left[ \int \left[ \kappa_s h + \alpha_s(\theta_s(x_s,h;\Gamma))x_s \right] \Omega_s(dx_s, dh) - p_h(\Gamma)Y_h \right. \\
\left. + \int \left[ -\kappa_b h + \alpha_b(\theta_b(x_b,h;\Gamma))x_b \right] \Omega_b(dx_b, dh) \right]
\]

subject to

\[
Y_h + \int h\alpha_s(\theta_s(x_s,h;\Gamma))\Omega_s(dx_s, dh) \geq \int h\alpha_b(\theta_b(x_b,h;\Gamma))\Omega_b(dx_b, dh)
\]

(consistency constraint: multiplier \( \mu(\Gamma) \))

Profit maximization implies that \( \mu(\Gamma) = p_h(\Gamma) \) and gives rise to the following submarket entry conditions:

\[
\kappa_b h \geq \alpha_b(\theta_b(x_b,h;\Gamma))(x_b - p_h(\Gamma)h) \quad \text{and} \quad \theta_b(x_b,h;\Gamma) \geq 0 \quad \text{with comp. slackness},
\]

(13)

\[
\kappa_s h \geq \alpha_s(\theta_s(x_s,h;\Gamma))(p_h(\Gamma)h - x_s) \quad \text{and} \quad \theta_s(x_s,h;\Gamma) \geq 0 \quad \text{with comp. slackness}.
\]

(14)

2.7.4 Financial Intermediaries

Each period, intermediaries choose capital \( K' \), issue bonds \( B' \) to households, and originate \( n_m(m',b',h,s) \) mortgages of type \( (m',b',h,s) \). The intermediary discounts period-\( t \) cash flows at the international bond interest rate \( i_t \). Long-duration assets on the intermediary’s balance sheet—namely, vintage mortgages and REO inventories—are priced-to-market. Therefore, intermediaries effectively sell their vintage mortgages and REO inventories at the beginning
of the period, distribute ex-post losses or gains, and then re-purchase their REO inventories and vintage mortgages at the end of the period.

Intermediary profit maximization implies the following:

\[ q_b(\Gamma) = \frac{1}{1 + i(\Gamma)} = \frac{1}{1 - \delta_c + \mathbb{E}_T(\Gamma')} \]  

(15)

\[ 1 + r_m(\Gamma) = (1 + i(\Gamma)) \frac{1 + \phi}{1 - \delta_h} \]  

(16)

with next period’s capital equaling the end-of-period sum of bond issuances minus the value of unsold REO inventories and new and vintage mortgages.

Mortgage prices satisfy the following recursive relationship:

\[ q_m^0(m', b', h, s, \Gamma) = \frac{1}{(1 + \zeta)(1 + r_m(\Gamma))} \mathbb{E} \{ p_s(\theta_s(x_s', h; \Gamma')) + [1 - p_s(\theta_s(x_s', h; \Gamma'))] \times \left\{ d\min \left\{ 1, \frac{J_{REO}(h, \Gamma')}{m'} \right\} + (1 - d') \right\} \left( \frac{m' - (1 + \phi)m''/(1 + r_m(\Gamma')) + \Pi(m'', b'', h, s', \Gamma')} {m'} \right) \} \]

(17)

\[ \Gamma' = G(\Gamma, z'_c) \]

where \( x_s', m'', b'', \) and \( d' \), stand in for the homeowner’s respective choices next period of selling price, new mortgage balance, bonds, and whether to default. Also, \( J_{REO} \) is the intermediary’s value function for repossessing the borrower’s house, and \( \Pi \) is the continuation value of the mortgage,

\[ \Pi(m'', b'', h, s', \Gamma') = q_m^0(m'', b'', h, s', \Gamma')(1 + \zeta)(1 + \phi)m''. \]
**Foreclosure Sales** Intermediaries sell their REO inventories by choosing submarket \((x_s, h)\). Intermediaries value a repossessed house of size \(h\) as \(J_{\text{REO}}(h, \Gamma)\),

\[
J_{\text{REO}}(h, \Gamma) = R_{\text{REO}}(h, \Gamma) + \frac{1 - \delta_h}{1 + i(\Gamma)} E J_{\text{REO}}(h, \Gamma')
\]

\[
R_{\text{REO}}(h, \Gamma) = \max \left\{ 0, \max_{x_s \geq 0} \lambda p_s(\theta_s(x_s, h; \Gamma)) \left[ (1 - \chi)x_s - \frac{1 - \delta_h}{1 + i(\Gamma)} E J_{\text{REO}}(h, \Gamma') \right] \right\}
\]

\[
\Gamma' = G(\Gamma, z_c').
\]

(18)

### 2.8 Block Recursivity in the Housing Market

This paper introduces the notion of **block recursivity** in the housing market with aggregate uncertainty, which greatly simplifies the characterization of equilibrium market tightnesses. As in [Hedlund (2013)](#), the shadow housing price \(p_h(\Gamma)\) is a sufficient statistic for the distribution of households over assets, debt, and labor efficiency states when calculating submarket tightnesses.

From equations (13) – (14), the dependence of \(\theta_s\) and \(\theta_b\) on \(\Gamma\) only enters through \(p_h(\Gamma)\). Therefore, calculating the entire distribution of submarket tightnesses simplifies to finding and substituting \(p_h\) into the submarket entry conditions. In other words, household heterogeneity still **matters**, but its interaction with search frictions is highly **tractable**.

#### 2.8.1 Determining the Shadow Housing Price

The shadow housing price \(p_h\) equates two Walrasian-like demand and supply equations. Housing supply \(S_h(p_h; \Gamma)\) equals the sum of new housing and existing houses sold by homeowners and intermediaries,

\[
S_h(p_h; \Gamma) = Y_h(p_h; \Gamma) + S_{\text{REO}}(p_h; \Gamma) + \int h p_s(\theta_s(x_s^*, h; p_h)) \Phi_{\text{own}}(dy, dm, dh, ds, df),
\]
where the first term is new housing, the second term is REO housing, and the third term is homeowner housing.

Housing demand \( D_h(p_h; \Gamma) \) equals housing purchased by matched buyers,\(^{12}\)

\[
D_h(p_h; \Gamma) = \int h^* p_h(\theta_h(x_b^*, h^*; p_h)) \Phi_{\text{rent}}(dy, ds, df)
\]

The equilibrium shadow housing price \( p_h(\Gamma) \) solves

\[
D_h(p_h(\Gamma); \Gamma) = S_h(p_h(\Gamma); \Gamma).
\]

\(19\)

### 2.9 Equilibrium

A recursive equilibrium consists of household value and policy functions, production firm policies, intermediary value and policy functions, market tightnesses, a shadow housing price, and prices for production factors, housing services, bonds, and mortgages, in addition to an aggregate law of motion. These elements must solve the household, firm, and intermediary optimization problems and must equilibrate the markets for housing, land, labor, capital, international bonds, and the consumption good. I solve the equilibrium using a hybrid of methods based off of Krusell and Smith (1998) and those used in the literature on equilibrium default. The detailed equilibrium definition and computational algorithm are in the appendix.

\(^{12}\)Equivalently, housing supply equals the left side of the real estate firm’s constraint while housing demand equals the right side.
3 Model Calibration

I calibrate the steady state of the model to match selected macroeconomic data from the 1990s, thus avoiding skewing the calibration with the recent extraordinary housing boom-bust and the Great Recession. First, I choose some parameters from the literature or from a priori information. I jointly calibrate the remaining parameters.

3.1 Model Specification

3.1.1 Households

Preferences Households have constant elasticity of substitution utility with constant relative risk aversion,

\[ u(c, c_h) = \left( \frac{\omega c^{\frac{\nu-1}{\nu}} + (1 - \omega)c_h^{\frac{\nu-1}{\nu}}}{1 - \sigma} \right)^{1-\sigma}. \]

I follow Kahn (2009) and Flavin and Nakagawa (2008) and set the intratemporal elasticity of substitution to \( \nu = 0.13 \). I determine the discount factor \( \beta \) and risk aversion \( \sigma \) jointly.

Labor Efficiency Log labor efficiency, \( \ln(e \cdot s) = \ln(s) + \ln(e) \), follows

\[ \ln(s') = \rho_s \ln(s) + \varepsilon' \]
\[ \varepsilon' \sim \mathcal{N}(0, \sigma_{\varepsilon}^2) \]
\[ \ln(e) \sim \mathcal{N}(0, \sigma_{\varepsilon}^2). \]

\[ \text{See also Li, Liu, Yang and Yao (2012). These papers find empirical evidence of a unit income elasticity for housing expenditures but a price elasticity substantially below one.} \]
I calibrate $\rho$, $\sigma_\varepsilon$, and $\sigma_\varepsilon$ following Storesletten, Telmer and Yaron (2004), with some modifications that I explain in the appendix. Computationally, I truncate $\ln(e)$ and discretize $\ln(s)$ with a three-state Markov chain using the Rouwenhorst method.

### 3.1.2 Production Sectors

I specify Cobb Douglas production functions in both sectors,

\[
Y_c = z_c A_c K^{\alpha_K} N_c^{1-\alpha_K} \quad Y_h = L^{\alpha_L} (S_h^{\alpha_S} N_h^{1-\alpha_S})^{1-\alpha_L}.
\]

I normalize mean quarterly earnings to 0.25 using $A_c$, and I set $\alpha_K = 0.26$, following Díaz and Luengo-Prado (2010). The shock $z_c$ follows

\[
\ln(z_{c_t}') = \rho_z \ln(z_c) + \varepsilon_{z_t}' \quad \varepsilon_{z_t} \sim \mathcal{N}(0, \sigma_{\varepsilon_z}^2)
\]

with standard values $\rho_z = 0.95$ and $\sigma_{\varepsilon_z}^2 = 0.007$. I discretize $z_c$ with a three state Markov chain using the Rouwenhorst method.

In the construction sector, I follow Favilukis et al. (2013) and set the structures share to $\alpha_S = 0.3$. I set the land share to $\alpha_L = 0.33$ based on data from the Lincoln Institute of Land Policy.\textsuperscript{14} Following Harding, Rosenthal and Sirmans (2007), I set $\delta_h = 0.00625$, which corresponds to a 2.5% annual housing depreciation rate. I determine the housing services technology $A_h$ jointly.

\textsuperscript{14}http://www.lincolnist.edu/subcenters/land-values/price-and-quantity.asp
3.1.3 Real Estate Sector

I specify constant elasticity of substitution matching functions. Therefore, buying \((j = b)\) and selling \((j = s)\) trading probabilities are

\[
p_j(\theta_j) = \min \left\{ \frac{A_j \theta_j}{(1 + \theta_j^{\gamma_j})^{1/\gamma_j}}, 1 \right\} \quad \text{and} \quad \alpha_j(\theta_j) = \frac{p_j(\theta_j)}{\theta_j}.
\]

The appendix gives the analytical characterization of trading probabilities for given \(p_h\). I jointly calibrate \(A_j\), \(\kappa_j\), \(\gamma_j\), and the utility cost \(\xi\).

3.1.4 Financial Sector

I set the mortgage origination cost to 2\% \((\zeta = 0.02)\), consistent with reports from the Federal Housing Finance Board of typical closing costs of 1\% – 3\%.\(^{15}\)

I set the servicing cost \(\phi = 4.15 \times 10^{-5}\) to achieve a 2.65\% spread between steady state mortgage interest rates \(1 + r_m = \frac{(1 + \phi)(1 + i)}{1 - \delta_h}\) and bond yields \(1 + i\). The annual capital depreciation rate is 10\%, implying quarterly \(\delta_c = 0.025\).

3.1.5 Foreclosures and Legal Environment

I set \(\gamma_f = 0.95\) to give an expected credit flag duration of 5 years.\(^{16}\) I jointly calibrate the REO sale loss \(\chi\) and search efficiency \(\lambda\).

3.2 Joint Calibration

Following Hedlund (2013), I divide the targets of the joint calibration into three categories: macroeconomic aggregates, household financial data, and housing market data. The calibration is summarized in table 1.

\(^{16}\)Fannie Mae and Freddie Mac do not generally underwrite mortgages to borrowers with foreclosure records until after 5 years.
Macroeconomic Aggregates  I target a 15% housing services-to-GDP ratio and, following Díaz and Luengo-Prado (2010), a non-residential capital-to-GDP ratio of 1.64.  

Household Financial Data  I use the 1998 Survey of Consumer Finances to target selected asset and debt statistics. I target mean homeowner housing wealth relative to normalized earnings of 3.62 and mean mortgage debt, conditional on having a mortgage, of 2.03, using $p_h$ to valuate housing wealth.

Housing Market Data  I target a 64% homeownership rate, an annual foreclosure rate of 1.4%, an average foreclosure price discount of 22% as reported by Pennington-Cross (2006), an average foreclosure house selling time of 52 weeks, and mean buyer and seller search durations of 10 weeks and 17 weeks, respectively. To calculate search durations, I assume that housing trades in period $t$ occur uniformly between $t$ and $t+1$, as in Caplin and Leahy (2011). Trading after $n$ periods corresponds to a search time of $(n + 0.5) \times 12$ weeks.

For buyers, I target a minimum buying premium $x_b(p_h)/p_h$ of 0.5% and a maximum buying premium $\bar{x}_b(p_h)/p_h$ of 2.5%, consistent with Gruber and Martin (2003). For sellers, I target a minimum selling discount where sellers are guaranteed to immediately sell, $(p_h - \bar{x}_s(p_h))/p_h$, of 6% to match direct realtor expenses in the data. I target a maximum selling discount $(p_h - x_s(p_h))/p_h$ of 20%, consistent with findings in Garriga and Schlaggenhauf (2009) and evidence from pre-foreclosure sales price discounts. 

---

17Housing services in the model equal $r_h c_h$ for renters and $r_h h$ for homeowners.
18Sources: The Census Bureau, the National Delinquency Survey, and the National Association of Realtors. The foreclosure selling duration implicitly includes any legal delays.
19RealtyTrac reports pre-foreclosure discounts ranging from 1.28% to 34.94%. Unlike REOs, which sell at a discount largely because of degradation caused by extended vacancy, pre-foreclosure houses are likely to sell at a discount because of financial urgency to sell.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Description</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.13</td>
<td></td>
<td>Intratemporal elasticity of substitution</td>
<td></td>
</tr>
<tr>
<td><strong>Stochastic Labor Endowment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.952</td>
<td></td>
<td>Autocorrelation of persistent shock</td>
<td></td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.49</td>
<td></td>
<td>Standard deviation of transitory shock</td>
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</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.17</td>
<td></td>
<td>Standard deviation of persistent shock</td>
<td></td>
</tr>
<tr>
<td><strong>Production Technologies</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
<td></td>
<td>Autocorrelation of technology shock</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_z$</td>
<td>0.007</td>
<td></td>
<td>Variance of technology shock</td>
<td></td>
</tr>
<tr>
<td>$\alpha_K$</td>
<td>0.26</td>
<td>26%</td>
<td>Non-residential capital share</td>
<td>26%</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>0.33</td>
<td>33%</td>
<td>Land share in construction</td>
<td>33%</td>
</tr>
<tr>
<td>$\alpha_S$</td>
<td>0.30</td>
<td>30%</td>
<td>Residential structures share</td>
<td>30%</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>0.025</td>
<td>10%</td>
<td>Annual non-residential capital depreciation</td>
<td>10%</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.00625</td>
<td>2.5%</td>
<td>Annual housing depreciation</td>
<td>2.5%</td>
</tr>
<tr>
<td><strong>Financial Sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>4.15e-5</td>
<td>2.65%</td>
<td>Mortgage interest rate spread</td>
<td>2.65%</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.02</td>
<td>2%</td>
<td>Mortgage origination fee</td>
<td>2%</td>
</tr>
<tr>
<td><strong>Legal Environment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>0.95</td>
<td>5</td>
<td>Average years duration of foreclosure flag</td>
<td>5</td>
</tr>
<tr>
<td><strong>Parameters determined jointly</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96397</td>
<td>1.64</td>
<td>Non-residential capital to GDP</td>
<td>1.62</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1.48e-7</td>
<td>3.62</td>
<td>Homeowner housing wealth to earnings</td>
<td>3.62</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4.70</td>
<td>2.03</td>
<td>Borrower mortgage debt to earnings</td>
<td>2.05</td>
</tr>
<tr>
<td><strong>Production Technologies</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.21609</td>
<td>0.25</td>
<td>Mean quarterly labor earnings</td>
<td>0.25</td>
</tr>
<tr>
<td>$A_h$</td>
<td>54.757</td>
<td>0.15</td>
<td>Housing services to GDP</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Housing Markets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>2.92</td>
<td>64%</td>
<td>Homeownership rate</td>
<td>64.7%</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>2.55</td>
<td>10</td>
<td>Buyer search duration in weeks</td>
<td>9.96</td>
</tr>
<tr>
<td>$A_b$</td>
<td>1.0065</td>
<td>0.5%</td>
<td>Minimum buying premium</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\kappa_b$</td>
<td>0.005</td>
<td>2.5%</td>
<td>Maximum buying premium</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.013</td>
<td>17</td>
<td>Seller search duration in weeks</td>
<td>16.96</td>
</tr>
<tr>
<td>$A_s$</td>
<td>2.2917</td>
<td>6%</td>
<td>Average realtor fees</td>
<td>6%</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>0.1375</td>
<td>20%</td>
<td>Maximum selling discount (incl. realtor fees)</td>
<td>20%</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>0.69</td>
<td>1.4%</td>
<td>Annual foreclosure rate</td>
<td>1.37%</td>
</tr>
<tr>
<td><strong>Foreclosure Sales</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.1199</td>
<td>22%</td>
<td>Foreclosure selling price discount</td>
<td>21.95%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.4051</td>
<td>52</td>
<td>REO time on the market in weeks</td>
<td>52.05</td>
</tr>
</tbody>
</table>
4 Results

I begin this section by describing the baseline model results. Next, I assess the dynamic effects of search frictions. Lastly, I analyze the impact on housing dynamics of a foreclosure law reform that makes all mortgages full recourse.

4.1 Baseline Results

To evaluate the performance of the baseline economy, I compare the dynamics of HP-filtered time series generated by the model to the equivalent HP-filtered series in the U.S. data from 1975 – 2010.\textsuperscript{20}

4.1.1 Housing and Foreclosure Dynamics

Table 2 reports the co-movement of existing homeowner sales with house prices, the foreclosure rate, and months supply, which proxies for average selling time on the market.\textsuperscript{21} In the data, sales exhibit significant positive co-movement with house prices and negative co-movement with months supply and the foreclosure rate. The baseline model successfully matches these co-movements, both qualitatively and quantitatively. Furthermore, both the model and the data feature procyclical prices and existing sales alongside countercyclical months supply and foreclosures, as shown in table 3.

To understand these dynamics, recall that productivity shocks are the source of fluctuations in the model. A positive shock to aggregate productivity increases incomes, which leads to higher demand for housing and a textbook

\textsuperscript{20}I omit 2011 – 2013 because of the recent spate of legal and industry practice changes in the housing and mortgage markets. Due to the protracted nature of housing booms and busts, I use a smoothing parameter of $10^8$ to avoid excessively removing variation.

\textsuperscript{21}I do not analyze new sales because the model does not include discrete sales or inventories of new houses. However, the value of new housing shows up as residential investment.
Table 2: Housing Co-Movements

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr(Sales, Prices)</td>
<td>0.50</td>
<td>0.59</td>
</tr>
<tr>
<td>Corr(Sales, Months supply)</td>
<td>−0.68</td>
<td>−0.74</td>
</tr>
<tr>
<td>Corr(Sales, Foreclosure rate)</td>
<td>−0.65</td>
<td>−0.48</td>
</tr>
</tbody>
</table>

Model sales consists of all sales by existing owners. Sales data is the existing sales series reported by the National Association of Realtors.

response of higher prices and sales. However, standard competitive models of housing cannot account for the decrease in months supply. Under perfect competition, homeowners can only respond to changing market conditions by deciding whether or not to sell their house at the prevailing market price. However, in a decentralized housing market with search frictions, homeowners have some price-setting power.

Figure 1 plots homeowners’ choice of selling price as a function of cash at hand. As the first panel of the figure shows, homeowners with low cash at hand do not wish to move and therefore do not put their house on the market. However, as cash at hand increases, homeowners gradually lower their selling price to sell more quickly.

When the shadow housing price $p_h$ increases, more real estate agents enter the market to match with sellers, driving up market tightnesses $\theta_s(x_s, h)$ and seller trading probabilities $p_h(\theta_s(x_s, h))$ at every listing price $x_s$. In response to the improvement in trade probabilities, homeowners sell more quickly and at a higher price because they increase $x_s$ by less than the change in $p_h$. Therefore, unlike in competitive models of housing, selling behavior with search frictions adjusts along both the price and selling time margins.

The countercyclicality of foreclosures can be attributed to three effects.
Table 3: Housing Dynamics

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sigma_x/\sigma_{output}$</th>
<th>$\rho_{x, output}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data Model</td>
<td>Data Model</td>
</tr>
<tr>
<td>House prices</td>
<td>2.07 1.90</td>
<td>0.50 0.95</td>
</tr>
<tr>
<td>Existing sales</td>
<td>3.93 4.35</td>
<td>0.73 0.13</td>
</tr>
<tr>
<td>Months supply</td>
<td>6.11 6.46</td>
<td>−0.44 −0.88</td>
</tr>
<tr>
<td>Foreclosure rate</td>
<td>4.96 16.32</td>
<td>−0.64 −0.88</td>
</tr>
</tbody>
</table>

Relative standard deviations and correlations with GDP of HP-filtered time series. See table 9 in the appendix for definitions and sources.

First, homeowners can better afford mortgage payments when they have higher incomes during economic expansions. Secondly, the increase in selling probabilities from higher $p_h$ makes it easier for distressed homeowners to sell their houses. Lastly, both higher incomes and higher trading probabilities loosen credit constraints, which makes refinancing easier. I delve into these effects in my discussion of the effects of search frictions.

Also consistent with the data, the model generates significant volatility in prices, sales, months supply, and foreclosures. The model’s almost exact matching of price, sales, and months supply volatilities represents a particular success, given the difficulty the literature has had in generating sufficient volatility for even a subset of these variables. As discussed recently by Chu (2013) and Kahn (2009), inelasticity of construction and a low elasticity of substitution between housing and consumption both contribute significantly to housing volatility. However, search frictions and fluctuations in credit constraints also play an important role in amplifying housing dynamics—channels which I explore momentarily.

Though the model generates significantly higher foreclosure volatility than
Figure 1: Selling price, selling probability, and time on the market (TOM) as a function of cash at hand for homeowners wishing to upgrade.

in the data, much of the apparent difference arises because of higher frequency fluctuations in the model, rather than larger absolute swings. During model simulations, the foreclosure rate fluctuates between 0.2% and 1.75%, which is in line with empirical foreclosure rate fluctuations before the Great Recession. Furthermore, if I expand the foreclosure rate to include all mortgages 90+ days late, the empirical relative volatility nearly doubles to 8.92.23

4.1.2 Consumption, Investment, and Portfolio Dynamics

Turning to standard business cycle variables, table 4 shows that the model generates consumption and investment dynamics that mimic those in the data. The model and empirical volatilities of aggregate consumption and investment correspond almost exactly, and the model matches the relative volatilities of each component of investment reasonably well. Of particular interest, the

22Many of these mortgages end in a short sale rather than an actual foreclosure sale—a distinction the model does not make.

23The foreclosure rate is also likely to be less volatile in the data because banks may attempt to work with borrowers rather than foreclose in the earlier stages of a housing bust. In the model, default always leads to immediate foreclosure.
Table 4: Consumption, Investment, and Portfolio Dynamics

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_x/\sigma_{\text{output}}$</th>
<th>$\rho_{x,\text{output}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.65</td>
<td>0.55</td>
</tr>
<tr>
<td>Composite</td>
<td>0.72</td>
<td>0.62</td>
</tr>
<tr>
<td>Housing</td>
<td>0.60</td>
<td>0.33</td>
</tr>
<tr>
<td>Investment</td>
<td>2.97</td>
<td>2.99</td>
</tr>
<tr>
<td>Non-residential</td>
<td>2.69</td>
<td>2.93</td>
</tr>
<tr>
<td>Residential</td>
<td>5.15</td>
<td>4.29</td>
</tr>
<tr>
<td>Financial assets</td>
<td>1.76</td>
<td>1.61</td>
</tr>
<tr>
<td>Mortgage debt</td>
<td>1.64</td>
<td>2.67</td>
</tr>
</tbody>
</table>

Relative standard deviations and correlations with GDP of HP-filtered time series. See table 8 in the appendix for definitions and sources.

model generates 83% of the empirical volatility in residential investment.

Volatile house prices largely drive these swings in residential investment—first, by causing fluctuations in the value of new housing, and second, by generating strong responses in construction, even with the constraining impact of fixed new land/permits. In fact, a moderate amount of inelasticity in construction actually contributes to higher residential investment volatility by magnifying house price movements.

Reflecting the importance of wealth and debt heterogeneity, both the model and the data demonstrate interesting cyclical behavior of household portfolios. Household net worth consists of financial assets, housing wealth, and mortgage debt, all of which are procyclical and more volatile than GDP. As table 4 demonstrates, the model almost exactly matches the relative volatility of assets. The model also generates procyclical, volatile mortgage debt—to excess, in fact—though the model performs well compared to the recent lit-
erature. For example, Iacoviello and Pavan (2013) generate almost four times the mortgage volatility as in the data.

The fact that households increase assets and debt during economic upturns implies that households do not single-mindedly use their improved financial position to deleverage. Instead, households take on increased mortgage debt to purchase more housing and simultaneously insure themselves against the future risk of an economic downturn. By borrowing more during periods with loose credit constraints, households use the funds to purchase assets for precautionary saving, rather than being forced to borrow to smooth consumption during downturns when credit constraints are tight.

4.2 The Dynamic Effects of Search

Search frictions greatly influence housing and foreclosure dynamics. To determine the effects of search, I compare the baseline economy to the limit economy with frictionless, competitive housing. Contrasting the dynamics of these two economies, three differences stand out. First, months supply does not fluctuate in the no-search economy because houses always sell instantly, and foreclosures almost disappear. Second, the co-movement between sales and prices decreases from 0.59 to 0.27. Third, the volatilities of residential investment and house prices decline substantially while existing sales volatility nearly doubles, as shown in table 5. Below, I explain the mechanisms behind these results as well as how search frictions help resolve other housing puzzles.

24The foreclosure rate fluctuates between 0% and 0.14% in the no-search economy.
4.2.1 Liquidity Spirals and Amplification

One of the major successes of the model is its ability to generate sufficient volatility in house prices and residential investment. Though other factors also contribute to these large swings, search frictions generate an additional 20% volatility in prices and 27% volatility in residential investment. This amplification primarily occurs because of liquidity spirals akin to those in Brunnermeier and Pedersen (2009) that arise from the interaction of search-based housing illiquidity with endogenous mortgage credit constraints.

To explain the nature of liquidity spirals, I appeal to the discussion in Hedlund (2013) that establishes a link between search risk and foreclosure risk. Figure 2 shows the optimal relative selling price $x_s/p_hh$, selling probability, and expected time on the market as a function of mortgage debt for sellers wishing to downsize or rent. Selling price is almost invariant to mortgage debt for low values of leverage but exhibits strong non-monotonicity as leverage approaches and exceeds an 80% loan-to-value ratio. When leverage hits moderately high levels, low asset homeowners trying to avoid financial insolvency become “distressed sellers” who sharply reduce their selling price to quickly
unload their house. These sellers have sufficient home equity to absorb large losses but are unable to extract equity through refinancing because intermediaries view them as risky borrowers. With even higher leverage, homeowners have insufficient equity to sharply lower their selling price. Instead, debt overhang forces these sellers to set high prices, which causes their houses to sit longer on the market.\footnote{Genesove and Mayer (2001) confirm this selling behavior empirically.} Eventually, some sellers default and enter foreclosure, and an influx of REO houses for sale occurs that depresses the housing market.

Banks anticipate this behavior and price higher default premia into new mortgages during times of lower prices and worse housing liquidity, thus exacerbating the debt overhang problem. These higher default premia tighten access to credit, which simultaneously makes refinancing more difficult and prevents new buyers from entering the housing market to prop up prices and liquidity. In short, search magnifies booms because higher prices increase selling probabilities, which reduces foreclosures, lowers default premia, and loosens credit constraints, resulting in even higher prices.
During simulations of the baseline economy, average default premia for newly originated mortgages with 80%+ leverage fluctuate between 0.3% and 2%, while average default premia fluctuate between 0.5% and 3.5% for 90%+ leverage mortgages and between 0% and 6% for 95%+ leverage mortgages.\textsuperscript{26} By contrast, less default risk and fewer high leverage borrowers in the no-search economy generate only trivial default premia, as in figure 3.

4.2.2 Momentum and Asymmetry in Housing Dynamics

Besides amplifying movements in house prices and residential investment, search frictions help resolve two important house price puzzles. First, house prices exhibit short-run momentum, as documented in Case and Shiller (1989), Capozza, Hendershott and Mack (2004), and several other papers. Second, house price busts tend to be slower and shallower than booms— with some notable exceptions— which suggests a degree of downward price stickiness.\textsuperscript{27}

\textsuperscript{26}In fact, these fluctuations actually understate the cyclicality of credit constraints because homeowners are unlikely to take out mortgages with exceptionally high default premia.

\textsuperscript{27}See, for example, Case and Quigley (2008) and Genesove and Mayer (2001).
The baseline model generates dynamics consistent with both of these phenomena, as shown in figure 4. Specifically, the model generates prolonged booms followed at times by mild slumps, as in panel 2, or else by sharp crashes and prolonged slumps, as in panel 4. These dynamics mimic the shallow U.S. housing bust in the early 1990s and the recent sharp, prolonged housing bust.

Search frictions help generate house price momentum in two ways. First, search frictions spread out the impact of economic shocks. Trading delays simultaneously reduce existing sales volatility from 8.15 to 4.35 and increase the co-movement of sales and prices from 0.27 to 0.59. Furthermore, these trading delays cause current house prices to improve current and future liquidity, which raises the re-sale value of housing and pushes up future prices. Second, liquidity spirals generate persistent price changes from the positive feedback loop of higher prices, less debt overhang, and looser credit.

Search frictions also help explain the asymmetry of booms and busts. During a boom, the virtuous cycle of higher prices, higher liquidity, and expanding credit combines with partially inelastic construction to generate large, persistent price increases. However, depending on the severity of the bust, most homeowners lack a strong incentive to sell their houses during times of de-
Table 6: Dynamics with Recourse Mortgages

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<th>Recourse</th>
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<td>1.68</td>
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<tr>
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<td>4.35</td>
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</tr>
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</tr>
<tr>
<td>Foreclosure rate</td>
<td>4.96</td>
<td>16.32</td>
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</table>

creasing prices. The combination of expected mean reversion, debt overhang, and long-term mortgages taken out during more favorable conditions causes most homeowners to resist substantially lowering their selling price. Sharp price declines— spurred partly by a spike in distressed and foreclosure sales— usually only occur after protracted booms followed by large productivity drops.

### 4.3 Reforming Foreclosure Laws

Currently, only twelve non-recourse states forbid financial institutions from suing borrowers when a foreclosure sale does not recover the remaining mortgage balance. The other thirty-eight recourse states permit such deficiency judgments, thus subjecting foreclosed borrowers to the additional penalty of having their other assets seized. However, conventional wisdom suggests that such deficiency judgments rarely occur due to high legal costs and low returns to pursuing borrowers after foreclosure.\(^{28}\) In this section, I analyze the impact

\(^{28}\)See Corbae and Quintin (2014) for further discussion. To match joint foreclosure and bankruptcy data, Mitman (2012) calibrates an 18% likelihood of deficiency judgments.
on housing dynamics of a foreclosure reform that makes all mortgages full recourse with a costless process for initiating deficiency judgments and allows lenders to seize up to 90% of a household’s financial assets.

4.3.1 Dynamic Effects of the Reform

Table 6 demonstrates the effects of the considered foreclosure reform on housing dynamics. The economy with costless, full recourse exhibits 12% less house price volatility, 17% less residential investment volatility, and 38% higher existing sales volatility than the baseline economy. Furthermore, fluctuations in months supply drop by over 85%, while foreclosures almost disappear.

The foreclosure reform reduces price, selling time, and foreclosure volatility primarily by reducing speculative borrowing. Increased borrower reluctance to default and higher recovery ratios drive down default premia, which increases the supply of credit. However, homeowners largely avoid taking out risky, high leverage mortgages. Simulated average default premia hover around 0%, even for those few borrowers who take out high leverage mortgages.

This reduction in risky borrowing, combined with the expansion of credit, prevents debt overhang and the emergence of liquidity spirals. Homeown-
ers with moderately high leverage no longer become “distressed sellers” because they can extract equity at low cost through refinancing. Furthermore, the lack of foreclosure activity and REO houses flooding the market during housing busts mediates price declines. Augmented by fewer distressed and debt-constrained sellers, the disappearance of countercyclical REO sales also explains the increased volatility and procyclicality of existing sales as well as the drastic drop in months’ supply volatility. The recourse model does, however, still generate protracted booms and busts, which confirms the importance of search frictions even without the amplification of liquidity spirals.

5 Conclusions

Search frictions in the housing market interact with endogenous credit constraints to produce quantitatively accurate housing, mortgage debt, and foreclosure dynamics. The liquidity spirals and gradual boom-bust dynamics generated by the model accord strongly with the data, making the model a good launching point for future theoretical and policy-related research. Furthermore, the tractable formulation of directed search in the housing market with rich, two-sided heterogeneity and aggregate uncertainty allows the model to address issues that affect housing simultaneously through financial and non-financial channels. For example, future work could look at the role of state variation in credit conditions, housing supply factors, and government policy in explaining different regional house price dynamics. Alternatively, the model provides a useful framework in which to analyze optimal monetary and fiscal policy with frictional housing.
References


Chu, Yongqiang, “Credit Constraints, Inelastic Supply, and the Housing Boom,” Review of Economic Dynamics, 2013, (0), –.


6 Appendix

Definition 1  A recursive equilibrium consists of:

- Household value and policy functions
- Production firm functions $K_c(\Gamma)$, $N_c(\Gamma)$, $L(\Gamma)$, $S_h(\Gamma)$, and $N_h(\Gamma)$
- Intermediary functions $J_{REO}(h, \Gamma)$, $R_{REO}(h, \Gamma)$, $x_s^{REO}(h, \Gamma)$, and $K(\Gamma)$
- Prices $r_h$, $q_b(\Gamma)$, $i(\Gamma)$, $r_m(\Gamma)$, $r(\Gamma)$, $w(\Gamma)$, $p_l(\Gamma)$, $p_h(\Gamma)$, and $q_m^0(m', b', h, s, \Gamma)$
- Market tightnesses $\theta_b(x_b, h; p_h(\Gamma))$ and $\theta_s(x_s, h; p_h(\Gamma))$
- An aggregate law of motion $\Gamma' = G(\Gamma, z'_c)$

such that

1. **Household Optimality:** The value/policy functions solve (20) – (29).
2. **Firm Optimality:** Conditions (6) – (11) are satisfied.
3. **Intermediary Optimality:** Conditions (15) – (18) are satisfied.
4. **Market Tightnesses:** $\theta_b$ and $\theta_s$ satisfy (13) – (14).
5. **Shadow Housing Price:** $D_h(p_h(\Gamma); \Gamma) = S_h(p_h(\Gamma); \Gamma)$.
6. **Land/Permits:** $L(\Gamma) = \bar{L}$.
7. **Labor Market Clears:** $N_c(\Gamma) + N_h(\Gamma) = \sum_{s \in S} \int_E e \cdot s F(de) \Pi_s(s)$.
8. **Capital Market Clears:** $K_c(\Gamma) = K(\Gamma)$.
9. **Bond Market Clears:** No active trading of international bonds.
10. **Resource Constraint:** Total use of the consumption good equals total production, $z_c F(K_c(\Gamma), N_c(\Gamma))$.
11. **Aggregate Law of Motion:** The law of motion $\Gamma' = G(\Gamma, z'_c)$ is consistent with the Markov process induced by the exogenous processes $\pi_s$, $\pi_z$, $F$, and all relevant policy functions.
Table 7: Standard Business Cycle Statistics

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42
Table 8: Lagged Correlations—Output, Consumption, and Investment

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<td>0.93</td>
<td>0.97</td>
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<td>0.89</td>
<td>0.89</td>
<td>0.88</td>
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</table>

Quarterly data come from NIPA table 1.5.5. and are deflated using the PCE index. Data and model output are detrended with $\lambda = 10^8$ using the HP filter.
Table 9: Lagged Correlations—House Prices, Sales, Months Supply, and Foreclosures

<table>
<thead>
<tr>
<th>x</th>
<th>$\sigma_x/\sigma_Y$</th>
<th>$x(-4)$</th>
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<td>Output (Y)</td>
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<td>0.78</td>
<td>0.89</td>
<td>0.96</td>
<td>1.00</td>
<td>0.97</td>
<td>0.90</td>
<td>0.82</td>
<td>0.73</td>
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<td>0.63</td>
<td>0.66</td>
</tr>
<tr>
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<td>0.73</td>
<td>0.78</td>
<td>0.81</td>
<td>0.80</td>
<td>0.73</td>
<td>0.66</td>
<td>0.60</td>
<td>0.56</td>
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</tr>
<tr>
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<td>-0.61</td>
<td>-0.57</td>
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<tr>
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<td>-0.68</td>
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<tr>
<td>Output (Y)</td>
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<td>0.87</td>
<td>0.90</td>
<td>0.93</td>
<td>0.97</td>
<td>1.00</td>
<td>0.97</td>
<td>0.93</td>
<td>0.90</td>
<td>0.87</td>
</tr>
<tr>
<td>House prices</td>
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<td>0.85</td>
<td>0.88</td>
<td>0.92</td>
<td>0.95</td>
<td>0.94</td>
<td>0.92</td>
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<td>-0.87</td>
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<tr>
<td>Output (Y)</td>
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<td>0.86</td>
<td>0.89</td>
<td>0.93</td>
<td>0.97</td>
<td>1.00</td>
<td>0.96</td>
<td>0.93</td>
<td>0.89</td>
<td>0.86</td>
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<tr>
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<td>0.82</td>
<td>0.85</td>
<td>0.89</td>
<td>0.92</td>
<td>0.96</td>
<td>0.95</td>
<td>0.94</td>
<td>0.92</td>
<td>0.91</td>
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<tr>
<td>Existing sales</td>
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<td>0.28</td>
<td>0.30</td>
<td>0.32</td>
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<td>0.25</td>
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<tr>
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<td>–</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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</tr>
<tr>
<td>Foreclosure rate</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<td>–</td>
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</tr>
<tr>
<td><strong>Recourse</strong></td>
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<td>0.86</td>
<td>0.90</td>
<td>0.93</td>
<td>0.97</td>
<td>1.00</td>
<td>0.97</td>
<td>0.93</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>House prices</td>
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<td>0.85</td>
<td>0.88</td>
<td>0.92</td>
<td>0.95</td>
<td>0.94</td>
<td>0.92</td>
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<tr>
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<td>-0.81</td>
<td>-0.84</td>
<td>-0.87</td>
<td>-0.83</td>
<td>-0.80</td>
<td>-0.77</td>
<td>-0.75</td>
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<tr>
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<td>–</td>
<td>–</td>
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</table>

Existing sales and months supply data span 1982 – 2010 and come from the National Association of Realtors. In the model, existing sales includes all REO sales and sales by owners. Foreclosure data is from the Mortgage Bankers’ Association and covers 1979 – 2010. For house prices I use the Freddie Mac House Price Index. Data and model output are detrended with $\lambda = 10^8$ using the HP filter.
Table 10: Lagged Correlations—Household Portfolios

<table>
<thead>
<tr>
<th>x</th>
<th>σ_x/σ_Y</th>
<th>x(-4)</th>
<th>x(-3)</th>
<th>x(-2)</th>
<th>x(-1)</th>
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<th>x(+2)</th>
<th>x(+3)</th>
<th>x(+4)</th>
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<tbody>
<tr>
<td>Output (Y)</td>
<td>1.00</td>
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<td>0.78</td>
<td>0.89</td>
<td>0.97</td>
<td>1.00</td>
<td>0.97</td>
<td>0.90</td>
<td>0.82</td>
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<tr>
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<td>0.55</td>
<td>0.66</td>
<td>0.74</td>
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<th>x(-2)</th>
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<td>0.90</td>
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<tr>
<td>Financial assets</td>
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<td>0.81</td>
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<td>0.77</td>
<td>0.80</td>
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<td>0.88</td>
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<td>0.90</td>
<td>0.91</td>
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<tr>
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<td>1.00</td>
<td>0.97</td>
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<td>0.74</td>
<td>0.77</td>
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<td>0.84</td>
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<td>0.89</td>
<td>0.89</td>
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<td>0.90</td>
</tr>
<tr>
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<tr>
<td>Housing wealth</td>
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<td>0.89</td>
<td>0.89</td>
<td>0.88</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Net worth, financial assets, housing wealth, and mortgage debt data come from table B.100 of the Federal Reserve Flow of Funds Accounts. Financial assets are defined as total financial assets plus consumer durables minus non-mortgage credit market liabilities. Housing wealth is defined as owner-occupied real estate at market values, and I use the home mortgages series for the definition of mortgage debt. I then define net worth as the sum of financial assets and housing wealth minus mortgage debt. Data and model output are detrended with $\lambda = 10^8$ using the HP filter.
SUPPLEMENTARY APPENDIX

A Supplementary Figures

A.1 Impulse Response Functions

Figure 6: Economic response to a small, 10-year increase in $z_c$. With the exception of months supply, the foreclosure rate, and the foreclosure sales share, each series is initially normalized to 1 and is plotted alongside output of the consumption good (the light curve) for reference.
A.2 Comparison of Boom-Bust Dynamics

Figure 7: Economic dynamics in a bust-to-boom transition caused by a permanent, positive $z_c$ shock. All variables except months supply and the foreclosure rate are initially normalized to 1.
Figure 8: Economic dynamics in a *boom-to-bust* transition caused by a permanent, *negative* $z_c$ shock. All variables except months supply and the foreclosure rate are initially normalized to 1.
B Household Value Functions

B.1 Subperiod 1 Value Functions

The value of not trying to sell is \( W(y, m, h, s, f, \Gamma) \), and the option value of trying to sell is \( R(y, m, h, s, f, \Gamma) \).

\[
R_{sell}(y, m, h, s, 0, \Gamma) = \max \{ 0, \max_{x_s \geq m} p_s(\theta_s(x_s, h; \Gamma)) [(V_{rent} + R_{buy})(y + x_s - m, s, 0, \Gamma) - W_{own}(y, m, h, s, 0, \Gamma)] - [1 - p_s(\theta_s(x_s, h; \Gamma))] \xi \} \tag{20}
\]

\[
R_{sell}(y, 0, h, s, 1, \Gamma) = \max \{ 0, \max_{x_s \geq 0} p_s(\theta_s(x_s, h; \Gamma)) [(V_{rent} + R_{buy})(y + x_s, s, 1, \Gamma) - W_{own}(y, 0, h, s, 1, \Gamma)] - [1 - p_s(\theta_s(x_s, h; \Gamma))] \xi \} \tag{21}
\]

\[
W_{own}(y, m, h, s, 0, \Gamma) = \max \{ (V_{rent} + R_{buy})(y + \max \{ 0, J_{REO}(h, \Gamma) - m \}, s, 1, \Gamma), V_{own}(y, m, h, s, 0, \Gamma) \} \tag{22}
\]

\[
W_{own}(y, 0, h, s, 1, \Gamma) = W_{own}(y, 0, h, s, 1, \Gamma) \tag{23}
\]

B.2 Subperiod 2 Value Functions

The option value of trying to buy is \( R_{buy}(y, s, f, \Gamma) \).

\[
R_{buy}(y, s, 0, \Gamma) = \max \{ 0, \max_{h \in H, x_b \leq y} p_b(\theta_b(x_b, h; \Gamma)) [V_{own}(y-x_b, 0, h, s, 0, \Gamma) - V_{rent}(y, s, 0, \Gamma)] \} \tag{24}
\]

\[
R_{buy}(y, s, 1, \Gamma) = \max \{ 0, \max_{h \in H, x_b \leq y} p_b(\theta_b(x_b, h; \Gamma)) [V_{own}(y-x_b, 0, h, s, 1, \Gamma) - V_{rent}(y, s, 1, \Gamma)] \} \tag{25}
\]

B.3 Subperiod 3 Value Functions

\[
V_{rent}(y, s, 0, \Gamma) = \max_{b' \in B, c \geq 0, c_h \in [0, h]} u(c, c_h) + \beta \mathbb{E} [(V_{rent} + R_{buy})(y', s', 0, \Gamma')] \tag{26}
\]

subject to

\[
c + r_h c_h + q_b(\Gamma) b' \leq y
\]

\[
y' = (1 - \tau(\Gamma'))(w(\Gamma') c' s' + b')
\]

\[
\Gamma' = G(\Gamma, z'_c)
\]
\[
V_{\text{rent}}(y, s, 1, \Gamma) = \max_{b' \in B, c \geq 0, c_h \in [0, h]} u(c, c_h) + \beta \mathbb{E} \left[ (V_{\text{rent}} + R_{\text{buy}})(y', s', f', \Gamma') \right]
\]
subject to
\[
c + r_h c_h + q_b(\Gamma)b' \leq y \\
y' = (1 - \tau(\Gamma'))(w(\Gamma')e's' + b') \\
\Gamma' = G(\Gamma, z'_c)
\]

(27)

\[
V_{\text{own}}(y, m, h, s, 0, \Gamma) = \max_{m' \in M, b' \in B, c \geq 0} u(c, h) + \beta \mathbb{E} \left[ (1 - \delta_h)(W_{\text{own}} + R_{\text{sell}})(y', m', h, s', 0, \Gamma') \right] + \delta_h(V_{\text{rent}} + R_{\text{buy}})(y', s', 0, \Gamma')
\]
subject to
\[
c + q_b(\Gamma)b' + [m - q_m(m', b', h, s, \Gamma)m'] \leq y \\
q_m(m', b', h, s, \Gamma) = \begin{cases} q^0_m(m', b', h, s, \Gamma) & \text{if } m' > m \\ \frac{1}{1 + r_m(\Gamma)} & \text{if } m' \leq m \end{cases} \\
y' = (1 - \tau(\Gamma'))(w(\Gamma')e's' + b') \\
\Gamma' = G(\Gamma, z'_c)
\]

(28)

\[
V_{\text{own}}(y, 0, h, s, 1, \Gamma) = \max_{b' \in B, c \geq 0} u(c, h) + \beta \mathbb{E} \left[ (1 - \delta_h)(W_{\text{own}} + R_{\text{sell}})(y', 0, h, s', f', \Gamma') \right] + \delta_h(V_{\text{rent}} + R_{\text{buy}})(y', s', f', \Gamma')
\]
subject to
\[
c + q_b(\Gamma)b' \leq y \\
y' = (1 - \tau(\Gamma'))(w(\Gamma')e's' + b') \\
\Gamma' = G(\Gamma, z'_c)
\]

(29)

C Calibration Details

C.1 Calibrating the Labor Efficiency Process

As explained in the calibration section, it is not possible to estimate quarterly income processes from PSID data because the PSID is only conducted annually. Instead, I start by specifying a labor process like that in Storesletten et al. (2004), except without life cycle effects or a permanent shock at birth. I adopt their values for the annual autocorrelation of the persistent shock and for the variances of the persistent and transitory shocks, transforming them to quarterly values.
C.1.1 Persistent Shocks

I assume that households play a lottery each period in which, with probability 3/4, they receive the same persistent shock as they did in the previous period, and with probability 1/4, they draw a new shock from a transition matrix calibrated to the persistent process in Storesletten et al. (2004) (in which case they still might receive the same persistent labor shock). This is equivalent to choosing transition probabilities that match the expected amount of time that households expect to keep their current shock. Storesletten et al. (2004) report an annual autocorrelation coefficient of 0.952 and a frequency-weighted average standard deviation over expansions and recessions of 0.17. I use the Rouwenhorst method to calibrate this process, which gives the following transition matrix:

\[ \tilde{\pi}_s(\cdot, \cdot) = \begin{pmatrix} 0.9526 & 0.0234 & 0.0006 \\ 0.0469 & 0.9532 & 0.0469 \\ 0.0006 & 0.0234 & 0.9526 \end{pmatrix} \]

As a result, the final transition matrix is

\[ \pi_s(\cdot, \cdot) = 0.75I_3 + 0.25\tilde{\pi}_s(\cdot, \cdot) = \begin{pmatrix} 0.9881 & 0.0059 & 0.0001 \\ 0.0171 & 0.9883 & 0.0171 \\ 0.0001 & 0.0059 & 0.9881 \end{pmatrix} \]

C.1.2 Transitory Shocks

Storesletten et al. (2004) report a standard deviation of the transitory shock of 0.255. To replicate this, I assume that the annual transitory shock is actually the sum of four, independent quarterly transitory shocks. I make use of the same identifying assumption that Storesletten et al. (2004) use, namely, that all households receive the same initial persistent shock. Any variance in initial labor income is then due to different draws of the transitory shock. Recall that the labor productivity process is given by

\[ \ln(e \cdot s) = \ln(s) + \ln(e) \]

Therefore, total labor productivity (which, when multiplied by the wage \( w \), is total wage income) over a year in which \( s \) stays constant is

\[ (e \cdot s)_{\text{year 1}} = \exp(s_0)[\exp(e_1) + \exp(e_2) + \exp(e_3) + \exp(e_4)] \]

For different variances of the transitory shock, I simulate total annual labor productivity for many individuals, take logs, and compute the variance of the
It turns out that quarterly transitory shocks with a standard deviation of 0.49 give the desired standard deviation of annual transitory shocks of 0.255.

### C.2 Calculating Submarket Trading Probabilities

Using (13) to solve for buyers’ trading probabilities gives

\[
p_b(\theta_b(x_b, h; p_h)) = \begin{cases} 
0 & \text{if } x_b < \underline{x}_b(p_h) \\
\frac{\left(\frac{A_b(x_b - p_h h)}{\kappa_b h}\right)^{1/\gamma_b} - 1}{x_b - p_h h} & \text{if } \underline{x}_b(p_h) \leq x_b \leq \overline{x}_b(p_h) \\
1 & \text{if } x_b > \overline{x}_b(p_h) 
\end{cases}
\]

where \(\underline{x}_b(p_h) = \left(p_h + \frac{\kappa_b}{A_b}\right) h\) and \(\overline{x}_b(p_h) = (p_h + \kappa_b(A_b^{-\gamma_b} - 1)^{-1/\gamma_b})h\).

Similarly, sellers’ trading probabilities come from solving (14), giving

\[
p_s(\theta_s(x_s, h; p_h)) = \begin{cases} 
0 & \text{if } x_s > \overline{x}_s(p_h) \\
\frac{\left(\frac{A_s(p_h h - x_s)}{\kappa_s h}\right)^{1/\gamma_s} - 1}{p_h h - x_s} & \text{if } \underline{x}_s(p_h) \leq x_s \leq \overline{x}_s(p_h) \\
1 & \text{if } x_s < \underline{x}_s(p_h) 
\end{cases}
\]

where \(\underline{x}_s(p_h) = \left(p_h - \frac{\kappa_s}{A_s}\right) h\) and \(\overline{x}_s(p_h) = (p_h - \kappa_s(A_s^{-\gamma_s} - 1)^{-1/\gamma_s})h\).

As this characterization of trading probabilities shows, search frictions give rise to endogenous adjustment costs. Matching fails if households try for too small of an adjustment cost, matching succeeds if they accept the maximum adjustment cost, and trade occurs probabilistically for intermediate values.
D The Intermediary’s Balance Sheet

Period-\(t\) cash flows are given by

\[
\pi_t = (1 - \delta_t + r_t)K_t - B_t + \sum_{n_{m,t}} \sum_{s_t} \int \left[ p_s(\theta_{s,t}(x_{s,t}, h_{t-1})) + [1 - p_s(\theta_{s,t}(x_{s,t}, h_{t-1}))] \right] \\
(1 - d_t^s)[m_t - (1 + \phi)m_{t+1}/(1 + r_{m,t}) + \Pi_t(m_{t+1}, h_{t+1}, h_t = h_{t-1}, s_t)] F(de) \pi_s(s_t|s_{t-1})n_{m,t} \\
+ \sum_{n_{m,t}} \sum_{s_t} \int [1 - p_s(\theta_{s,t}(x_{s,t}, h_{t-1}))]d_t^s J_{REO,t}(h_{t-1})F(de) \pi_s(s_t|s_{t-1})n_{m,t} + \sum_{h \in H} J_{REO,t}(h)H_{REO,t}(h) \\
- \sum_{n_{m,t}} \sum_{s_t} \int [1 - p_s(\theta_{s,t}(x_{s,t}, h_{t-1}))]d_t^s J_{REO,t}(h_{t-1}) - p_s(\theta_{s,t}(x_{s,t}, h_{t-1}))x_{s,t}^{REO}(h_{t-1})]F(de) \pi_s(s_t|s_{t-1})n_{m,t} \\
- \sum_{h \in H} [J_{REO,t}(h) - p_s(\theta_{s,t}(x_{s,t}^{REO}, h))x_{s,t}^{REO}(h)]H_{REO,t}(h) + q_{s,t}B_{t+1} \\
- \sum_{n_{m,t+1}} q_{m,t}(m_{t+1}, b_{t+1}, h_t, s_t)(1 + \zeta)m_{t+1}n_{m,t+1} - K_{t+1}
\]

The first three lines represent the intermediary’s beginning-of-period profits resulting from capital (+), households’ redemption of bonds (−), revenues from mortgage payments (+), and the sale of continuation mortgages and REO inventories to other intermediaries (+). The last three lines represent intermediary expenses at the end of the period. The intermediary purchases unsold REO inventories (−), sells bonds (+), issues new mortgages (−), purchases vintage mortgages (−), and chooses next period’s capital (−).

When prices are in equilibrium, next period’s capital stock equals the sum of the other subperiod 3 expenses. In other words, higher bond issuances \(B_{t+1}\) increase next period’s capital stock, while higher mortgage origins lower the capital stock. Furthermore, the mortgage pricing condition (17) and REO value function (18) ensure zero ex-ante beginning-of-period profits.

E Computing the Model

In the spirit of Krusell and Smith (1998), I compute a bounded rationality equilibrium where households approximate the aggregate state space when forming expectations. As in Favilukis et al. (2013), I use capital \(K\) and the shadow housing price \(p_h\) as the approximating variables. Therefore, the aggregate state in this bounded rationality economy is \(\hat{\Gamma} = (z_c, p_h, K)\). I posit the
following forecasting functions:

\[ p'_h(z_c, p_h, K, z'_c) = a^p_0(z_c, z'_c) + a^p_1(z_c, z'_c) p_h + a^p_2(z_c, z'_c) K \]  \hspace{1cm} (30)

\[ K'(z_c, p_h, K, z'_c) = a^K_0(z_c) + a^K_1(z_c) p_h + a^K_2(z_c) K \]  \hspace{1cm} (31)

\[ \tau'(z_c, p_h, K, z'_c) = a^\tau_0(z_c, z'_c) + a^\tau_1(z_c, z'_c) p_h + a^\tau_2(z_c, z'_c) K \]  \hspace{1cm} (32)

In an approximating equilibrium, the forecasting function coefficients maximize predictive accuracy relative to simulated time series of \( p'_h, K', \) and \( \tau' \).

The entire computational algorithm is outlined below:

1. Solve for equilibrium submarket tightnesses \( \{ \theta_b(x_b, h; p_h) \} \) and \( \{ \theta_s(x_s, h; p_h) \} \) for each value of the housing shadow price, \( p_h \), using (13) – (14).

2. **Loop 1** – Make an initial guess of coefficients \( \vec{a}^{p,0}, \vec{a}^{K,0}, \) and \( \vec{a}^{\tau,0}. \)

   (a) Solve for \( w(\Gamma'), r(\Gamma'), q_b(\Gamma), \) and \( r_m(\Gamma) \) using the aggregate laws of motion implied by the coefficients above, the equilibrium conditions for the firm’s problem, (6) – (7) and (10) – (11), the intermediary conditions (15) – (16), and the market clearing conditions for land/permits and labor. In practice, this procedure involves solving a simple fixed point problem in the amount of labor employed in the consumption good sector and then substituting to calculate the remaining quantities and factor prices.

   (b) **Loop 2a** – Make an initial guess for the mortgage company’s REO value function, \( J^0_{\text{REO}}(h, \Gamma) \).

      i. Substitute \( J^0_{\text{REO}} \) into the right hand side of (18) and solve for \( J_{\text{REO}}(h, Z) \).

      ii. If \( \sup(|J_{\text{REO}} - J^0_{\text{REO}}|) < \epsilon_J \), then exit the loop. Otherwise, set \( J^0_{\text{REO}} = J_{\text{REO}} \) and return to (i).

   (c) **Loop 2b** – Make an initial guess of mortgage prices \( q^0_{m,n}(m', b', h, s, \Gamma) \) for \( n = 0 \).

      i. Calculate the lower bound of the budget set for homeowners with good credit entering subperiod 3, \( \underline{y}(m, h, s, \Gamma) \), by solving

      \[ \underline{y}(m, h, s, \Gamma) = \min_{m', b'}[q_0(\Gamma)b' + m - q_m(m', b', h, s, \Gamma)m'] \], where

      \[ q_m(m', b', h, s, \Gamma) = \begin{cases} q^0_m(m', b', h, s, \Gamma) & \text{if } m' > m \\ \frac{1}{1 + r_m(\Gamma)} & \text{if } m' \leq m \end{cases} \]

      ii. **Loop 3** – Make an initial guess for \( V^0_{\text{rent}}(y, s, f, \Gamma) \) and \( V^0_{\text{own}}(y, m, h, s, f, \Gamma) \).
A. Substitute \( V_{\text{rent}}^0 \) and \( V_{\text{own}}^0 \) into the right hand side of (24) – (25) and solve for \( R_{\text{buy}} \).

B. Substitute \( V_{\text{rent}}^0 \), \( V_{\text{own}}^0 \), and \( R_{\text{buy}} \) into the right hand side of (22) – (23) and solve for \( W_{\text{own}} \).

C. Substitute \( W_{\text{own}} \), \( V_{\text{rent}}^0 \), and \( R_{\text{buy}} \) into the right hand side of (20) – (21) and solve for \( R_{\text{sell}} \).

D. Substitute \( W_{\text{own}} \), \( V_{\text{rent}}^0 \), \( R_{\text{sell}} \), and \( R_{\text{buy}} \) into the right hand side of (26) – (29) and solve for \( V_{\text{rent}} \) and \( V_{\text{own}} \).

E. If \( \sup(|V_{\text{rent}} - V_{\text{rent}}^0|) + \sup(|V_{\text{own}} - V_{\text{own}}^0|) < \epsilon_V \), then exit the loop. Otherwise, set \( V_{\text{rent}}^0 = V_{\text{rent}} \) and \( V_{\text{own}}^0 = V_{\text{own}} \) and return to A.

iii. Substitute \( q_{m}^{0,n} \), \( J_{\text{REO}} \), and the household’s policy functions for bonds, mortgage choice and selling and default decisions into the right hand side of (17) and solve for \( q_{m}^{0} \).

iv. If \( \sup(q_{m}^{0} - q_{m}^{0,n}) < \epsilon_q \), then exit the loop. Otherwise, set \( q_{m}^{0,n+1} = (1 - \lambda_q)q_{m}^{0,n} + \lambda_q q_{m}^{0} \) and return to (i).

(d) **Loop 4** – Initialize the distribution \( \Phi \) of homeowners and renters, the capital stock \( K \), and the stock \( H_{\text{REO}}^0 \) of REO houses.

i. Draw an initial shock \( z_{c,0} \) from the stationary distribution \( \Pi_z \) followed by a sequence of aggregate shocks \( \{z_{c,t}\}_1^T \) using the Markov transition matrix \( \pi_z(z'_{c}|z_c) \).

ii. Simulate the economy for \( T \) periods\(^{29}\) using the decision rules of the households and intermediaries. In each period, compute the tax rate that properly distributes the intermediary’s ex-post profits/losses, calculate tomorrow’s capital stock (equal to bond issuances minus the value of new and vintage mortgages and unsold REO inventories), and solve for the equilibrium housing shadow price \( p_{h,t} \) that satisfies (19).

(e) Run the following regressions:

\[
\begin{align*}
    p_{h,t+1} &= a_{0}^{p}(z_{c,t}, z_{c,t+1}) + a_{1}^{p}(z_{c,t}, z_{c,t+1})p_{h,t} + a_{2}^{p}(z_{c,t}, z_{c,t+1})K_{t} \\
    K_{t+1} &= a_{0}^{K}(z_{c,t}) + a_{1}^{K}(z_{c,t})p_{h,t} + a_{2}^{K}(z_{c,t})K_{t} \\
    \tau_{t+1} &= a_{0}^{\tau}(z_{c,t}, z_{c,t+1}) + a_{1}^{\tau}(z_{c,t}, z_{c,t+1})p_{h,t} + a_{2}^{\tau}(z_{c,t}, z_{c,t+1})K_{t}
\end{align*}
\]

which gives new coefficients \( \vec{a}^p \), \( \vec{a}^K \), and \( \vec{a}^\tau \).

\(^{29}\)I simulate the economy for 5,600 periods and ignore the first 600 periods for the regressions.
(f) If \( \sup(|\vec{a}^p - \vec{a}^{p,n}|) + \sup(|\vec{a}^K - \vec{a}^{K,n}|) + \sup(|\vec{a}^r - \vec{a}^{r,n}|) < \epsilon_a \), then the algorithm is complete. Otherwise, set \( \vec{a}^{p,n+1} = (1 - \lambda_a)\vec{a}^{p,n} + \lambda_a \vec{a}^p, \vec{a}^{K,n+1} = (1 - \lambda_a)\vec{a}^{K,n} + \lambda_a \vec{a}^K, \vec{a}^{r,n+1} = (1 - \lambda_a)\vec{a}^{r,n} + \lambda_a \vec{a}^r \), and go to (a).

### F Accuracy of Approximate Equilibria

I try several modifications of the regressions explained in the approximating equilibrium section. In particular, I impose various identifying restrictions on the coefficients to see if doing so affects the \( R^2 \) of the regressions or the stability of the computational algorithm. Eliminating the house price terms in the capital stock regression and combining terms for large and small shocks (e.g. lumping transitions \( (z_1, z' = z_2) \) and \( (z_1, z' = z_3) \) into one term, \( (z_1, z' > z_1) \)) gives the most accurate and stable solution. The converged results are reported below for the baseline, no search, and recourse economies.

Table 11: Baseline Equilibrium

<table>
<thead>
<tr>
<th>( (z_1, z_1) )</th>
<th>( (z_1, z'_1 &gt; z_1) )</th>
<th>( (z_2, z_1) )</th>
<th>( (z_2, z_2) )</th>
<th>( (z_2, z_3) )</th>
<th>( (z_3, z'_1 &lt; z_3) )</th>
<th>( (z_3, z_3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^p_0 )</td>
<td>0.0308</td>
<td>0.0590</td>
<td>0.0256</td>
<td>0.0465</td>
<td>0.0813</td>
<td>0.0099</td>
</tr>
<tr>
<td>( a^p_1 )</td>
<td>0.9646</td>
<td>0.9646</td>
<td>0.9480</td>
<td>0.9480</td>
<td>0.9480</td>
<td>0.9628</td>
</tr>
<tr>
<td>( a^p_2 )</td>
<td>-0.0007</td>
<td>-0.0007</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>-0.0007</td>
</tr>
</tbody>
</table>

\( R^2 = 0.998 \)

| \( a^K_0 \) | 0.0613 | 0.0613 | 0.0652 | 0.0652 | 0.0652 | 0.1072 | 0.1072 |
| \( a^K_1 \) | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| \( a^K_2 \) | 0.9728 | 0.9728 | 0.9739 | 0.9739 | 0.9739 | 0.9600 | 0.9600 |

\( R^2 = 0.99997 \)

| \( a^r_0 \) | 0.0001 | -0.0018 | 0.0026 | 0.0009 | -0.0007 | 0.0020 | 0.0005 |
| \( a^r_1 \) | -0.0009 | -0.0009 | -0.0015 | -0.0015 | -0.0015 | -0.0005 | -0.0005 |
| \( a^r_2 \) | 0.0003 | 0.0003 | 0.0002 | 0.0002 | 0.0002 | -0.0000 | -0.0000 |

\( R^2 = 0.988^* \)

*Simulated taxes/subsidies usually \( \sim 0.00x\% \) and never exceed \( 0.x\% \) (when \( z \) switches values).
### Table 12: No Search Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>((z_1, z_1))</th>
<th>((z_1, z' &gt; z_1))</th>
<th>((z_2, z_1))</th>
<th>((z_2, z_2))</th>
<th>((z_2, z_3))</th>
<th>((z_3, z' &lt; z_3))</th>
<th>((z_3, z_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0^p)</td>
<td>0.0229</td>
<td>0.0418</td>
<td>0.0037</td>
<td>0.0246</td>
<td>0.0583</td>
<td>0.0291</td>
<td>0.0518</td>
</tr>
<tr>
<td>(a_1^p)</td>
<td>0.9822</td>
<td>0.9822</td>
<td>0.9853</td>
<td>0.9853</td>
<td>0.9853</td>
<td>0.9594</td>
<td>0.9594</td>
</tr>
<tr>
<td>(a_2^p)</td>
<td>-0.0039</td>
<td>-0.0039</td>
<td>-0.0049</td>
<td>-0.0049</td>
<td>-0.0049</td>
<td>-0.0048</td>
<td>-0.0048</td>
</tr>
</tbody>
</table>

\[ R^2 = 0.998 \]

<table>
<thead>
<tr>
<th></th>
<th>((z_1, z_1))</th>
<th>((z_1, z' &gt; z_1))</th>
<th>((z_2, z_1))</th>
<th>((z_2, z_2))</th>
<th>((z_2, z_3))</th>
<th>((z_3, z' &lt; z_3))</th>
<th>((z_3, z_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0^K)</td>
<td>0.0686</td>
<td>0.0686</td>
<td>0.0467</td>
<td>0.0467</td>
<td>0.0467</td>
<td>0.1238</td>
<td>0.1238</td>
</tr>
<tr>
<td>(a_1^K)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>(a_2^K)</td>
<td>0.9676</td>
<td>0.9676</td>
<td>0.9799</td>
<td>0.9799</td>
<td>0.9799</td>
<td>0.9515</td>
<td>0.9515</td>
</tr>
</tbody>
</table>

\[ R^2 = 0.99997 \]

\*Simulated taxes/subsidies never exceed 0.0\% (when \(z\) switches values).

### Table 13: Full Recourse Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>((z_1, z_1))</th>
<th>((z_1, z' &gt; z_1))</th>
<th>((z_2, z_1))</th>
<th>((z_2, z_2))</th>
<th>((z_2, z_3))</th>
<th>((z_3, z' &lt; z_3))</th>
<th>((z_3, z_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0^p)</td>
<td>0.0344</td>
<td>0.0583</td>
<td>0.0290</td>
<td>0.0473</td>
<td>0.0800</td>
<td>0.0104</td>
<td>0.0440</td>
</tr>
<tr>
<td>(a_1^p)</td>
<td>0.9558</td>
<td>0.9558</td>
<td>0.9431</td>
<td>0.9431</td>
<td>0.9431</td>
<td>0.9697</td>
<td>0.9697</td>
</tr>
<tr>
<td>(a_2^p)</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.0016</td>
<td>-0.0050</td>
<td>-0.0050</td>
</tr>
</tbody>
</table>

\[ R^2 = 0.999 \]

<table>
<thead>
<tr>
<th></th>
<th>((z_1, z_1))</th>
<th>((z_1, z' &gt; z_1))</th>
<th>((z_2, z_1))</th>
<th>((z_2, z_2))</th>
<th>((z_2, z_3))</th>
<th>((z_3, z' &lt; z_3))</th>
<th>((z_3, z_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0^K)</td>
<td>0.0651</td>
<td>0.0651</td>
<td>0.0701</td>
<td>0.0701</td>
<td>0.0701</td>
<td>0.1006</td>
<td>0.1006</td>
</tr>
<tr>
<td>(a_1^K)</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>(a_2^K)</td>
<td>0.9715</td>
<td>0.9715</td>
<td>0.9723</td>
<td>0.9723</td>
<td>0.9723</td>
<td>0.9630</td>
<td>0.9630</td>
</tr>
</tbody>
</table>

\[ R^2 = 0.99997 \]

\*Simulated taxes/subsidies never exceed 0.0\% (when \(z\) switches values).