Money, Interest Rates and Output Revisited

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Abstract: There is a long tradition in economic research that studies the relationship between money, interest rates and output. In this paper, we specify VARs using cyclical measures of monetary aggregate, interest rates, and output to assess whether money has marginal predictive content for output. Because there is no consensus on how to identify the cyclical component, we consider four alternatives. One goal is to re-examine the result that when the interest rate variable is included, the marginal predictive content of money become insignificant. In addition, we decompose the monetary aggregates into base money and money multiplier components. In this way, we can determine whether inside money has marginal predictive content for output. We can also assess whether the interest rate has marginal predictive content for the money multiplier. The evidence suggests that with the M2 aggregate, movements in money do temporally precede movements in output. However, the evidence is strong that movements in interest rates temporally precede movements in output. The evidence is mixed regarding the movements in interest rates and future movements in the money multiplier.

Keywords: Detrending methods, marginal predictive content, inside money vs. outside money, time-varying VAR

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1. Introduction

There is a long tradition in economic research that studies the relationship between money, interest rates and output. In an extraordinary clear analysis, Friedman and Schwartz (1963) carefully documented movements in the money supply as they related to turning points in output. Later, Sims (1972, 1980) provided evidence that movements in the monetary aggregates temporally preceded movements in output. Based on such evidence, researchers developed monetary models that could explain output fluctuations at business-cycle frequencies.²

In this paper, we reexamine the relationship between money, interest rates and output at business-cycle frequencies. Following Lucas (1977), movements about trend capture the main qualitative features of what researchers call business cycles. Therefore, we study the lead-lag relationships at business-cycle frequencies using detrended measures of money, interest rates, and output. In this way, we avoid commingling cyclical and trend measures that are present in “raw” measures of economic time series. This, of course, leads to another measurement issue: What is the best way to decompose the time series and obtain the “correct” measure of the cyclical components? Rather than take a stand on that question, we follow Canova (1998), considering four different decomposition methods to identify the cyclical component.³ Thus, the chief contribution of our work lies in asking further whether the answers to these questions are sensitive to method used to decompose variables into trend and cyclical components.

There is a long history of economic research that has looked at the relationships between money, interest rates, and output. The findings were used to distinguish between monetary models of the business cycle and real business cycle theory. Sims’ (1980) work was offered as support for the real business cycle view; specifically, when one includes an interest rate variable, there is no explanatory power left in the monetary variables. The underlying idea is that movements in the interest rate corresponded to movements in productivity. Litterman and Weiss (1985) further developed this theme by constructing measures of ex ante real interest rates. Both Sims (1980) and Litterman and Weiss (1985) report results from VARs in which innovations in interest rates could account for a larger fraction of the forecast error.

² In addition, Christiano and Ljungqvist (1988) presented evidence that money does Granger cause output in bivariate settings. Stock and Watson (1989) extended Sims’ work to look at monthly observations with M1 and industrial production. In contrast to the existing literature, Stock and Watson find that innovations in M1 have marginally significant predictive power for industrial production even after including an interest rate variable.

³ When analyzing the MIO relationship with, say, log levels, it is implicitly assumed that the coefficient on the trend component is equal to the coefficient on the cyclical component. By Wold’s Representation Theorem, however, the trend component is orthogonal to the cyclical component. So, the estimated coefficients are generally not the same.
variance of output. Innovations in money supply accounted for a much smaller fraction of the forecast error variance of output.\(^4\)

Within the monetary models of business cycles, there are a subset of questions arising that are developed further in this paper. For instance, Friedman and Schwartz present evidence on the relationship between broad monetary aggregates and output. Researchers subsequently asked whether the evidence was picking up a relationship between movements in output and movements in outside (base) money, or inside (deposit) money, or both. The idea is that there are at least two branches of the monetary models of business cycles; for example, one could imagine constructing a different model economy depending on the empirical regularity observed. If movements in base money were found to have marginal predictive power for output but movements in inside money were not, one could imagine building a model economy that stressed the importance of monetary policy. In contrast, if movements in the money multiplier were found to have marginal predictive power for output, but movements in outside money were not, perhaps the model economy would stress the role of financial intermediaries. King and Plosser (1984) present evidence that is consistent with the notion that it is movements in inside money, not base money, that are systematically related to movements in future output. In addition, work by Freeman and Huffman (1991) and Freeman and Kydland (2000) built model economies that were modified versions of the canonical real business cycle model. These models could account for movements in inside money that temporally preceded movements in output.

In this paper, we focus on temporal precedence. In particular, we directly assess whether movements in money or interest rates have marginal predictive content for output. Alternatively, does money or the interest rate, or both, Granger cause output. In a four-variable VAR, we differentiate between outside and inside money, specifying base money, the interest rate, the money multiplier and output. This specification allows us to directly examine two questions. Is there marginal predictive power in either base money or the money multiplier for output? Is there marginal predictive power in the interest rate for the money multiplier? Together, these two questions extend the work that King and Plosser conducted; is there predictive information in the money multiplier for output, even when one includes an interest rate variable in the VAR. Even if the answer is no, we can gain some insight that can be used to account for the relationships. If the interest rate has marginal predictive power for the money multiplier, there is empirical support for using model economies along the lines of King and Plosser, Freeman and

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\(^4\) The results are not uniform across the studies. Sims (1972, 1980), for example, presents evidence that indicates that money temporally precedes output and interest rates temporally precede output and those results are not sensitive to using base money or M1 as the measure of money. Note that Sims (1972) estimates the relationship for the 1947-69 sample and Sims (1980) uses 1949-1975 data.
Huffman and Freeman and Kydland. Such evidence is consistent with the notion that movements in the money multiplier reflect endogenous responses to movements that are driving interest rates.

In this paper, our analysis seeks to answer the following set of five questions:

In bivariate analysis,

- Do movements in monetary aggregates temporally precede movements in output?

In trivariate analysis,

- Do movements in the interest rate temporally precede movements in output?
- Do movements in the monetary aggregates temporally precede movements in output after including the interest rate variable?

In quadrivariate analysis,

- Do movements in the money multiplier temporally precede movements in output?
- Do movements in the interest rate temporally precede movements in the money multiplier?

We study these five questions in two different structures. We consider different lag length selection criteria. One is ad hoc, fixing the lag length at four. The other approach uses a likelihood ratio test to determine the “optimal” lag length. In addition, we extend the fixed coefficient setting to consider time-varying coefficients and stochastic volatility.

With focus on these five questions, the results are summarized as follows. First, it matters which monetary aggregate one uses in the analysis of bivariate VARs. With M2, we find that money has predictive content for output in a majority of the decomposition methods. In contrast, we find that money does not have marginal predictive power for output when M1 is the money measure.\(^5\)

Next, when we include the interest rate variable, the evidence indicates that movements in the interest rate do temporally precede movements in output. In addition, for most of the decomposition methods, the trivariate VARs indicates that the monetary aggregate matters. In the trivariate settings, movements in M1 temporally precede movements in output in three of the four decomposition methods. Here, we also see that the results are sensitive to lag length selection. For cases in which we fix the lag length at four, movements in M2 temporally precede movements in output in three of the four

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\(^5\) First differences are measures of the cyclical components under certain conditions. Diba and Oh (1991), for example, analyzed the MIO relationship using growth rates of money and output. But, this is only one decomposition approach.
decomposition methods. However, there is only one case in which movements in M2 temporally precede output when the lag length is selected optimally. Lastly, we see that movements in the interest rate temporally precede movements in the monetary aggregates in almost all the decomposition methods.

Next, we decompose the monetary aggregates, estimating a quadrivariate VAR. Movements in the interest rate temporally precede movements in output in every decomposition method. In fewer than half the cases, there is evidence that either movements in the money multiplier or movements in base money temporally precede movements in output. In three-fourths of the cases, movements in the interest rate temporally precede movements in the money multiplier. We consider impulse response functions to further enlighten the evidence. We see that interest rate shocks generate a cyclical response, initially resulting in higher levels of output, then a contractionary effect two years later. With respect to the monetary aggregates, the responses are generally not significant, thus weakening any temporal precedence evidence.

Lastly, we extend the analysis to consider time-varying parameters depending on the decomposition method and the lag length selected. Because of the number of estimated parameters, we estimate the time-varying parameters with fewer lags than we do in the time-invariant settings. Correspondingly, we see the results generally weaken in terms of finding significant marginal predictive power for output. We do see periods in which shocks to interest rates play important roles in accounting for future output movements. In these cases, interest rate movements also account for future movements in money multipliers. Together, these results are consistent with the Freeman-Huffman hypothesis; there is a transmission mechanism in which productivity shocks are initially reflected as movements in interest rates. Subsequently, endogenous responses in quantities—both those inside financial institutions and production—occur. This line of inquiry, of course, deserves far more attention.

Thus, we find that interest rate movements do temporally precede movements in output. Moreover, movements in interest rates temporally precede movements in the monetary aggregates. The decomposition method does not matter much with respect to these two findings. To account for those results can be challenging. When we look at the impulse response functions, we do see how the unique features of each decomposition method affect the timing of output responses to interest rate shocks. In particular, there is evidence of a phase shift in the output response when one uses the Unobservable Components decomposition compared with the other three decomposition methods. In the time-varying VARs, we also see differences in the significance of Granger causality results; for the Unobservable Components and the Beveridge-Nelson decompositions, Granger causality is not uniformly significant over the entire period.
The results presented in this paper provide a richer set of correlations that economic theory can account for. The sensitivity analysis provides greater confidence that the relationship between the interest rate and output is invariant to decomposition methods. With small-dimensional VARs, the time-varying results could be pointing to predictive content that is associated with a set of different unobservable shocks that have hit the U.S. economy during the postwar period. For example, sometimes the “true” underlying shock operates initially through movements in the interest rate, leading to future movements in money measures then eventually output. The sensitivity of the time-varying results to the decomposition method, however, could be pointing different kinds of shocks at different points in time; sometime the interest rate shocks are correlated with the true underlying cause and sometimes movements in interest rates do not temporally precede output.\textsuperscript{6}

The paper is organized as follows. In Section 2, we briefly review the alternative methods used to extract the cyclical component from the time series. In Section 3, we report the results of the lag-length selection process. The sensitivity of the money-output relationship under the different measures of the cyclical components is reported in Section 4. To get more insight into the relationship, we plot the impulse response functions for the cases in Section 5. Section 6 presents the results when we consider time-varying coefficient VARs with stochastic volatility. Finally, Section 7 provides a brief summary of our findings.

2. A review of four different decomposition methods

In this section, we begin with a brief overview of the alternative decomposition methods. The analysis is obviously influenced by Canova (1998, 2007) and Morley, Nelson, and Zivot (2003). Following the implementation by these authors, we assume that the series have been treated for any seasonality properly so that only trend and cyclical components are present. Throughout our analysis, we use superscript “$c$” and “$\tau$” to denote the cyclical component and trend component, respectively.

2.1 The Beveridge-Nelson (BN) Decomposition

Canova (2007) classifies the BN decomposition as a member of the class of statistical methods. Morley (2010) characterizes the BN decomposition as calculating the trend and cyclical components for an I(1) time series. Because the BN decomposition calculates the optimal long-run forecast, BN does not...

\textsuperscript{6} McCallum (1986), for example, offered some simple explanations why Granger causality tests are limited in what they actually say about the causes of business cycle correlations. Indeed, McCallum points out that Granger causality tests may not be relevant at all for investigations into the cause of business cycle fluctuations. The correlation-causality point is also made in Freeman and Huffman.
require additional identifying restrictions to become operative. In other words, the BN trend uses ARMA process that captures the autocovariance structure of \( \Delta y_t \) where \( y_t \) is the I(1) process.

In this paper, we follow approaches described in Morley (2002) and Morley, Nelson, and Zivot (2003) to implement the BN decomposition. First, identify the ARIMA\( (p,d,q) \) model for a time series \( y_t \) by plotting the sample ACF (autocorrelation function) and PACF (partial autocorrelation function). Second, convert the ARIMA\( (p,d,q) \) model into its state-space companion form. If \( y_t \) follows an ARIMA\( (2,1,2) \) process, its first difference \( \Delta y_t \) follows an ARMA\( (2,2) \) process, which can be described as

\[
(1 - \phi_1 L - \phi_2 L^2)(\Delta y_t - \mu) = (1 + \theta_1 L + \theta_2 L^2)e_t \quad \text{where} \quad e_t \sim \text{i.i.d.} N(0, \sigma_e^2) \quad \text{and} \quad \mu \quad \text{is the mean of} \quad \Delta y_t .
\]

The state-space companion form of the ARIMA\( (2,1,2) \) model is expressed as

\[
\begin{pmatrix}
\Delta y_t - \mu \\
\Delta y_{t-1} - \mu \\
e_t \\
e_{t-1}
\end{pmatrix} = 
\begin{bmatrix}
\phi_1 & \phi_2 & \theta_1 & \theta_2 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{pmatrix}
\Delta y_{t-1} - \mu \\
\Delta y_{t-2} - \mu \\
e_{t-1} \\
e_{t-2}
\end{pmatrix} + 
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} e_t \quad \text{or} \quad \beta_t = F \beta_{t-1} + v_t \quad (1)
\]

The BN trend at any given date can be determined as

\[
y^{\text{BN}}_t = y_t + [1 \quad 0 \quad 0] F (I - F)^{-1} \beta_{t|t} \quad (2)
\]

where \( \beta_{t|t} = E_t [\beta_t] \) can be obtain using the Kalman filter. The BN cycle can be obtained as the difference between the time series and its BN trend, i.e.,

\[
y^{\text{c,BN}}_t = y_t - y^{\text{BN}}_t \quad (3)
\]

### 2.2 The Unobserved Component (UC) Decomposition

Canova (2007) also classifies the UC decomposition as a statistical method. He further notes that the cyclical components computed by the UC decomposition enjoy certain optimality properties that make it preferred to other statistical methods. The chief advantages of the UC decomposition is that (i) it specifies a flexible structure for trend, cyclical, and other components; and (ii) the data are allowed to select characteristics of the trend and cyclical components and diagnostic testing can be used to examine what is left.
Following Kim and Nelson (1999) and Morley, Nelson, and Zivot (2003), the UC decomposition is applied by setting up a special state-space representation. In particular, suppose that an observed time series $y_t$ consists of two unobserved components: a trend component and a cyclical component such that

$$y_t = y_{t, UC}^r + y_{t, UC}^c$$  (4)

The UC trend is assumed to follow a random walk with drift $\mu$ such that

$$y_{t, UC}^r = y_{t-1, UC}^r + \mu + \eta_t \quad \text{where } \eta_t \sim \text{i.i.d. } N(0, \sigma^2)$$  (5)

And the UC cycle is assumed to be stationary and follow an invertible $\text{ARMA}(p, q)$ process:

$$\varphi_p(L)y_{t, UC}^c = \theta_q(L)e_t \quad \text{where } e_t \sim \text{i.i.d. } N(0, \sigma^2)$$  (6)

In addition, the innovations to UC cycle are assumed to be uncorrelated with trend innovations contemporaneously and cross time, i.e., $\text{Cov}(\eta_t, e_{t+k}) = 0$ for all $k$. The equations above form a state-space model, which can be estimated via the Kalman filter.

To illustrate the implementation, suppose the UC cyclical component follows an $\text{ARMA}(2, 0)$ or $\text{AR}(2)$ process, the state-space model can be written as

**Observation Equation:**

$$y_t \equiv [1 \ 1 \ 0] \begin{bmatrix} y_{t, UC}^r \\ y_{t, UC}^c \\ y_{t-1, UC}^c \end{bmatrix} \quad \text{or } y_t = H\beta_t$$  (7)

**State Equation:**

$$\begin{bmatrix} y_{t, UC}^r \\ y_{t, UC}^c \\ y_{t-1, UC}^c \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1, UC}^r \\ y_{t-1, UC}^c \\ y_{t-2, UC}^c \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \eta_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} \quad \text{or } \beta_t = \bar{\mu} + F\beta_{t-1} + v_t$$  (8)

Because the system (7) and (8) has a state-space format, the unknown parameters and the unobservable components can be estimated by maximum likelihood methods, recursively, using the Kalman filter.

**2.3 The Hodrick-Prescott (HP) Filter**
Here, Canova (2007) classifies the HP filter as a hybrid decomposition method. The objective is to compute the trend so as to minimize a function consisting of the squared deviations from the observed series and a smoothed difference in the trend values over time. Formally, Hodrick and Prescott (1997) write the objective function in terms of the HP trend component \( \{y_{i\cdot,HP}^T\}_{i=1} \) that minimizes the following expression:

\[
\sum_{t=1}^{T} \left( y_i - y_{i,HP}^T \right)^2 + \lambda \sum_{t=2}^{T-1} \left[ \left( y_{t+1} - y_{t,HP}^T \right) - \left( y_{t} - y_{t-1,HP}^T \right) \right]^2
\]

(9)

Following Hodrick and Prescott (1997), we set \( \lambda = 1600 \) for quarterly data. Once the HP trend is extracted from an observed time series, the residual is the HP cycle.\(^7\)

### 2.4 The Band-Pass (BP) Filter

The BP filter treats problems that arise because other filters can exaggerate the amount of variability that is present at high frequencies. The BP filter allows the researcher to specify the range over which business cycles, for example, exist. As such, there is information about business cycles that is embedded in the filtering process.

Canova (2008) classifies the BP filter as a hybrid decomposition method. The BP filter became more popular as a decomposition method after work by Canova (1998), Baxter and King (1999) and Christiano and Fitzgerald (2003). Here, we most closely follow the methods laid out in Baxter and King. An ideal band-pass filter which passes frequencies within the range \([\omega, \bar{\omega}]\) can be constructed by two ideal low-pass filters with cutoff frequencies \( \omega \) and \( \bar{\omega} \):

\[
B_{j, BP} (\omega, \bar{\omega}) = B_{j, LP} (\bar{\omega}) - B_{j, LP} (\omega) = \frac{\sin(j \bar{\omega}) - \sin(j \omega)}{j \pi}
\]

(10)

for \( j = \pm 1, \pm 2, \ldots \) and \( B_{0, BP} (\omega, \bar{\omega}) = (\bar{\omega} - \omega)/\pi \). The resulting cyclical component of a time series \( y_t \) can be obtained by

\[
y_{i, BP}^T = B_{0, BP} (\omega, \bar{\omega}) y_i = \sum_{j=0}^{\infty} B_{j, BP} (\omega, \bar{\omega}) L^j y_i = \sum_{j=0}^{\infty} B_{j, BP} (\omega, \bar{\omega}) y_{i-j}
\]

(11)

\(^7\) See also Ravn and Uhlig (2002) and De Jong and Sakarya (2015) for more detailed expositions of the HP filter.

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7 See also Ravn and Uhlig (2002) and De Jong and Sakarya (2015) for more detailed expositions of the HP filter.
Equation (11) implies that an ideal band-pass filter is a special form of MA filter of infinite order. With a finite amount of data, the ideal band-pass filter is not implementable in practice. Hence, approximations are necessary and an approximate filter \( \hat{B} \) is optimal if it minimizes its “distance” from the ideal filter \( B \).

There are two measures of the “distance” between the approximate and the ideal filters. Baxter and King (1999) measure the “distance” as the squared approximation errors at all relevant frequencies. By contrast, Christiano and Fitzgerald (2003) measure the “distance” as the mean squared error between the approximate and the ideal filters. In this paper, we focus on the Baxter-King approach. So, the optimal BP filter \( \hat{B}_{j_{BP}}(\omega, \bar{\omega}) \) is given as

\[
\hat{B}_{j_{BP}}(\omega, \bar{\omega}) = B_{j_{BP}}(\omega, \bar{\omega}) - \frac{\sum_{j=-J}^{J} B_{j_{BP}}(\omega, \bar{\omega})}{2J + 1} \tag{12}
\]

According to Canova (2007), the optimal truncation point \( J \) is 12 if one wants to extract business cycles with periodicity between 6 and 32 quarters (or between 1.5 and 8 years). With this optimal -BP filter, the BP cycle can be determined as well.

3. Lag-length selection

Both Batten and Thornton (1985) and Lee (1997) argue that there are costs associated with using rules of thumb to arbitrarily select lag lengths for VARs. Given the influence of lag structure specification on statistical inferences on estimated VARs, we begin by identifying the optimal lag length of each VAR. The data are the quarterly values of real GDP, source base, M1, M2, the nominal interest rate on 3-month Treasury bill. For the monetary aggregates, we use the last month of each quarter as the measure of the quarterly value. The data span the period 1959:Q1 through 2007:Q4. Since the truncation in the BP filter results in a loss of 12 quarters of data at the beginning and the end of the sample, the cyclical components obtained cover the period 1962:Q1 through 2004:Q4.

To find the optimal lag length, we use a likelihood ratio (LR) test. Specifically, we perform likelihood ratio tests for VARs with longer lags versus VARs with shorter lags. VARs with longer lags are treated as an unrestricted model and VARs with shorter lags are viewed as the restricted model. Then we can construct a LR statistic to test whether imposing the restrictions is statistically significant in the sense that an unrestricted model improves the fit enough to select a longer lag length. If the restrictions

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8 Source base is the measure of liabilities of the Federal Reserve System; namely, the sum of currency in circulation and reserves held by banks at the Federal Reserve. This measure does not include any adjustments for changes in reserve requirements.
lead to a significant reduction in model fit, the restricted model is rejected and the unrestricted longer-lag
model is retained. Formally, the LR test statistic is computed as

\[ LR = (T - c) \left( \log |\Sigma_r| - \log |\Sigma_u| \right) \]  (13)

where \( T \) is the sample size, \( c \) is a degrees of freedom correction factor suggested by Sims(1980a), which
equals the number of lag coefficients in each equation of the unrestricted VAR. \( |\Sigma_r| \) denotes the
determinant of the error variance matrix for the restricted model and \( |\Sigma_u| \) is the determinant of the error
variance matrix for the unrestricted model. The test statistic follows a chi-squared distribution with
degrees of freedom equal to the number of restrictions. According to Canova (2007), an upper bound of
lag length can be set to 8 for quarterly data when conducting LR tests. Thus, we implement a sequence of
LR tests starting from a maximum lag length (which is set to 8) down to a minimum lag length (which is
set to 1).

In our analysis, we consider two different broad monetary aggregates. Both M1 and M2 are used in
the analysis. When we get the quadrivariate VAR, the money multiplier is defined as the ratio of the
broad monetary aggregate to source base. We employ the following process to select the lag length for the
VARs. In each setting, the variables are the cyclical components obtained from one of the four
decomposition methods. We begin with a bivariate VAR including output (y) and a monetary aggregate,
M1 and M2, and so that there is a pair of bivariate VARs. Once the lag length is selected for that VAR,
we add one more variable to the system so that the trivariate VAR includes output, the broad monetary
aggregate and the nominal interest rate (i), repeating the lag length selection process. Finally, the
quadrivariate VAR includes output, source base (MB), the money multiplier, both MM1 and MM2,
corresponding the M1 and M2 monetary aggregate, respectively, and the nominal interest rate. In short,
we build two sets of VARs corresponding to two measures of money stock, and each set consists of a
bivariate, a trivariate, and a quadrivariate VAR, resulting in 24 separate VARs estimated.

Table 1 reports the lag length selection for the 12 VARs that are estimated with M1 as the monetary
aggregate. Note that the results are obtained using the Sims’ (1980) degrees of freedom correction factor.
From Table 1, the ‘statistically optimal’ lag length is 8 in almost every VAR. The lone exception is the
bivariate VAR estimated with the BP filter which selects seven lags as the optimal lag length. Table 2
reports the results for the 12 VARs in which M2 is the monetary aggregate. In half of the cases, the lag
length selected is less than eight lags when M2 is monetary variable.
The purpose of this section is to estimate the structure of the VARs that will be used in the Granger causality analysis. The leg length selection process does hint at the joint sensitivity of our results to the decomposition approach and the measure of the monetary aggregate. As we see in Table 1, the leg length selection is virtually invariant to the decomposition method when M1 is the monetary aggregate. In contrast, we observe some variation in lag length selection when M2 is the monetary aggregate, though eight lags is the mode of lag length selected in the M2 case. We observe variation in the bivariate specification as the lag length is as short as two (HP) and as long as six (BP). Based on the results in Table 2, it seems as if the HP filter results in fewer lags contributing explanatory power to the system.

4. **Granger causality test results**

In this section, we examine the central question. Specifically, do movements in the cyclical component of the money supply temporally precede movements in the cyclical component of output? Is this relationship sensitive to either the interest rate or the inside and outside money?

As a quick recap, the Granger causality test examines whether movements in one variable (say, money) temporally precede movements in another variable (say, output). More concretely, do movements in lagged values of, say, money help to predict current and future movements in output? The Granger causality test statistic is computed under the null hypothesis that in the estimated output equation, coefficients on lagged values of money are jointly equal to zero. A standard F-test is used to accept or reject the null hypothesis at the appropriate significance level. Thus, money Granger causes output if the null hypothesis is rejected or if the test statistic exceeds the critical value at the significance level. Throughout this analysis, we adopt the following short-hand: movements in money, for example, temporally precede movements in output means that movements in the cyclical components of money temporally precede movements in the cyclical component of output.

Our analysis considers two different monetary aggregates. In Table 3, we report VARs estimated with M1 as the monetary aggregate. Panel A reports the p-values for Granger causality tests for VARs with the optimal lag length. In our discussion, we move from the bivariate cases to the trivariate and then to the quadrivariate cases. In the bivariate setting, the evidence indicates that movements in M1 do not temporally precede movements in output. This result is invariant to the decomposition method used. In the trivariate setting, movements in the interest rate temporally precede movements in output. Other results depend on the decomposition method. Moreover, there is no systematic observation in the sense that one decomposition method deviates from the others in terms of not indicating Granger causality. For example, movements in the interest rate temporally precede movements in M1, except for the UC decomposition method. Movements in M1 temporally precede movements in output, except for the BP
As we move to the quadrivariate VAR, movements in the interest rate temporally precede movements in output in all four decomposition methods. In this setting, however, movements in the interest rate temporally precede movements in M\textsubscript{1}, except for the HP filter decomposition. Generally, we see that movements in base money do not temporally precede movements in output. There is one case—the HP filter decomposition—in which movements in the M\textsubscript{1} money multiplier temporally precede movements in output. Based on this evidence, the evidence supports the view that the interest rate Granger causes output, regardless of the decomposition method implemented.

We also point out one other result in Table 3. The evidence points to a relationship between the interest rate and future values of the money multiplier. Indeed, in three of the four decomposition methods, we observe that movements in the interest rate temporally precede movements in money multiplier. This result is suggestive of the transmission mechanism; as the interest rate shock occurs, we observe reactions in future actions of the financial institutions and future levels of production.

Panel B addresses the robustness of the results when one adopts an ad hoc lag selection approach. In the bivariate setting, we see that with four lags, movements in M\textsubscript{1} temporally precede movements in output and this result is invariant to the decomposition method. In Panel B, there are additional cases in which movements in money variables temporally precede movements in output. For example, movements in the M\textsubscript{1} multiplier and movements in base money temporally precede movements in output when there are four lags and the BP filter decomposition is used. What does not change is that lag length differences do not affect the most robust result; namely, movements in interest rates temporally precede movements in output.

Next, we consider the same questions, using M\textsubscript{2} as the monetary aggregate. The results are reported in two panels of Table 4. Following the pattern in Table 3, Panel A estimates VARs with the optimal lag length and Panel B estimates VARs with four lags. In the bivariate VARs, movements in M\textsubscript{2} temporally precede movements in output and that result is invariant to the decomposition method. In the trivariate setting, the evidence indicates that movements in the interest rate temporally precede movements in both M\textsubscript{2} and output. These two results are invariant to the decomposition method. There is only one case—the BP filter decomposition—in which movements in the M\textsubscript{2} temporally precede movements in output. In the quadrivariate VARs, movements in the interest rate temporally precede movements in output. Movements in the interest rate temporally precede movements in the M\textsubscript{2} money multiplier except in the UC decomposition case. Further, there are few cases in which the M\textsubscript{2} money multiplier and base money Granger cause output. Movements in the M\textsubscript{2} money multiplier temporally precede output but only in the BP filter decomposition case. Movements in base money temporally
precede movements in output when the UC decomposition and the BP filter decomposition methods are used. As we observed in Table 3, there is evidence supporting the view that movements in the interest rate temporally precede movements in the M2 money multiplier.

Panel B reports the results for the VARs with four lags. In the trivariate VARs, we see that movements in M2 temporally precede movements in output in three of the four decomposition methods. In Panel A, we see that M2 Granger causes output only when the BP filter decomposition is implemented. In the quadrivariate VARs, the HP filter decomposition indicates that the M2 money multiplier Granger causes output. In the optimal lag selection VARs, it is the BP filter decomposition that indicates the M2 money multiplier Granger causes output. Otherwise, a column-by-column comparison indicates the results are robust with respect to this particular change to lag structure.

Overall, the Granger causality results support findings previously reported in the literature. Most notably, movements in the interest rate temporally precede movements in output. Our findings differ from those reported by Litterman and Weiss in two specific ways. First, we use the nominal interest rate instead of a measure of the ex ante real rate. Second, we estimate the relationships using several methods to obtain measures of the cyclical components instead of the levels or log levels. Other results are mixed, depending on either the monetary aggregate or the decomposition method. It is true that with M2, we observe movements in the broad monetary aggregate temporally precede movements in output in the bivariate setting. Because M2 contains all the information in M1, such evidence suggests that the researcher needs the “correct” measure of broad money to uncover the Granger causality result. After including the interest rate variable in the VAR, the relationship between money and output is mixed. Though there is some evidence that movements in interest rates Granger cause movements in the monetary aggregates. We further see that movements in base money are not systematically related to future movements in output except for a few isolated decomposition methods. Lastly, there is little evidence to support the view that movements in the money multipliers temporally precede movements in output. Thus, the evidence points to movements in the interest rate help to predict future movements in output. However, movements in money help to predict movements in output, but it depends on which decomposition method the researcher chooses. There is no robust result regarding money and output after one extracts the trend.

The results of the Granger causality tests offer insight to questions that accept yes/no answers. Either the null hypothesis is rejected or not. In the next section, we examine the relationship between movements in the unpredictable parts of money and the interest rate and the impulse responses by output. In this way, we can get insight into the quantitative relationship. More importantly, we can assess more
easily whether the different decomposition methods yield quantitative differences in the impulse responses. So, in mixed cases we can see whether one decomposition method indicates that Granger causality exists while the other methods do not. In such cases, the impulse responses will tell us whether there are material quantitative differences across the decomposition methods.

5. Impulse response functions

In this section, we plot the impulse response functions for the different VARs with the optimal lag length. In each case, we consider the response to a one standard deviation shock to a monetary variable. In the trivariate and quadrivariate specifications, we also consider how a one standard deviation shock to the interest rate affects the other variables. We perform this analysis for each of the four decomposition methods. Following Blanchard and Quah (1989), we impose zero long-run restrictions so that all disturbances are orthogonal. In each plot, the solid line is the impulse response while the dotted lines represent 90% confidence bands obtained by bootstrapping with 100 draws. In all the figures, we focus on the output response, but include other responses.

Figure 1 plots the impulse responses for a shock to M1 for each of the four decomposition methods based on the estimated bivariate VARs. Focus on the response by output to a shock to M1. In each of the four cases, we observe qualitative differences to the impulse responses across the different decomposition methods. However, based on the 90-percent confidence bands, the evidence is consistent with the notion that none of the responses are significantly different from zero. Thus, the evidence from the impulse responses in bivariate VARs is consistent with the evidence from the Granger causality tests. Specifically, there is no systematic relationship between movements in M1 and movements in future output.

Figure 2 presents the impulse responses to an M1 shock in the estimated trivariate VARs. With the interest rate variable included, the evidence indicates there is virtually no systematic, significant response in output, given a shock to M1. In the lone case in which a positive shock to M1 results in a significant output response, the impact is negative. Figure 3 plots the impulse responses for a shock to the interest rate in the trivariate VARs. In general, the evidence indicates there is a significant, positive effect on output. Here, we see that with the UC decomposition, output initially declines in response to an interest rate shock. About a year after the shock, we observe a positive output response. For the other three decomposition cases, the timing is very similar. Initially, output responds positively to an interest rate shock, eventually the response is negative. In all four cases, the length of the cycle is approximately three years. Morley, et al. noted that the UC decomposition exhibits different autocorrelation properties compared with the BN decomposition. This feature could account for why the cyclical pattern is reversed in response to an interest rate shock.
We present the impulse responses to an interest rate shock in Figure 4 for the quadrivariate VARs. Noticeably, the interest rate responses in the quadrivariate cases are identical to those presented in the trivariate VARs. In the VARs with M1 as the monetary aggregate, we observe evidence that is broadly consistent with the evidence from the Granger causality tests. There is no evidence that movements in future output are significantly related to shocks in the monetary variables. In addition, shocks to the interest rate are significantly and initially positively related to future movements in output. With respect to the impulse responses to an interest rate shock, differences across the decomposition methods are qualitatively the same as those in the trivariate VARs.

Figures 5 through 8 present the same impulse response functions for VARs with the optimal lag length estimated with M2 as the monetary aggregate. Figure 5 plots the impulse response functions for the four decompositions methods, given a shock to M2. In each case, except the BN decomposition, we observe a significant response in output. As such, the evidence is consistent with the Granger causality tests; namely, movements in M2 are systematically related to future movements in output. There are differences in the sign and time of the response. With the HP decomposition, output declines immediately after the M2 shock, followed by a positive response within two years after the shock. In contrast, the UC decomposition and BP filter indicate a lagged positive response to the M2 shock, occurring sometime between a half and two years after the money shock.

In Figure 6, we see that the response by output to an M2 shock is significant for the HP and BP filters even with the interest rate variable included in the VAR. However, for the BN and UC decomposition methods, the effect of the money shock on output is not significant. Figure 7 presents the impulse response functions to an interest rate shock in the trivariate VARs. The results indicate that there is a significant, positive output response immediately after interest rate shocks except for the UC decomposition. For the UC decomposition, output initially declines and then reaches a peak about one year after the shock. In addition, the output response exhibits a cyclical pattern in each of the four decomposition methods.

Figure 8 presents the impulse response functions to an interest rate shock in the quadrivariate VARs. As we observe in the trivariate VARs, there is a cyclical pattern in the impulse response functions. Except for the UC decomposition, there is a significant, positive response that occurs after the interest rate shock. For the UC decomposition, a negative response occurs before the positive response. Now, both the HP filter and the BP filter methods indicate a significant, negative response is observed about two years after the interest rate shock occurs. Thus, a cyclical response is significant for at least some of the decomposition methods.
Overall, the results reported in Figures 1 through 8 correspond with the Granger causality test results. There is strong evidence that movements in the interest rate are systematically related with future movements in output. Movements in the M1 monetary variable are not systematically related to future movements in output, but there is evidence that movements in the M2 measure are systematically related to future movements in output. This is true even when movements in interest rates are included in the VARs. The impulse response functions shed additional light on the pattern of the response. The evidence indicates that for at least one decomposition method, an interest rate shock results in a cyclical output response that occurs over a three-year period.

6. Results from time-varying coefficient VARs with stochastic volatility

There are concerns that the results are generated from VARs that do not satisfy the basic conditions for temporal stability. In this section, we implement time-varying estimation methods to assess the robustness of our results with respect to changes that may be affecting the data-generating process over time.

With time-varying methods, there are a large number of parameters estimated. To deal with this concern, we specify four lagged values in all bivariate VARs and two lagged values in all trivariate and quadrivariate VARs estimated. A necessary and sufficient condition for the VAR to be stable is that all the eigenvalues lie outside the unit circle. [Because the largest eigenvalue of the companion matrix in full sample VARs has modulus no less than one for all VARs including variables detrended by the BP filter and the quadrivariate VAR containing the cyclical components of output, base money, the M2 money multiplier, and the interest rate identified by the UC decomposition.] Therefore, we set the lag length to 1 for these VARs. There may be a cost associated with the time-varying approach. By reducing the number of lagged values in the VAR, the error terms will be larger.

To estimate the time-varying parameter VARs with stochastic volatility, we apply the modelling strategy and the Markov chain Monte Carlo (MCMC) algorithm proposed by Primiceri (2005). The simulations are based on 20,000 draws using the Gibbs sampler, discarding the first 4000 for convergence and skipping every other draw in what is retained.⁹ Accordingly, 8000 draws are used for the computation of posterior medians and confidence bands of parameters of interest. Following Primiceri (2005), we use the first 40 observations (10 years, from 1962:Q1 to 1971:Q4) as a training sample to calibrate the prior distribution.

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⁹ The RATS program and procedure written by Todd Clark are used to estimate the VARs with time-varying parameters and stochastic volatility.
Once we allow VAR lag coefficients to vary over time, Granger causality results may also be sensitive. Thus, we propose a strategy to estimate time-varying Granger causality between money, interest rates, and output based on the Bayesian estimates of the time-varying parameters and variance-covariance matrix of the innovations. Specifically, for each draw retained, we simulate dependent variables in each VAR at each date using estimated lag coefficients and variance-covariance matrix at that date. These simulated series can be interpreted as the realization of the data had the model parameters been the ones used to generate the series. We then conduct F-tests for the output equation in simulated VARs to see whether the set of lag coefficients on monetary measures are jointly equal to zero. In this way, we can obtain an F-statistic and a corresponding p-value for every test at each date for every draw. Then, we can compute the medians and the 16th and 84th percentiles of the F-statistic and p-value across the 8000 draws.\textsuperscript{10} By plotting the median and 68\% band over time, we can visually detect any time variation in the Granger causality between money, interest rates, and output over different business cycles.

Figure 9 plots the p-values for the bivariate VARs with M1 as the monetary aggregate. We use “*” values to designate the cases in which the p-values are less than five percent, indicating cases in which movements in M1 temporally precede movements in output. Figure 10 plots the cases in the bivariate VARs with M2 as a measure of the money supply. In bivariate VARs including the cyclical components of output and M1, over the period between 1972:Q1 and 2004:Q4, the evidence generally indicates that M1 does not Granger cause output before 1985 but does Granger cause output afterwards across the decomposition methods. There are a few, short-lived exceptions. In the case of the BN decomposition in 1990, we fail to reject the null hypothesis that M1 does not Granger cause output. For the BP filter, there are a few short periods (1990 – 1992, 1994 – 1995, 1998 – 1999) over which M1 does not Granger cause output. In the case of the VARs with M2 as the money measure, there is even stronger evidence that M2 Granger causes output since 1984 across the decomposition methods. In addition, the HP filter and BP filter decompositions suggest the existence of Granger causality from M2 to output over periods before the mid-1980s. Specifically, in the case of the HP filter, M2 temporally precedes output over the period between 1972 and 1977; in the case of the BP filter, M2 temporally precedes output over almost the entire sample period.

Interestingly, the periods over which monetary aggregates temporally precede output for the majority of decomposition methods coincide with the years of the Great Moderation, over which macroeconomic activity became less volatile. The reduction in output volatility is confirmed by the Bayesian estimates of stochastic volatility; the standard deviation of structural residuals in the output

\textsuperscript{10} With the assumption of normality, the 16\textsuperscript{th} and 84\textsuperscript{th} percentiles correspond to the lower and upper bounds of a one-standard-deviation confidence interval.
equation started to decline since the mid-1980s. Our findings further indicate that there is opposing changes in the volatility of monetary aggregates: M1 began to increase in volatility since the mid-1980s, while M2 began to decline in volatility since the mid-1980s for all the decomposition methods except the BP filter. Such evidence is consistent with the notion that while short-term deposits became more volatile, the less liquid instruments included in M2 were less volatile since the mid-1980s.\(^\text{11}\)

Next, we add the interest rate variable. Here, the results are mixed. In particular, with time-varying lag coefficients, the decomposition matters in terms of whether movements in the interest rate temporally precede movements in output. Figure 11 plots the p-values for the null hypothesis that there is no relationship between movements in the interest rate and future movements in output. With the HP filter and the BP filter decompositions, the evidence indicates that one would reject the null hypothesis over the entire sample period. However, with the BN and with the UC decompositions, the null hypothesis can only be rejected over certain periods. Specifically, in the case of the BN decomposition, movements in the interest rate predict future movements in output in the early 1980s and early 2000s. With the UC decomposition, the interest rate temporally precedes output over more than half of the sample period – before 1988 (except 1976 – 1977) and in the early 2000s. As Figure 11 also shows, there is little evidence to support the view that movements in M1 temporally precede movements in output, except for the early 2000s with the HP filter decomposition and since the early 1990s with the BP filter decomposition. Figure 11 also provide strong evidence that the interest rate temporally precedes M1. The Granger causality is significant over the entire sample period with the HP filter, over the entire sample period except the late 1980s with the BP filter, over the period between 1972 and the early 1990s and in the early 2000s with the BN decomposition, and over the period before the early 1980s with the UC decomposition.

Figure 12 reports the p-values for the trivariate VARs in which M2 is the money measure. With M2, the results similar as those in the M1 case with a few exceptions. First, with the BP filter decomposition, movements in the interest rate do not temporally precede movements in output in the 1990s. Second, with the BN, HP filter, and BP filter decompositions, movements in M2 temporally precede movements in output over a longer period compared to the M1 case. Finally, with nearly all the decomposition methods, movements in the interest rate temporally precede movements in output over the period of the Great Moderation and in 2002.

\(^{11}\) Perhaps less volatility in the broader monetary aggregate could help explain why output was less volatile during the Great Moderation. For example, if the Federal Reserve manipulates the nominal interest rate (and thus the money supply) aiming to reduce macroeconomic volatility, we could account for why monetary aggregates temporally precede output over the years of the Great Moderation.
Figure 13 plots the p-values for the quadriavariate VARs with M1 as the money measure. Here, the evidence is consistent with the trivariate VARs. More specifically, the decomposition method seemingly plays a crucial role in terms of whether movements in the interest rate temporally precede movements in output. Once being separated from M1, movements in base money temporally precede movements in output over the entire sample period except the late 1970s with the BN decomposition and except the late 1970s and the early 1980s with the HP filter decomposition. In the case of the BP filter decomposition, base money Granger causes output only over the period since 1995.

In cases in which M2 is the money measure, we report the p-values under the null hypothesis that movements in interest rates do not temporally precede movements in output for the quadriavariate VARs in Figure 14. The evidence is consistent with the results reported in the trivariate cases. For both the HP filter and BP filter decompositions, movements in the interest rate temporally precede output over the entire sample period. The results are stronger than those reported in the trivariate case. In addition, the decomposed measures of the money supply temporally precede output over certain periods except for the BP filter decomposition. Specifically, base money temporally precedes output in 2003 and 2004 with the BN decomposition and since the late 1990s with the UC decomposition. On the other hand, the M2 money multiplier temporally precedes output over the entire sample period with the BN decomposition and since 1993 with the HP filter decomposition.

In this paper, we focus on null hypothesis that movements in money or interest rates temporally precede movements in output. Compared with the time-invariant coefficients, the results are sometimes weaker when we use time-varying estimation. Based on the comparison, it appears that shortening the lag length for the time-varying VARs can account for why interest rates Granger cause output in the fixed coefficient estimation but infrequently in the time-varying estimation cases. By specifying “too few” lags in the time-varying estimation, the standard errors are inflated.

7. Summary and conclusions

In this paper, we answer five questions centered on the relationship between money, interest rates, and output. Specifically, we ask whether money, or interest rates, or both have marginal predictive power for output. With respect to money, we ask whether inside money or outside money, or both have marginal predictive content for output. We are not the first to look at questions. Indeed, both monetary shocks and real shocks are offered as alternative sources of business cycle fluctuations. What we do differently is measurement; in particular, we examine these questions using cyclical components of these economic variables. The literature is dominated by estimating VARs in which the log levels of money aggregates and output are the units of measurement. Log levels, however, are a mixture of trend and
cyclical components. Therefore, we estimate VARs using the cyclical components as the unit of measurement. Because there is not a generally accepted decomposition method, we use four alternative approaches to identify the cyclical component: Beveridge-Nelson, Unobserved Components, HP filter, and a Band-Pass filter. In addition, we look at both M1 and M2 monetary aggregates. Because the monetary aggregates are a combination of movements in base money and inside money, we consider a VAR in which the monetary aggregate is decomposed into its base money and money multiplier components. In this way, we also investigate which type of money is related to movements in output.

We find that movements in interest rates do temporally precede movements in output. This finding is invariant to the decomposition method. When we look at the impulse response functions, we see that the decomposition does affect the output response to an interest rate shock. In all four decomposition methods, there is a cyclical response to an interest rate shock. Compared with the other three decomposition methods, there is a phase shift in the impulse response of output when the Unobserved Components method is used. The initial response with the UC method is a contraction cycle followed by an expansionary period. In direct contrast, the other three decomposition methods report an initial positive response for output followed by a contractionary period. This result is consistent with the differences in the autocorrelation function for output when comparing the cyclical components of the Beveridge-Nelson method and the Unobserved Components method.

We do find that whether money has marginal prediction power for output does depend on the decomposition method. Movements in the monetary aggregate temporally precede movements in output for some of the decomposition methods. We further observe that there are differences between M2 and M1 monetary aggregates. When we decompose the monetary aggregates into the outside and inside money components, the decomposition methods do not tell a clear story about which type of monetary aggregate has marginal predictive content for output.

To check the stability of the results over business cycles, we estimate time-varying coefficient versions of the VARs. There is some difficulty when directly comparing the results from the time-varying coefficient VARs and the time-invariant coefficient VARs. In particular, the large number of estimated parameters in the time-varying coefficient VARs causes us to shorten the lag length in order to preserve degrees of freedom. For some decomposition methods, the marginal predictive power of the interest rate for output is present over the entire sample period, but for other decomposition methods, the interest rate has the ability to predict future output only over certain periods in the sample. The standard errors are larger in the time-varying estimation and this is at least partially due to lag length selection.
Overall, the results provided in this paper point to several fruitful extensions. First, we see that results are consistent with the notion that monetary models of the business cycle generally fail to indicate that base money has marginal predictive content for output. Second, the time-varying results are consistent with the notion that there is a special relationship between money, interest rates and output during the Great Moderation. Third, we excluded the data from the 2008 Financial Crisis. In our view, this event created many different types of monetary policies, including, for example, paying interest on reserves. Fourth, another avenue to pursue would involve the Fisher Equation; that is, are the results affected by decomposing the nominal interest rate into its expected inflation and ex post real interest rate component.
References


### Table 1. Optimal Lag Length for VARs with M1 as the Monetary Aggregate

<table>
<thead>
<tr>
<th>Decomposition Method</th>
<th>Bivariate VAR including y, M1</th>
<th>Trivariate VAR including y, M1, i</th>
<th>Quadrivariate VAR including y, MB, MM1, i</th>
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<tr>
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<td>HP</td>
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<td>8</td>
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<tr>
<td>BP</td>
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**Notes:** BN denotes the Beveridge-Nelson decomposition; UC denotes unobserved-component decomposition allowing for correlation between trend and cycle innovations; HP denotes the Hodrick-Prescott filter; BP denotes the band-pass filtered proposed by Baxter and King (1999); M1 = the M1 money stock; y = real GDP; i = the 3-Month Treasury bill rate; MB = base money; MM1 = M1 money multiplier defined as the ratio between M1 and MB.

### Table 2. Optimal Lag Length for VARs M2 as the Monetary Aggregate

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<th>Decomposition Method</th>
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<th>Trivariate VAR including y, M2, i</th>
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<td>BP</td>
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**Notes:** See Table 1 for notations; M2 = the M2 money stock; MM2 = M2 money multiplier defined as the ratio between M2 and MB.
Table 3. \( p \)-values Associated with Granger-causality Test Results for VARs

Panel A: M1 as the monetary aggregate and optimal lag length

<table>
<thead>
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<th>Decomposition Method</th>
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<th>Trivariate</th>
<th>Quadrivariate</th>
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<td>( G ) i( \rightarrow )M1</td>
<td>( G ) i( \rightarrow )y</td>
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Panel B: M1 as the monetary aggregate and 4 lags

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<th>Quadrivariate</th>
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Notes: See Table 1 for notations; \( G \) M1\( \rightarrow \)y denotes the F-test that M1 Granger-causes output, and the like; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.
Table 4. \(P\)-values Associated with Granger-causality Test Results for VARs

Panel A: M2 as the monetary aggregate and optimal lag length

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</table>

Panel B: M2 as the monetary aggregate and 4 lags

<table>
<thead>
<tr>
<th>Decomposition Method</th>
<th>Bivariate (G)</th>
<th>Trivariate (G)</th>
<th>Quadrivariate (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(M2 \rightarrow y)</td>
<td>(i \rightarrow M2)</td>
<td>(i \rightarrow y)</td>
</tr>
<tr>
<td>BN</td>
<td>0.042**</td>
<td>0.001***</td>
<td>0.005***</td>
</tr>
<tr>
<td>UC</td>
<td>0.015**</td>
<td>0.010***</td>
<td>0.000***</td>
</tr>
<tr>
<td>HP</td>
<td>0.003***</td>
<td>0.031**</td>
<td>0.000***</td>
</tr>
<tr>
<td>BP</td>
<td>0.006***</td>
<td>0.068*</td>
<td>0.001***</td>
</tr>
</tbody>
</table>

Notes: See Table 1 and Table 2 for notations; \(M2 \rightarrow y\) denotes the F-test that M2 Granger-causes output, and the like; *** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.
Figure 1

Bivariate VARs including \( y \) and M1

IRF to a shock to M1 - BN Decomposition

IRF to a shock to M1 - UC Decomposition

IRF to a shock to M1 - HP Filter

IRF to a shock to M1 - BP Filter
Figure 2

Trivariate VARs including y, M1, and i: Shocks to M1
Figure 3
Trivariate VARs including $y$, M1, and $i$: Shocks to interest rate
Figure 4

Quadrivariate VARs including $y$, $mb$, $mm1$, and $i$: Shocks to interest rate
IRF to a shock to Interest Rate - BP Filter

Output to Interest Rate

MB to Interest Rate

MM1 to Interest Rate

Interest Rate to Interest Rate
Figure 5

Bivariate VARs including y and M2

IRF to a shock to M2 - BN Decomposition

IRF to a shock to M2 - UC Decomposition

IRF to a shock to M2 - HP Filter

IRF to a shock to M2 - BP filter
Figure 6

Trivariate VARs including y, M2, and i: Shocks to M2
Figure 7

Trivariate VARs including $y$, M2, and $i$: Shocks to interest rate
Figure 8

Quadrivariate VARs including $y$, $mb$, $mm2$, and $i$: Shocks to interest rate
Figure 9

P-values for Granger causality tests: M1 bivariate VARs

BN Decomposition

UC Decomposition
Figure 10

P-values for Granger causality tests: M2 bivariate VARs

BN Decomposition

UC Decomposition
**Figure 11**

**P-values for Granger causality tests: trivariate VARs with M1 as the money measure**

**BN decomposition**
UC decomposition

Interest rate Granger causes M1?

Interest rate Granger causes output?

M1 Granger causes output?

HP filter decomposition

Interest rate Granger causes M1?

Interest rate Granger causes output?

M1 Granger causes output?
BP filter decomposition

Figure 12

P-values for Granger causality tests: trivariate VARs with M2 as the money measure

BN decomposition
UC decomposition

Interest rate Granger causes M2?

P-value

Interest rate Granger causes output?

M2 Granger causes output?

HP filter decomposition

Interest rate Granger causes M2?

P-value

Interest rate Granger causes output?

M2 Granger causes output?
Figure 13

P-values for Quadrivariate VARs with M1 as money measure

BN decomposition
HP filter decomposition

BP filter decomposition
Figure 14

P-values for Quadrivariate VARs with M2 as money measure

BN decomposition

[Graphs showing p-values for different Granger causality tests over time]

UC decomposition

[Graphs showing p-values for different Granger causality tests over time]
HP filter decomposition

BP filter decomposition